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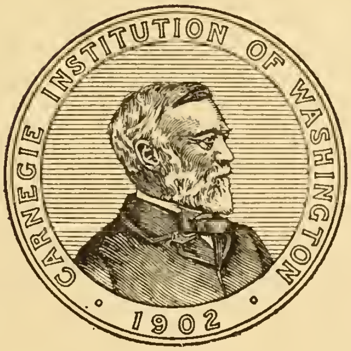

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A NEW METHOD OF ESTIMATING STREAM-FLOW
BASED UPON
A NEW EVAPORATION FORMULA

By J. A. Folse
Research Associate, Carnegie Institution of Washington

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Washington, 1929

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"Nearly all the grandest discoveries of science have been but the rewards of accurate measurement and patient, long-continued labor in the minute sifting of numerical results."

Lord Kelvin.

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ACKNOWLEDGMENT

In addition to those made in the body of the text, it is appropriate at this place to make a more complete acknowledgment. Whatever merit this work possesses should be credited to the late Dr. John F. Hayford. His untimely death in March 1925 brought this research to an abrupt termination, after it had been in progress continuously, under his direction and inspiration, since the fall of 1912, except during two years of the Great War.

I first became connected with this research as a computer in 1913, and so served, intermittently, during summer vacations and other spare time, while an undergraduate in the School of Engineering of Northwestern University, of which Dr. Hayford was then Director. After graduating in 1920 I served full time on these studies, and became the principal assistant on them. At the end of 1924 the studies were temporarily halted because of exhaustion of financial support. They were to have been resumed again early in 1925, with the aid of a further grant from the Carnegie Institution, but Dr. Hayford's affliction and death, coming at this time, brought the project to an end.

Early in 1926, at the request of the Carnegie Institution, I resumed work on the studies, but primarily with the object of rounding out in form for publication the work already accomplished by Dr. Hayford. At the time of his death, the evaporation formula as hereinafter presented had been derived, and its application to the investigation of the laws of flow of Stream A, Wagon Gap, Colorado, had been practically completed. The evaporation formula, and most of the numerical results obtained on Stream A together with the principles developed in connection therewith, are strictly those of Dr. Hayford. For the completion of the application of the principles developed on Stream A to Stream B; for the completion of the flood-flow study on Stream A, and the making of a similar one on Stream B, I, alone, am responsible. The completion of these items rendered possible a more precise determination of the freezing-melting constants, C, F and M. Studies with this object in view were also carried out by me in 1926–1927 to the extent possible in the time available. The Appendix to Part I and, in Part II, the application of the Pearsonian Generalized Probability curves to the determination of the "Relation of S_c to Mean, Median and Mode of Frequency Distribution" are entirely my own, although, in the latter case, the fundamental idea was furnished by Dr. Hayford.

Except for some notes made in connection with the least-square computations, the studies as left by Dr. Hayford were all in numerical form, mostly as least-square computations. The principal task in connection with rounding out this work for publication, therefore, consisted in abstracting from these least-square solutions the theories, principles and numerical results which they embodied, and presenting them in logical order. This I have attempted to do in the pages which follow. This part of the work in addition to that stated in the preceding paragraph, is also entirely my own, and for which I assume full responsibility.

J. A. Folse.

Evanston, Illinois, October 29, 1929.

Part I. A New Evaporation Formula

A NEW EVAPORATION FORMULA

INTRODUCTION

The investigation of the laws of evaporation and stream-flow, which is the subject of this publication, under the general title "A New Method of Estimating Stream-flow based upon a New Evaporation Formula," was brought to a stop in the latter part of December 1924 at which time Dr. John F. Hayford, who was conducting the investigation, was stricken with apoplexy, which resulted in his death in March 1925. In June 1926 the investigation was resumed under the direction of the writer, with a view principally to bringing it to a stage of publication as soon as possible with due regard to the work already accomplished by Dr. Hayford, and without undertaking any new studies beyond those deemed necessary for rounding out in good form the work already done. The results presented in this publication are therefore those achieved mainly by Dr. Hayford, with the writer as his principal assistant. At the time of his death in March 1925, the evaporation formula as hereinafter presented had been derived, and its application to the investigation of the laws of flow of Stream A, Wagon Wheel Gap, Colorado, had been practically completed. Since June 1926 the studies on Stream A have been completed and similar studies on Stream B have been made by the writer. In addition, during this time interval, the constants C, F and M in Part II of this publication were increased in accuracy and reliability.

The ultimate object of this investigation was to obtain a much better formulation than the engineering profession now has of the laws governing the amount of stream-flow. The engineer is confronted at the outset in his design of works for controlling the waters of a river for power, flood-control, sanitation and navigation, with the necessity for knowing as accurately as possible the future flow of the river and its variations. The more accurately this can be determined at the outset, the more accurately it is possible to design the works with economy and safety.

It appeared at the beginning of this investigation that progress toward the formulation of better laws governing the amount of stream-flow probably hinged largely upon obtaining a better knowledge of the laws of evaporation from large water and land surfaces. In order to secure this better knowledge of the laws of evaporation, it seemed advisable to supplement the numerous intensive studies of evaporation which had been made from evaporation pans a few square feet in area by a study made on the full scale of nature and under natural conditions. For this purpose it was proposed to consider each of the Great Lakes in turn as an evaporation pan and to evaluate from day to day (1) the change of content, (2) the income, and (3) the outgo, including evaporation. These evaluations have been made with sufficient accuracy to segregate that part of the outgo which is evaporation, and the laws which control the rate of evaporation have been determined with a fair degree of accuracy. The laws so determined have been applied in the formulation of better laws governing the amount of stream-flow with a confidence based upon the fact that the experimental evidence in establishing the evaporation laws, which form the basis of the stream-flow study, has been on the full scale of nature and under natural conditions.

The principal obstacle at first toward formulating the laws of evaporation from observations on the Great Lakes was the difficulty of evaluating with sufficient accuracy the change of content of the lake from day to day. The change of content of any one of the Great Lakes is measured by the change of elevation of the mean lake surface from day to day, since the area remains substantially constant. Each recording gage measures the change in elevation of the surface of the lake at the point at which the gage is located. At the beginning of this investigation the only feasible way to fix the elevation of the mean surface of the lake was to take it as equal to the arithmetic mean of the elevations at the one or more points on the lake at which first-class gages were operating. It soon became apparent, in the course of the investigation, that the most serious errors encountered were those due to the effects of winds and of barometric pressures in causing the recorded elevation of the lake surface at the gage to be different from that of the mean elevation of the whole lake surface. Accordingly, the first part of the investigation centered upon devising methods of evaluating the effects of winds and of barometric pressures upon the lake surface. This part of the investigation was published in 1922 by the Carnegie Institution as Publication No. 317, entitled *Effects of Winds and of Barometric Pressures on the Great Lakes* by John F. Hayford. For the complete exposition of that part of the investigation the reader is referred to that publication.

As an outcome of that part of this investigation contained in Publication No. 317, the mean elevation of the surface of the whole lake may be determined on any day by applying the known corrections for wind effects and barometric effects at any gage to the observed elevation for that gage. The proper weighted mean for the several gages may be taken and the few abnormal values may be detected and rejected by a definite criterion. In the place, then, of the former values of mean elevation of the whole lake surface, of a certain degree of accuracy, as an outcome of the part of this investigation contained in Publication No. 317, one has now available values of a much higher degree of accuracy. It appeared at the time of publishing Publication No. 317 that from observations at Mackinaw alone the mean elevation of the whole of Lake Michigan-Huron on any day may possibly be determined with a probable error less than ±0.010 foot. (See page 113, Publication No. 317.) From the three stations, Milwaukee, Harbor Beach and Mackinaw together, it was estimated that this probable error may be reduced still more. It appeared that the change in elevation of the mean surface of the whole of Lake Michigan-Huron in one day might possibly be determined with a probable error less than ±0.007 foot. This estimate, ±0.007 foot, was made at the culmination of Publication No. 317. It was not later to be realized. It was found in the evaporation study, the results of which are hereinafter presented, that the accuracy actually achieved is as follows: from readings at the three gages Milwaukee, Harbor Beach and Mackinaw, corrected for wind and barometric effects, and taking account of evaporation, the change in elevation from one day to the next, of the mean surface of the whole of Lake Michigan-Huron, can be determined with a probable error of ±0.014 foot, that is, the actual mean elevation on any day can be determined with a probable error of $(\pm 0.014 \div \sqrt{2} =)$ ±0.010 foot.

It was estimated early in this investigation that the evaporation from Lake Michigan-Huron in a single day, measured in depth of water taken off the whole surface of the lake, probably varied between limits which are approximately 0.000 foot and 0.021 foot. With the new accuracy obtained by applying corrections for winds and barometric effects, it became possible to evaluate the change in content of the lake from day to day with sufficient accuracy to segregate that part of the outgo which is evaporation. This has been done on Lake Michigan-Huron, and with a lesser degree of success also on Lake Superior.

Part I of this publication deals with the investigation of the laws of evaporation from observations on Lakes Michigan-Huron and Superior. It shows how the investigation was made; how the final evaporation formula was derived. It necessarily overlaps, to a certain extent, Publication No. 317, and represents the outcome of one application of the results of that part of this investigation contained in that publication.

Using the evaporation formula derived from observations on the Great Lakes, the attack on formulating the laws of stream-flow was begun. This part of the investigation is presented in Part II of this publication.

For the reader, therefore, who is interested in the theory of wind and barometric effects and the method and results of applying those effects to correct gage readings on Lakes Erie and Michigan-Huron, Publication No. 317 should be consulted. If the reader is interested only in the method of deriving the evaporation formula, assuming the corrections for winds and barometric pressures to be known, he is referred to Part I of this publication. If the reader is interested only in this new method of estimating stream-flow, assuming the evaporation formula to be known, Part II of this publication should be consulted.

DATA USED AND ACKNOWLEDGMENTS

No new observations were made in this investigation. The observations used covered the period July to October inclusive of 1909 and May to October, inclusive, of each of the years of 1910, 1911, 1912 and 1913. The observations used covered the 28 months named on each of Lakes Michigan-Huron and Superior. They were necessarily confined to the warmer months of each year. The observations consisted of the following items:

(1) The daily mean elevations of the water surface at each of the gages Milwaukee, Harbor Beach and Mackinaw on Lake Michigan-Huron, and at Marquette on Lake Superior. Each daily mean was the mean of 24 hourly values for that day obtained from the records of the automatic gages which had operated at the stations named.

(2) The observed barometric pressures at six points near Lakes Michigan-Huron and Superior were read from the isobars (lines of equal barometric pressure) as shown on the daily forecast maps as prepared at the U. S. Weather Bureau at Chicago for the use of the forecaster at that station. Two such maps are made for each day, one showing the facts at 8 a. m., 75th meridian time, and the other the facts at 8 p. m., 75th meridian time. The barometric pressures used in this investigation were for the two times named on each day at each of six stations for the 28 months named.

(3) The observed mean daily wind velocity which was used had been obtained from recording anemometers as operated at the U. S. Weather Bureau stations at Chicago (Ill.), Milwaukee (Wis.), Grand Haven (Mich.), Green Bay (Wis.), Escanaba (Mich.), Sault Ste. Marie (Mich.), Alpena (Mich.), Port Huron (Mich.), and at the Canadian Meteorological stations of Parry Sound and Saugeen, all near Lake Michigan-Huron. On Lake Superior the observed mean daily wind velocities which were used had been similarly obtained at the U. S. Weather Bureau Stations at Sault Ste. Marie (Mich), Marquette (Mich.), Houghton (Mich.), Duluth (Minn.), and at the Canadian Meteorological station at Port Arthur. The mean daily wind velocity for each lake was taken as the arithmetic mean of the velocities observed at each of the stations named on each lake for each day of the 28-month period mentioned.

(4) The observed mean daily air temperature had been obtained at the U. S. Weather Bureau stations and the Canadain Meteorological stations mentioned in item (3). The mean daily temperature at each station was taken as the mean of the maximum and minimum at that station. The mean temperature for the lake was taken as the arithmetic mean of that at the several stations, 10 near Lake Michigan-Huron and 5 near Lake Superior.

(5) The observed vapor-pressure had likewise been obtained at the U. S. and Canadian meteorological stations mentioned in item (3), on each day at 8 a. m. and 8 p. m., 75th meridian time. The mean vapor-pressure at each station for each day was taken as the mean of that recorded at the two times mentioned. The mean vapor-pressure for the lake for each day was taken as the arithmetic mean of the mean daily values at the several stations, 10 near Lake Michigan-Huron and 5 near Lake Superior.

(6) The daily rainfall on the drainage areas of the two lakes had been observed at the regular meteorological stations already mentioned in the United States and Canada, and also at additional stations on these areas maintained by cooperative observers. Most of the rainfall data which was used in this investigation appears recorded in the Monthly Weather Review for the 28 months stated, in District No. 4.

(7) The daily water levels at the Harbor Beach and St. Clair Flats Canal gages from which the daily discharges of the St. Clair River were computed, as well as the daily water levels at the S. W. Pier, Sault Ste. Marie, from which the daily discharges of the St. Mary's River were computed, had been observed and recorded by the U. S. Lake Survey (Survey of Northern and Northwestern Lakes).

Grateful acknowledgment is hereby extended to the organizations which have furnished the data used in this investigation; to Professor Charles F. Marvin, Chief of the United States Weather Bureau; to Professor Henry J. Cox, Senior Meteorologist in charge of the Chicago office of the U. S. Weather Bureau; to Sir R. F. Stupart, Director of the Meteorological Service of Canada; to various officers of the United States Lake Survey; and to various officers of the Corps of Engineers of the United States Army.

METHODS USED IN AND SCOPE OF THIS INVESTIGATION

The methods used in this part of this investigation are the same as those used in the study of wind effects and of barometric effects on the Great Lakes. A full statement of these methods appears on pages 6 and 7 of Publication No. 317, and will not here be repeated.

In the broad investigation of wind effects, barometric effects and evaporation, 98 least-square solutions and their corresponding studies were made. These 98 solutions include the 74 reported upon in Publication No. 317; therefore since the issue of that publication, 24 additional solutions were made in the investigation of the laws of evaporation proper and of the run-off into the lake. These 24 solutions each contained from 2 to 9 unknowns and each included from 30 to 800 observation equations, each observation equation having for its absolute term the observed rise of the mean surface of the whole lake, corrected for the various influences for which it was possible to apply reliable corrections.

Of the 24 solutions mentioned in the preceding paragraph, only 2 were directly used in establishing the constants in the final evaporation formula as hereinafter presented. The other 22 solutions served to give general information about the run-off into each lake from its adjacent drainage area, and to lead the investigator gradually to the final form adopted for the evaporation formula.

In addition to the 98 least-square solutions mentioned in the second preceding paragraph, a total of 69 solutions and their accompanying studies was made in the stream-flow part of this investigation. A more complete statement concerning the extent of these 69 solutions is given in the appropriate context in Part II (see pages 146 and 147).

The total expenditure of time on the complete investigation of wind effects, barometric effects, evaporation and stream-flow, excluding over 2,000 hours spent on the non-routine parts by Dr. John F. Hayford, amounts to over 32,000 man-hours of labor. A total of 41 persons has been engaged on the computations from time to time since the investigation began in 1913. Of these, not more than 10 persons have been engaged upon it at any one time.

The preceding statements indicate that this was an extensive investigation. It is probably safe to say that there have been few times, if any, when an equivalent amount of time and energy has been expended in the derivation of an evaporation formula, including the by-products, the method of evaluating the effects of winds and barometric pressures on free, open surfaces of water, and including the studies toward formulating the laws of stream-flow.

OUTCOME OF THE INVESTIGATION

The outcome of this part of this investigation may be briefly stated as follows:

(1) An evaporation formula has been derived, based upon over 700 observations independently on each of two of the Great Lakes, Michigan-Huron and Superior. This formula enables one to estimate the daily rate of evaporation from any free, open surface of water (such as from the surface of a lake, reservoir or river) anywhere in the world, in terms of the meteorological elements of air temperature, vapor-pressure and wind velocity as ordinarily observed at the standard U. S. Weather Bureau stations.

(2) An estimate has been made of the constant part of the run-off into each of Lakes Michigan-Huron and Superior from their respective land draingage areas, and certain general knowledge with reference to the variable parts of the run-off in each case has been obtained.

(3) As a direct sequel to the results already published in Publication No. 317, and as a necessary precursor of Item (1), the following two items of knowledge have been derived:

(*a*) A reasonably accurate numerical expression has been obtained for the effects of barometric pressures on the elevation of the water surface at the Marquette station on Lake Superior. With this expression one may compute, from the distribution of the barometric pressures ordinarily shown on the forecast maps of the Weather Bureau, the disturbances in elevation of the water surface thereby produced at Marquette.

(*b*) The numerical constants (the Σ's) in the general formula for wind effects stated as equation (69), page 63, Publication No. 317, have been derived for the Marquette station on Lake Superior. These constants enable one to compute the hourly or daily effect of a wind of any velocity and direction upon the elevation of the water surface at Marquette, Michigan.

(4) The relationship of the new knowledge gained in (1) and (2) to various important problems in science and engineering has become increasingly evident. Some of these problems are:

(*a*) The problem of estimating the evaporation from the surface of each of the Great Lakes as a basis for more accurate estimates of the hydrological phase of the problem of regulating the surfaces of the lakes by controlling works in their connecting channels, and the use of this available knowledge of the daily evaporation in such regulation after the controlling works are constructed.

(*b*) The problem of estimating the evaporation from other large open surfaces of water, such as the surfaces of reservoirs, lakes and rivers, in the design of controlling works for power, irrigation, navigation and sanitation.

(*c*) The problem of estimating the evaporation from large land surfaces as a basis for estimating the run-off from such surfaces for use in various engineering and scientific problems. The uses to which run-off estimates are put in engineering problems need not be enlarged upon. As one example of the use of such an estimate in a scientific problem is the use made of it in the problem of formulating the laws of stream-flow, in Part II of this publication.

ORDER OF PRESENTATION

The order of presentation of the material in Part I of this publication is as follows:

(1) The final form of the observation equation used which served to determine the evaporation from Lakes Michigan-Huron and Superior is presented. The complete list of 755 daily observation equations on Lake Michigan-Huron is given, together with the residuals from them. These equations and their residuals served to fix the constants and their probable errors in the evaporation formula as derived from the observations on Lake Michigan-Huron. The principal facts from the two final least-square solutions for determining the evaporation are given, and the adopted weighted-mean values of the constants in the final evaporation formula as developed in this investigation are presented.

(2) Some of the evidence, which led to the final adopted form of the formula for expressing evaporation, is presented.

(3) The method of evaluating the constant part of the run-off into the lake is stated, and a brief discussion is given of the effect upon the final results of neglecting the variable part of the run-off into the lake.

(4) A brief discussion of the accuracy of the computed evaporation is given, together with a comparison of the formula developed in this investigation with others (Appendix to Part I).

FINAL FORM OF OBSERVATION EQUATION USED FOR DETERMINING EVAPORATION

The final form of observation equation used for determining the rate of evaporation from the surfaces of the Great Lakes is the following:

$$+eE_1+e\left[\left(\frac{w}{100}-x\right)\right]E_2+I=v \qquad (1)$$

The current day is the day to which the observation equation is credited in listing the equations; the next earlier day will be called the preceding day. The meanings of the quantities in equation (1) are as follows:

e = mean difference for the lake between (1) the saturation vapor-pressure corresponding to the mean air temperature for the lake for the two days ending at midnight at the end of the day to which the observation equation refers; and (2) the mean actual vapor-pressure for the 24-hours ending at 12:30 p. m. on the date to which the observation equation refers. The unit is 0.01 inch of mercury. The difference (1) − (2) may, for convenience, be called the vapor-pressure potential.

E_1 = that part of the evaporation which is proportional to the vapor-pressure potential.

w = average travel of the wind, for winds greater than x, in miles per 24-hours, over the whole lake surface for the two days ending at midnight at the end of the date to which the observation equation refers, the mean being taken without any regard to the direction of the wind.

x = that value of the wind travel, w, in units of 100 miles per day, below which the evaporation from the lake surface is inappreciably affected by the wind.

E_2 = that part of the evaporation which is proportional to the product of the vapor-pressure potential and the wind velocity, for winds greater than x.

$+I = +I_1 - I_3 + I_4 - I_2 - I_c$ = that part of the directly observed *rise* in the mean lake surface which is unaccounted for in terms of I_3, I_4, I_2 and I_c. In other words I is that part of the directly observed rise which is to be accounted for as much as possible by the evaporation from the lake surface.

$+I_1$ = (weighted mean elevation of the water surface of the lake on the current day, from readings on all of the available, reliable gages, corrected for wind and barometric effects) *minus* (weighted mean elevation of the water surface of the lake on the preceding day, from readings on all of the available, reliable gages, corrected for wind and barometric effects). In other words $+I_1$ is the rise in the mean surface of the whole lake from the preceding to the current day, the elevation of the mean surface on each day being determined as accurately as possible by applying corrections for the effects of winds and barometric pressures. Unit = 0.001 foot.

I_3 = Computed rise of the lake surface due to inflow into the lake from the next lake above in the chain of lakes. Unit = 0.001 foot.

I_4 = Computed fall of the lake surface due to outflow from the lake into the next lake below in the chain of lakes. Unit = 0.001 foot.

I_2 = Computed rise of lake surface, noon to noon, due to rainfall on the lake surface. It is assumed to be the mean between the rise as computed from the rainfall on the preceding day, and the rise as computed from the rainfall on the current day. Unit = 0.001 foot.

I_c = Estimated constant part of the run-off into the lake in one day. It is the run-off when the ground-water is at its average level and the quantity (rainfall on land) *minus* (estimated evaporation from land) *minus* (estimated run-off) has been zero for a long time. Unit = 0.001 foot.

In equation (1), only positive values of the expression in brackets were used. The full explanation for this will be fully stated in appropriate places.

The right-hand member of equation (1), v, is a residual. It represents the discrepancy between the observed rise of the lake surface corrected for the various influences, $+I$, and the computed fall of the lake surface due to evaporation, $+eE_1 + e\left[\left(\frac{w}{100} - x\right)\right]E_2$. If the agreement between the theory and the facts were perfect, and if there were no errors in the observations and computations, the residuals would all be zero. The closer this agreement and the smaller the errors of the kind stated, the smaller the v's tend to become, and the more closely they approach the law of distribution of accidental errors.

The least-square solutions serve to determine the most probable values of the two unknowns E_1 and E_2 directly, and of the third unknown, x, indirectly.

The division of the wind velocity, w, by 100 was made simply for the purpose of making the average coefficient of E_2 the same numerical size as the average coefficient of E_1 and the absolute term, I. Such approximate equality facilitates the least-square computation by providing certain checks against numerical error.

The use of equation (1) involves an important assumption which will be stated at this point, and designated as Assumption No. 5. (For the other four principal assumptions made in the investigation of evaporation, see Publication No. 317, pages 12, 16, 23 and 43.)

Assumption No. 5—It is assumed that the mean temperature of the surface water of the lake for a day, during the months of May to October inclusive, is the same as that of the free-air above, as measured at the U. S. and Canadian Weather Bureau stations nearest the lake.

Assumption No. 5 was necessarily made in the absence of, and the practical impossibility of obtaining, data on the mean daily temperature of the surface waters of the Great Lakes. It was made for convenience both in the study of evaporation and in the subsequent study of stream-flow.

The temperature of the surface water of a lake changes more slowly than that of the air above, hence it will in general be colder during the day and warmer during the night than the overlying air in the summer months. The mean day and night temperature of the surface water will, in general, tend to be the same as that of the overlying air.

In a lake of the size and depth of any of the Great Lakes, the volumetric temperature of the lake will tend to lag behind the air temperature; that is, in the early spring and summer months the air temperature will generally be warmer than the temperature of the whole mass of water in the lake, and in the late summer and fall the reverse will, in general, be true. The statements made in Assumption No. 5, and in the preceding paragraph, refer to the surface film of water from which evaporation takes place, and not to the whole mass of water in the lake.

METEOROLOGICAL STATIONS NEAR LAKES MICHIGAN-HURON AND SUPERIOR

The regular U. S. Weather Bureau and Canadian Meteorological Service stations at which observations of wind velocity, vapor-pressure and air temperature were recorded are shown in Table 1 for Lake Michigan-Huron and in Table 2 for Lake Superior. These stations are also shown plotted on Plates I and 4 respectively.

TABLE 1—*Regular U. S. Weather Bureau and Canadian Meteorological Service Stations near Lake Michigan-Huron*

Station	Latitude	Longitude	Elevations in feet			
			Barometer above sea	Thermometer above ground	Rain gage above ground	Anemometer above ground
In the United States:						
Chicago	41° 53′	87° 37′	823	140	133	310
Milwaukee	43 02	87 54	681	122	116	140
Grand Haven	43 05	86 13	632	54	49	89
Green Bay	44 31	88 00	617	49	43	86
Escanaba	45 48	87 05	612	40	33	82
Sault Ste. Marie	46 30	84 21	614	40	35	61
Alpena	45 05	83 30	609	13	4	92
Port Huron	43 00	82 26	638	70	63	120
In Canada:						
Parry Sound	45° 19′	80° 00′	635	4.5	1	40*
Saugeen	44 30	81 21	656	4.5	1	44*
Means	....	....	652	54	48	106

* Approximate elevation of ground above mean lake level, 42 feet and 62 feet, respectively.

The average elevation of all the anemometers above the ground at these 15 stations is about 100 feet, which is about the same as the average elevation of the anemometers at all of the regular U. S. Weather Bureau stations in the United States.

INFLOW INTO LAKE MICHIGAN-HURON, (I_3); OUTFLOW FROM LAKE SUPERIOR (I_4)

The inflow into Lake Michigan-Huron from Lake Superior through the St. Mary's River, the I_3 of equation (1) for Lake Michigan-Huron, was computed from the formula

$$\text{Discharge of St. Mary's River} = 46{,}730 + 15{,}300 \times (\text{S. W. Pier} - 600.0) \text{ cu. ft. per second} \ldots\ldots (2)$$

in which "S. W. Pier" is the elevation above mean sea-level of the water surface at the S. W. Pier at Sault Ste. Marie. The flow through the power plants and locks is included in the above formula. Equation (2) converted to units of 0.001 foot of depth per day on Lake Michigan-Huron, the unit used in equation (1), is

Table 2—*Regular U. S. Weather Bureau and Canadian Meteorological Service Stations near Lake Superior*

Station	Latitude	Longitude	Elevations in feet			
			Barometer above sea	Thermometer above ground	Rain gage above ground	Anemometer above ground
In the United States:						
Sault Ste. Marie	46° 30′	84° 21′	614	11	3	52
Marquette	46 34	87 24	734	77	70	111
Houghton	47 07	88 34	668	62	57	99
Duluth	46 47	92 06	1133	11	3	47
In Canada:						
Porto Arthur	48° 27′	89° 12′	644	4.5	1	100*
Means			759	33	27	82

* Approximate elevation of ground above mean lake level, 40 feet.

$$\left.\begin{array}{l}\text{Discharge of St. Mary's River}\\ \text{in 0.001 ft. of depth per day}\\ \text{on Lake Michigan-Huron}\end{array}\right\} = 3.20 + 1.05\,(\text{S. W. Pier} - 600.0) \ldots\ldots (3)$$

The outflow from Lake Superior into Lake Michigan-Huron through the St. Mary's River, the I_4 of equation (1) for Lake Superior, is also equation (2) if expressed in cubic feet per second. If expressed in 0.001 foot of depth per day on Lake Superior, the unit used in equation (1), it is

$$\left.\begin{array}{l}\text{Discharge of St. Mary's River}\\ \text{in 0.001 ft. of depth per day on}\\ \text{Lake Superior}\end{array}\right\} = 4.53 + 1.49\,(\text{S. W. Pier} - 600.0) \ldots\ldots (4)$$

Lake Superior being the highest lake in the chain of Great Lakes, there is no inflow into it from a lake above, so I_3 for Superior is zero.

Formula (2) was taken from the U. S. Lake Survey Records.

OUTFLOW FROM LAKE MICHIGAN-HURON (I_4)

The outflow from Lake Michigan-Huron into Lake St. Clair through the St. Clair River was computed from the formula

$$\left.\begin{array}{l}\text{Discharge of}\\ \text{St. Clair River}\\ \text{in cu. ft. per}\\ \text{second}\end{array}\right\} = 3{,}820\left[(H'-567.50)+1.135\;(h-567.50)\right](H'-h)^{\frac{1}{2}} \ldots\ldots\ldots\ldots (5)$$

in which H' is the elevation above sea-level at the Harbor Beach gage, and h the elevation at the St. Clair Flats Canal gage. (See "Diversion of Water from the Great Lakes and Niagara River," Letter from the Secretary of War, Government Printing Office, 1921, page 355.) In the units used in the observation equation,

$$\left.\begin{array}{l}\text{Discharge of St.}\\ \text{Clair River in}\\ \text{0.001 ft. depth on}\\ \text{Lake Michigan-}\\ \text{Huron}\end{array}\right\} = 0.262\left[(H'-567.50)+1.135\,(h-567.50)\right](H'-h)^{\frac{1}{2}} \ldots\ldots\ldots (6)$$

TABLE 3.—*Rainfall stations, adjacent areas, and factors for converting rainfall in inches of depth to units of 0.0001 foot rise of surface of Lake Michigan-Huron*

Station	Lake area adjacent to station	Factor	Station	Lake area adjacent to station	Factor
	(sq. mi.)			*(sq. mi.)*	
Chicago, Ill.	946	17.4	Jeddo, Mich.	505	9.3
Whiting, Ind.	512	9.4	Sarnia, Canada	232	4.3
Saint Joseph, Mich.	1,195	22.0	Clinton, Canada	669	12.3
South Haven, Mich.	501	9.2	Point Clark, Canada	1,008	18.5
Ganges, Mich.	614	11.3	Lorne Park, Canada	698	12.8
Holland, Mich.	143	2.6	North Bruce, Canada	39	0.7
Grand Haven, Mich.	745	13.8	Saugeen, Canada	1,059	19.5
Muskegon, Mich.	189	3.5	Wiarton, Canada	3,302	60.8
Montague, Mich.	872	16.0	Owen Sound, Canada	46	0.8
Hart, Mich.	402	7.4	Meaford, Canada	367	6.8
Ludington, Mich.	801	14.7	Collingwood, Canada	216	4.0
Manistee, Mich.	619	11.4	Midland, Canada	394	7.2
Frankfort, Mich.	1,592	29.3	Parry Sound, Canada	2,184	40.2
Traverse City, Mich.	86	1.6	Bruce Mines, Canada	1,201	22.1
Old Mission, Mich.	829	15.3	Detour, Mich.	974	17.9
Charlevoix, Mich.	723	13.3	Mackinac Is., Mich	244	4.5
Petoskey, Mich.	120	2.2	St. Ignac, Mich.	240	4.4
St. James, Mich.	2,472	45.5	Escanabe, Mich.	817	15.0
Mackinaw, Mich.	248	4.6	Menominee, Mich.	361	6.6
Cheboygan, Mich.	360	6.6	Plum Is., Wis.	2,319	42.7
Alpena, Mich.	3,130	57.6	Sturgeon Bay, Wis.	921	16.9
Harrisville, Mich.	1,662	30.6	Oconto, Wis.	281	5.1
East Tawas, Mich.	344	6.3	Green Bay, Wis.	102	1.8
Omer, Mich.	212	3.9	Kewanee, Wis.	765	14.0
Bay City, Mich.	229	4.2	Manitowoc, Wis.	600	10.9
Hayes, Mich.	303	5.6	Sheboygan, Wis.	901	16.5
Pt. Austin, Mich.	918	16.9	Port Washington, Wis.	793	14.5
Harbor Beach, Mich.	1,072	19.7	Milwaukee, Wis.	609	11.1
			Racine, Wis.	1,647	30.2
Total 57 Stations				45,333	833.3

NOTE—In the above table, Factor $= a/A \times \frac{10000}{12}$, in which a is the lake area adjacent to a rainfall station, A is the area of Lake Michigan-Huron, viz, 45,314 square miles, and the factor is to convert the observed rainfall at the station in inches of depth to units of 0.0001 foot rise of surface of Lake Michigan-Huron.

This area was obtained from Report of International Waterways Commission on the Regulation of Lake Erie (1910), page 9, and was the best value then available. The area given in "Diversion of Water from Great Lakes and Niagara River" (1921), page 367, is 45,410 square miles.

In the computation of the outflow from Lake Michigan-Huron, the diversion at Chicago was added to the quantity obtained from equation (6). On Lake Michigan-Huron, 0.001 foot of depth per day is equivalent to 14,650 cubic feet per second, therefore the diversion at Chicago was taken as 0.0005 foot per day and constant, with sufficient accuracy for this purpose.

RAINFALL ON THE LAKE (I_2)

The rainfall on the lake area was assumed to be the same as that observed at the shore stations nearest the lake.[1] All of the rainfall stations existing at the time covered by the data used in this investigation, 1909 to 1913 inclusive, were used, including both the regular U. S. and Canadian Weather Bureau stations and the stations maintained by cooperative observers. The rainfall of each station was weighted according to the proportion of lake area adjacent to it. The complete list of rainfall stations used for Lake Michigan-Huron is shown in Table 3 and on Plate 1. The corresponding complete list of rainfall stations for Lake Superior is shown in Table 4 and on Plate 4.

TABLE 4.—*Rainfall stations, adjacent areas, and factors for converting rainfall in inches of depth to units of 0.0001 foot rise of surface of Lake Superior*

Station	Lake area adjacent to station	Factor	Station	Lake area adjacent to station	Factor
	(*sq. mi.*)			(*sq. mi.*)	
Two Harbors, Minn.	721	18.81	Chatham, Mich.	82	2.19
Duluth, Minn.	73	1.96	Munissing, Mich.	637	16.62
Superior, Wis.	139	3.67	Grand Marias, Mich.	3,449	89.73
Bayfield, Wis.	3,829	99.61	Whitefish Pt., Mich.	3,096	80.55
Ashland, Wis.	127	3.36	Sault Ste. Marie, Mich.	153	4.04
Houghton, Mich.	1,844	48.00	Deer Park, Mich.	1,221	31.81
Calumet, Mich.	569	14.85	Schreiber, Canada	5,180	134.74
Eagle Harbor, Mich.	3,974	103.38	Port Arthur, Canada	1,025	26.71
Baraga, Mich.	236	6.20	Isle Royal, Canada	3,768	98.03
Marquette, Mich.	1,885	49.07			
Total 19 Stations				32,008	833.33

NOTE—Factor $= a/A \times \frac{10000}{12}$, in which a is the lake area adjacent to a rainfall station, A is the area of Lake Superior, viz, 32,060 square miles, and the factor converts observed rainfall at a station in inches of depth to units of 0.0001 foot rise of surface of Lake Superior. The best available value of the area of Lake Superior at the time these factors were computed was 32,060 square miles, as given on page 9 of "Report of the International Waterways Commission on Regulation of Lake Erie" (1910). In "Diversion of Water from the Great Lakes and Niagara River" (1921), page 367, the area is given as 31,810 square miles.

RAINFALL ON LAND DRAINAGE AREA ADJACENT TO LAKE (RELATED TO I_c)

The estimate of the constant part of the run-off into each lake from its adjacent land drainage area, the I_c of equation (1), involved a knowledge of the rainfall on that land drainage area. Each rainfall station for estimating the rainfall on land was weighted in proportion to the land area adjacent to it. The complete list of rainfall stations used in this investigation for estimating the rainfall on the land part of the drainage area of Lake Michigan-Huron is given in Table 5 and the locations of these stations are shown on Plate 1. Note that the stations near the

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[1] The accuracy of this assumption was tested by a series of least-square solutions. The method used, and results obtained, are not presented for want of space. Briefly, it may be said that, the departure—if any—of the assumption from the truth could not be detected from the observations by the method employed.

Table 5—*Rainfall stations, adjacent areas, and factors for converting rainfall in inches of depth to units of 100 inch-miles on land part of drainage area (excluding lake area) of Lake Michigan-Huron*

Station	Lake area adjacent to station	Factor
	(sq. mi.)	
In United States:		
Chicago, Ill.	155	1.55
Whiting, Ind.	65	.65
Hammond, Ind.	68	.68
South Bend, Ind.	376	3.76
Elkhart, Ind.	470	4.70
Howe, Ind.	829	8.29
St. Joseph, Mich.	384	3.84
Cassapolis, Mich.	532	5.32
Wasepi, Mich.	558	5.58
Coldwater, Mich.	569	5.69
Hillsdale, Mich.	175	1.75
Bloomingdale, Mich.	336	3.36
South Haven, Mich.	180	1.80
Ganges, Mich.	148	1.48
Allegan, Mich.	469	4.69
Kalamazoo, Mich.	529	5.29
Battle Creek, Mich.	572	5.72
Concord, Mich.	382	3.82
Olivet, Mich.	348	3.48
Charlotte, Mich.	438	4.38
Holland, Mich.	320	3.20
Grand Haven, Mich.	192	1.92
Grand Rapids, Mich.	706	7.06
Sarnac, Mich.	771	7.71
Stanton, Mich.	687	6.87
St. Johns, Mich.	401	4.01
Ovid, Mich.	273	2.73
Lansing, Mich.	311	3.11
Agrl. College, Mich.	181	1.81
Webberville, Mich.	342	3.42
Jackson, Mich.	384	3.84
Grass Lake, Mich.	141	1.41
Muskegon, Mich.	295	2.95
Croton, Mich.	737	7.37
Big Rapids, Mich.	588	5.88
Reed City, Mich.	405	4.05
Montague, Mich.	281	2.81
Hart, Mich.	548	5.48
Ludington, Mich.	224	2.24
Thornville, Mich.	222	2.22
Hayes, Mich.	470	4.70
Pt. Austin, Mich.	182	1.82
Harbor Beach, Mich.	463	4.63
Jeddo, Mich.	146	1.46
Detour, Mich.	282	2.82
Sault Ste. Marie, Mich.	283	2.83
Mackinac Is., Mich.	22	.22
St. Ignace, Mich.	1,405	14.05
Escanaba, Mich.	877	8.77
Maple Ridge, Mich.	2,286	22.86
Humboldt, Mich.	1,084	10.84
Powers, Mich.	788	7.88
Iron River, Mich.	1,124	11.24
Iron Mountain, Mich.	853	8.53
Menominee, Mich.	865	8.65
Florence, Wis.	762	7.62
Plum Island, Wis.	93	.93
Sturgeon Bay, Wis.	373	3.73
Oconto, Wis.	389	3.89
Manistee, Mich.	398	3.98
Luther, Mich.	577	5.77
Cadillac, Mich.	735	7.35
Ivan, Mich.	574	5.74
In United States—Continued:		
Benzonia, Mich.	540	5.40
Frankfort, Mich.	42	.42
Traverse City, Mich.	565	5.65
Mancelona, Mich.	495	4.95
Old Mission, Mich.	207	2.07
Charlevoix, Mich.	219	2.19
Petoskey, Mich.	466	4.66
Mackinaw, Mich.	168	1.68
St. James, Mich.	45	.45
Cheboygan, Mich.	314	3.14
Onaway, Mich.	906	9.06
Gaylord, Mich.	588	5.88
Alpena, Mich.	699	6.99
Harrisville, Mich.	513	5.13
Woodlawn, Mich.	998	9.98
Grayling, Mich.	408	4.08
Roscommon, Mich.	600	6.00
East Tawas, Mich.	413	4.13
West Branch Mich.	806	8.06
Omer, Mich.	459	4.59
Harrison, Mich.	700	7.00
Gladwin, Mich.	751	7.51
Mt. Pleasant, Mich.	552	5.52
Midland, Mich.	571	5.71
Bay City, Mich.	266	2.66
Saginaw, Mich.	113	1.13
Saginaw, W. S., Mich.	303	3.03
Alma, Mich.	544	5.44
Vassar, Mich.	701	7.01
Arbela, Mich.	407	4.07
Lapeer, Mich.	476	4.76
Flint, Mich.	325	3.25
Durand, Mich.	320	3.20
Owosso, Mich.	325	3.25
Howell, Mich.	435	4.35
Green Bay, Wis.	637	6.37
Kewanee, Wis.	385	3.85
Manitowoc, Wis.	351	3.51
Sheybogan, Wis.	170	1.70
Plymouth, Wis.	441	4.41
Pt. Washington, Wis.	332	3.32
Crandon, Wis.	1,521	15.21
Cecil, Wis.	1,865	18.65
Waupaca, Wis.	826	8.26
New London, Wis.	610	6.10
Appleton, Wis.	351	3.51
Menasha, Wis.	193	1.93
Chilton, Wis.	514	5.14
Oshkosh, Wis.	322	3.22
Fond du Lac, Wis.	515	5.15
Ripon, Wis.	1,049	10.49
Pine River, Wis.	1,068	10.68
Milwaukee, Wis.	158	1.58
Monominee Falls, Wis.	438	4.38
Racine, Wis.	151	1.51
In Canada:		
Sarnia	295	2.95
Clinton	1,013	10.13
Lucknow	984	9.84
Point Clark	95	.95
Lorne	168	1.68
North Bruce	294	2.94

TABLE 5—*Continued*

Station	Lake area adjacent to station	Factor	Station	Lake area adjacent to station	Factor
	(*sq. mi.*)			(*sq. mi.*)	
In Canada—Continued:			In Canada—Continued.		
Saugeen	149	1.49	Coldwater	198	1.98
Wiarton	677	6.77	Midland	350	3.50
Owen Sound	583	5.83	Gravenhurst	690	6.90
Meaford	321	3.21	Beatrice	1,526	15.26
Collingsood	395	3.95	Parry Sound	1,743	17.43
Shelburne	850	8.50	Uplands	2,217	22.17
Uxbridge	460	4.60	Lake Talon	1,359	13.59
Barnie	714	7.14	Copper Cliffe	11,040	110.40
Orillia	795	7.95	Bruce Mines	6,817	68.17
Total 142 Stations				92,493	924.93

NOTE—In the above table, Factor=a/100, in which a is the land area adjacent to a rainfall station, and the factor is to convert the observed rainfall at the station in inches of depth to units of 100 inch-miles on the land drainage area adjacent to the rainfall station.

The total land part of the drainage area of Lake Michigan-Huron shown, 92,493 square miles, was the best value obtainable at the time.

shore were used for estimating the rainfall on both the land and lake areas. The corresponding list of stations for the Lake Superior land drainage area is given in Table 6, and the locations of the stations are shown on Plate 4.

The factors shown in Tables 5 and 6 convert the observed rainfall at the station in inches of depth to units of 100 inch-miles on land, an inch-mile being defined as

TABLE 6 —*Rainfall stations, adjacent areas, and factors for converting rainfall in inches of depth to units of 100-inch-miles on land part of drainage area (excluding lake area) of Lake Superior*

Station	Lake area adjacent to station	Factor	Station	Lake area adjacent to station	Factor
	(*sq. mi.*)			(*sq. mi.*)	
Virginia	1,309	13.09	Ishpenning	426	4.26
Two Harbors	3,039	30.39	Marquette	273	2.73
Duluth	463	4.63	Chatham	180	1.80
Cloquet	767	7.67	Munissing	142	1.42
Floodwood	839	8.39	Grand Marais	323	3.23
Superior	846	8.46	Deer Park	324	3.24
Bayfield	327	3.27	Whitefish Point	272	2.72
Ashland	1,428	14.28	Newberry	835	8.35
Ironwood	624	6.24	Sault Ste. Marie	266	2.66
Thomaston	1,070	10.70	Isle Royal	186	1.86
Watersmeet	1,014	10.14	Sault Ste. Marie, Canada	2,807	28.07
Baraga	1,241	12.41	White River, Canada	8,066	80.66
Houghton	740	7.40	Schreiber, Canada	8,796	87.96
Calumet	312	3.12	Port Arthur, Canada	5,039	50.39
Eagle Harbor	232	2.32	Kakabika Falls, Canada	5,127	51.27
Total 30 Stations				47,313	473.13

NOTE—In the above table, Factor=a/100, in which a is the land area adjacent to a rainfall station, and the factor is to convert the observed rainfall at the station in inches of depth to units of 100 inch-miles on the land drainage area adjacent to the rainfall station.

It has been subsequently discovered that the above table of stations and factors is not the one used in this investigation. A thorough search for the one actually used proved futile. Incidentally this is the only piece of computation definitely known to be lost at the time of publishing this material, fourteen years after labors were begun upon it.

one inch of rainfall on one square mile of area. The question might well be asked, why use factors to convert the rainfall into units of 100 inch-miles, when the unit used in the observation equations is 0.001 foot of depth on the lake surface? The answer is that this unit, 100 inch-miles, was used in the very early stages of the investigation in an attempt to derive the law of run-off of the rainfall into the lake. When the unit was subsequently changed to 0.001 foot of depth on the lake surface, the factors were not changed, as complete use of them had already been made. Instead, the proper conversion factor was applied to the rainfall computed in the original unit to reduce it to the final unit used.

FIRST APPROXIMATE VALUES OF I_c FOR LAKES MICHIGAN-HURON AND SUPERIOR

It is proposed to state briefly the manner of obtaining the first approximations to I_c for each of Lakes Michigan-Huron and Superior. At a subsequent place, the manner of revising these first approximations from the least-square computations will be set forth.

For the land part of the drainage area of a lake, the following equation is true for the average year:

(Rainfall on land) − (evaporation from land) − (run-off) = 0 . . . . (7)

from which the run-off from land can be computed, and this, converted to depth on the lake surface and divided by 365, gives the first estimate of I_c for the lake; that is, the run-off into the lake in one day when (rainfall on land) − (evaporation from land) − (run-off) has been zero for a long time and the ground water is at its average level, expressed in depth on the lake area.

In order to estimate I_c from equation (7), it is necessary to know what the evaporation from the land surface is. At first this is not known, but there exist in the literature estimates of evaporation from the water surfaces of the Great Lakes in inches of depth per year. If one can determine the ratio of the rate of evaporation from the land surface to the rate of evaporation from the lake surface, the product of this ratio and the estimated evaporation from the water surface equals the evaporation from land. Call this ratio $\frac{E_l}{E_w}$ in which E_l is the evaporation from each unit of area of land expressed in depth of water evaporated per unit of time, and E_w the corresponding evaporation from the water surface.

In order to calculate the ratio $\frac{E_l}{E_w}$ for the Great Lakes region, we make use of the following equation:

(Rainfall on drainage area) − (evaporation from land) − (evaporation from water) + (inflow from lakes above) − (outflow to lakes below) + (decrease in storage in lakes and streams and ground-water) = 0 . . . . (8)

By choosing the average year as the unit of time, the last term on the left-hand side of equation (8) vanishes; that is, for the average year there is no decrease in storage in the lakes and streams and ground-water of the land part of the drainage area. The remaining unknown, the evaporation from land, can then be determined in terms of the other four quantities, estimates of which are available in the literature on the hydrology of the Great Lakes drainage basin.

It is now proposed, first, to use equation (8) in evaluating the ratio $\frac{E_l}{E_w}$, second to apply this ratio in equation (7) to obtain I_c. Equation (8) will be applied in three cases—

Case 1: Involving Lakes Superior, Michigan-Huron, Erie and Ontario; and the average year.
Case 2: Involving Lakes Superior, Michigan-Huron and Erie; and the average year.
Case 3: Involving Lakes Superior and Michigan-Huron; and the average year.

The estimates of rainfall, evaporation, inflow and outflow for use in equation (8) for each case were made from sources partly external to this investigation and need not be given in detail. The necessary land and water areas are shown in Table 7.

TABLE 7—*Areas of land and water surfaces on the Great Lakes drainage basin.* (*The unit is 1 square mile.*)

Lake	Area of lake surface	Drainage area including lake surface	Land area excluding lake area	Ratio of lake to drainage area
Superior	32,060	76,134	44,074	1:2.37
Michigan	22,336	65,799	43,463	1:2.95
Huron	22,978	72,008	49,030	1:3.13
St. Clair	503	6,194	5,691	1:12.31
Erie	9,968	34,573	24,605	1:3.47
Ontario	7,243	32,980	25,737	1:4.55
Total	95,088	287,688	192,600	1:3.02

From "Report of International Waterways Commission on Regulation of Lake Erie" (1910), page 9. The values given in the above table were the best available at the time. In subsequent literature more accurate values, no doubt, are given. Where feasible and advisable, the latest values were used in this investigation; that is, the computations based upon the early values were corrected, but in analyses where the results would be affected very little by the later values, the original computations were allowed to stand.

Consider the above three cases in turn, and let the unit be one foot of depth over the whole land area considered in each case.

Case 1:

The total land area = 192,600 square miles.
The total area including water surface = 287,688 square miles.
The total water area = 95,088 square miles.
The total rainfall for a year = 34.7 inches over the whole area. In the adopted unit the total rainfall for a year is $\left(\frac{34.7}{12}\right)\left(\frac{287{,}688}{192{,}600}\right)=4.31$.
The evaporation from the water surface = 31.1 inches per year over the whole water area. In the adopted unit it is $\left(\frac{31.1}{12}\right)\left(\frac{95{,}088}{192{,}600}\right)=1.27$.
The run-off down the St. Lawrence River in one year is 39.7 feet on the surface of Lake Ontario (an area of 7,243 square miles). In the adopted unit, it is $39.7\times\left(\frac{7243}{192{,}600}\right)=1.49$.

Hence, from equation (8), Lake Superior being the highest lake in the area considered and there being no decrease in storage in the average year, in the adopted unit, evaporation from land $=E_l=4.31-1.27-1.49=1.55$. This value, 1.55, is the evaporation from each unit of area of land expressed in feet of depth of water evaporated per year.

The corresponding rate of evaporation from the water surface $=\frac{31.1}{12}=2.59$ feet, as that is the depth evaporated from each unit of water area per year. Hence for Case 1, $\frac{E_l}{E_w}=\frac{1.55}{2.59}=0.60$.

Case 2:

The total land area = 166,863 square miles.
The total area including water surface = 254,708 square miles.
The total water area = 87,845 square miles.
The total rainfall for the average year = 34.7 inches over the whole area considered in Case 2. In the adopted unit the total rainfall for the average year is $\left(\frac{34.7}{12}\right)\left(\frac{254,708}{166,863}\right)=4.42$.
The total evaporation for a year for this area is estimated at 30.5 inches. In the adopted unit the total evaporation for a year is $\left(\frac{30.5}{12}\right)\left(\frac{87,845}{166,863}\right)=1.34$.
The run-off through the Niagara River for the average year = 24.1 feet on the surface of Lake Erie, an area of 9,968 square miles. In the adopted unit, therefore, the run-off for Case 2 is $24.1\left(\frac{9,968}{166,863}\right)=1.44$.

Hence from equation (8), Lake Superior being the highest lake in the chain of Great Lakes and there being no decrease in storage in the average year, in the adopted unit evaporation from land $=E_l=4.42-1.34-1.44=1.64$. Hence, for Case 2, $\frac{E_l}{E_w}=\frac{1.64}{2.59}=0.63$.

Case 3:

The total land area = 136,567 square miles.
The total area including water surface = 213,941 square miles.
The total water area = 77,374 square miles.
The total rainfall for the average year = 34.7 inches, or in the adopted unit the total rainfall is $\left(\frac{34.7}{12}\right)\left(\frac{213,941}{136,567}\right)=4.51$.
The total annual evaporation from the water surfaces of Lakes Michigan-Huron and Superior is estimated at 25.0 inches, or, in the adopted unit, it is $\left(\frac{25.0}{12}\right)\left(\frac{77,374}{136,567}\right)=1.18$.
The run-off through the Detroit River for the average year = 5.13 feet on the surface of Lake Michigan-Huron. In the adopted unit it is $5.13\left(\frac{45,314}{136,567}\right)=1.70$.

Hence, from equation (8), Lake Superior being the highest lake in the series and there being no decrease in storage in the average year, in the adopted unit evaporation from land $= E_l = 4.51 - 1.18 - 1.70 = 1.63$. Hence, for Case 3, $\frac{E_l}{E_w}=\frac{1.63}{2.59}=0.63$.

The adopted value for the ratio $\frac{E_l}{E_w}$ for the Great Lakes region is taken as the mean of the three values obtained in the above three cases or $\frac{E_l}{E_w}=0.62$.

VALUE OF I_c FOR LAKE MICHIGAN-HURON

Using equation (7) and the adopted value of $\frac{E_l}{E_w}$ the expression for run-off becomes

Run-off = (Rainfall on land) − 0.62 (evaporation from water) .... (9)

For Lake Michigan-Huron it was estimated from sources partly external to this investigation that the rainfall on the land drainage area is 34 inches per year. The annual evaporation from the surface of Lake Michigan-Huron was similarly estimated at 28 inches per year. Hence from equation (9) the run-off into the lake in terms of the land drainage area is $\left[\frac{34}{12}-\frac{28}{12}(0.62)\right]=1.39$ feet per year. In terms of the rise of the surface of Lake Michigan-Huron, the annual run-off is $1.39\left(\frac{92{,}493}{45{,}314}\right)=2.84$ feet per year, or $(2.84\div365=)$ 0.008 feet per day, which is the first approximation to I_c for Lake Michigan-Huron used in this investigation.

VALUE OF I_c FOR LAKE SUPERIOR

For Lake Superior, the rainfall on the land drainage area was estimated at 27 inches per year. The evaporation from the surface of Lake Superior was estimated at 23 inches per year. Hence from equation (9) the run-off into the lake from the adjacent drainage area, expressed in terms of the land area, is $\left[\frac{27}{12}-\frac{23}{12}(0.62)\right]=1.06$ feet per year. In terms of the rise of the surface of Lake Superior, the annual run-off is $\left[1.06\left(\frac{44{,}074}{32{,}060}\right)=\right]$ 1.45 feet per year, or $(1.45\div365=)$ 0.004 foot per day, which is the first approximation to I_c for Lake Superior used in this investigation.

RISE IN THE MEAN SURFACE OF THE WHOLE LAKE (I_1)

Up to this point, the methods of computing all of the known quantities which enter into equation (1), except the quantity I_1, have been stated in general terms. It is now proposed to state briefly the method of evaluating I_1. The exposition involved in this will necessarily overlap Publication No. 317 to some extent, for in that publication the theory involved in the evaluation of wind and barometric effects was set forth, as well as some results of the application of corrections for those effects. It is not the intention to here restate the theory nor any of the discussions of the results in Publication No. 317, but rather to carry over certain parts of it only for convenience in the present discussion, and only those parts dealing with Lake Michigan-Huron. The material pertaining to Lake Superior to be set forth here is herewith published for the first time.

BAROMETRIC POINTS FOR LAKES MICHIGAN-HURON AND SUPERIOR

The location of the points from which the barometric pressures were read from the forecast maps is given in Table 8, and shown on Plate 2.

HOURLY BAROMETRIC EFFECTS

The effect, E_1, of unequal barometric pressures on the surfaces of the Great Lakes in causing the recorded elevation of the lake surface at the gage for any hour to be different from the mean elevation of the whole lake surface for that hour, is given by the following four equations for the gages specified, the first three on Lake Michigan-Huron, and Marquette on Lake Superior.

For Milwaukee, E_1 for any hour $=(4-5)(-4.97)+(3-6)(+8.44)$, in which $(4-5)$ must be taken for 7 hours earlier and $(3-6)$ for 4 hours earlier than the hour for which the effect is being computed .... (10)

For Harbor Beach, E_1 for any hour $=(4-5)(+6.94)+(3-6)(-0.44)$, in which $(4-5)$ must be taken for 6 hours earlier and $(3-6)$ for 6 hours earlier than the hour for which the effect is being computed .... (11)

For Mackinaw, E_1 for any hour $=(4-5)(-2.54)+(3-6)(-4.38)$, in which $(4-5)$ must be taken for 2 hours later and $(3-6)$ for 1 hour later than the hour for which the effect is being computed .... (12)

For Marquette, E_1 for any hour $=(1-3)(+2.67)+(2-4)(+2.38)$, in which $(1-3)$ must be taken for 3 hours earlier and $(2-4)$ for 1 hour earlier than the hour for which the effect is being computed .... (13)

In the above equations, E_1 is in units of 0.001 foot, and the quantities $(1-3)$, $(2-4)$, $(4-5)$ and $(3-6)$ are expressed in units of 0.01 inch of mercury.

TABLE 8.—*Location of barometric points*

Point	Lat.	Long.
	°	°
1	47.5	92.5
2	Same as Port Arthur	87.5
3	47.5	85
4	45	87.5
5	45	80
6	42.5	85

In equations (10), (11) and (12), $(4-5)$ is the difference between the barometric pressure at the point marked 4 on Plate 2, as read from the weather forecast maps for any hour, and the pressure at point marked 5. Similarly, $(3-6)$ is the barometric pressure at point 3 for any hour minus the pressure at point 6. In equation (13) the differences $(1-3)$ and $(2-4)$, similarly, are the differences in hourly pressure at those points around Lake Superior.

The barometric pressure is recorded on the forecast maps at the Chicago office of the U. S. Weather Bureau at only two times per day, namely, 8 a. m. and 8 p. m., 75th meridian time. In using equations (10) to (13), the barometric pressure at the particular points 1, 2, 3, 4, 5 and 6 are interpolated from the isobars (lines of equal barometric pressure) at the two times named each day. The pressure for any hour of the day is then obtained by linear interpolation between the 8 a. m. and 8 p. m. pressures. The substitution of the proper differences in the hourly barometric pressures so obtained at the points shown into equations (10) to (13), taking account of the lag stated in each case, enables one to compute the hourly effect, E_1, of the barometric pressure on the elevation of the water surface at the four gages named.

To obtain hourly elevations of the mean surface of the lake from the hourly elevations recorded at a gage, one must subtract the barometric effect, E_1, computed for each hour from the gage reading for that hour.

The theory involved in equations (10) to (13) is adequately set forth in Publication No. 317, as is also the method of deriving the constants. Equations

(10), (11) and (12) are the same as equations (43), (44) and (45), respectively, in Publication No. 317, page 34.

CHANGES IN DAILY BAROMETRIC CORRECTIONS

The following formulas express the change in the barometric correction from one day to the next at the stations named:

At Milwaukee:

$$-1.78\,b_{w0}-4.34\,b_{w1}-2.92\,b_{w2}-0.90\,b_{w3}+1.99\,b_{n0}+6.53\,b_{n1}+6.34\,b_{n2}+2.03\,b_{n3}\ \ldots\ldots(14)$$

At Harbor Beach:

$$+2.81\,b_{w0}+5.29\,b_{w1}+5.10\,b_{w2}+0.68\,b_{w3}-1.00\,b_{n0}-0.84\,b_{n1}+0.49\,b_{n2}+0.47\,b_{n3}\ \ldots\ldots(15)$$

At Mackinaw:

$$-0.73\,b_{w0}-0.91\,b_{w1}-2.05\,b_{w2}-1.38\,b_{w3}+0.38\,b_{n0}-2.89\,b_{n1}-4.70\,b_{n2}-1.55\,b_{n3}\ldots\ldots(16)$$

At Marquette:

$$+1.03\,b_{w0}+1.50\,b_{w1}+2.20\,b_{w2}+0.61\,b_{w3}+0.56\,b_{n0}+1.16\,b_{n1}+1.67\,b_{n2}+1.37\,b_{n3}\ldots\ldots(17)$$

Equations (14), (15) and (16) are the same as equations (48), (49) and (50), respectively, in Publication No. 317, page 36.

The definitions of b_{w0}, b_{w1}, b_{w2}, b_{w3}, b_{n0}, b_{n1}, b_{n2}, b_{n3}, for Lake Michigan-Huron are as follows, the current day being defined as the day to which the equation is assigned:

$b_{w0}=(4-5)$ at 8 p. m. on the day before the day preceding the current day *minus* $(4-5)$ at 8 a. m. on that preceding day.

$b_{w1}=(4-5)$ at 8 a. m. of the preceding day *minus* $(4-5)$ at 8 p. m. of that day.

$b_{w2}=(4-5)$ at 8 p. m. of the preceding day *minus* $(4-5)$ at 8 a. m. of the current day.

$b_{w3}=(4-5)$ at 8 a. m. of the current day *minus* $(4-5)$ at 8 p. m. of the current day.

$b_{n0}=(3-6)$ at 8 p. m. on the day before the day preceding the current day *minus* $(3-6)$ at 8 a. m. on that preceding day.

$b_{n1}=(3-6)$ at 8 a. m. on the preceding day *minus* $(3-6)$ at 8 p. m. of that day.

$b_{n2}=(3-6)$ at 8 p. m. of the preceding day *minus* $(3-6)$ at 8 a. m. of the current day.

$b_{n3}=(3-6)$ at 8 a. m. of the current day *minus* $(3-6)$ at 8 p. m. of the current day.

The quantity $(4-5)$, as in the equations for hourly effects, is the barometric pressure at the point marked 4 on Plate 2, minus the barometric pressure at the point marked 5 on that plate, in 0.01 inch of mercury.

Similarly, $(3-6)$ is the barometric pressure at the point marked 3 *minus* the barometric pressure at the point marked 6.

For Lake Superior, (Marquette), the definitions of b_{w0}, b_{w1}, . . . b_{w3}, b_{n0}, b_{n1}, . . . b_{n3}, are the same as stated above except that $(1-3)$ and $(2-4)$ should be substituted for $(4-5)$ and $(3-6)$, respectively.

The expressions, (14) to (17) inclusive, give the change in barometric correction at the gages stated from one day to the next in units of 0.001 foot. The barometric correction is of opposite sign to the barometric effect.

METHOD OF COMPUTING DAILY BAROMETRIC CORRECTION

The mean barometric correction for a day at each of the four gages stated may be computed from equations (10) to (13) inclusive, by first computing the effect for each hour of the day, summing them up, dividing by 24, then changing the sign of the result so obtained. To compute, in this manner, the daily barometric correction for each of the 1,500-odd days of observation used at each of the four gages would be a long, tedious process.

The method of computation actually used to secure the barometric corrections for each day consisted of five steps, as follows:

(1) The hourly barometric effects were computed by equations (10) to (13) inclusive for the first, the last, and a few intermediate days of the long series of days under consideration. The daily barometric correction was obtained for each of these selected days, by taking the mean of the 24 hourly effects for that day and changing the sign.

(2) The change in barometric correction from day to day through the whole series was computed from the appropriate one of the formulæ (14) to (17) inclusive.

(3) The changes from day to day computed in step (2) were applied one by one to the computed correction for the first day of the series obtained in step (1) to secure the barometric correction for each day in turn up to and including the next selected day on which the barometric correction has been computed in step (1).

(4) At this point, the second selected day, a discrepancy appeared between the barometric correction as computed for the day by step (1) and as computed by step (3). This discrepancy was distributed proportionally to time between the first and the second selected day, and all intermediate values (computed by step (2)) corrected, so as to make the discrepancy disappear at the second selected day. The values as corrected were adopted as being sufficiently accurate.

(5) Steps (3) and (4) were taken from the second to the third selected day, from the third to the fourth selected day, and so on to the end of the series.

The rule ordinarily observed in regard to selected days was to place them not more than one month apart in any case and not more than 10 days apart over any interval through which the discrepancy to be distributed exceeded 0.070 foot.

The discrepancy in question is due to two causes: (*a*) to omitted decimal places in the computations, and (*b*) to the fact that the process of computing the hourly barometric effects, and thence the daily barometric effects and corrections, by step (1) involves a smoothing out of the discrepancies between the value of the constants in equations (14) to (17) inclusive as derived from the least-square solution, and therefore does not agree exactly with the computations made from formulæ (14) to (17). The rule stated in the preceding paragraph was adopted, as a result of experience, as probably giving the best balance between extreme accuracy, on the one hand, and large expenditure of time in computation, on the other hand.

WIND EFFECTS

The wind, in blowing across the surface of a lake, piles the water up on the leeward shore. The slope of the water surface due to this cause, after a steady régime has been established, is downward to windward. The effect, W, of a wind in causing the water-surface at a shore gage for an hour to read higher or lower than the mean elevation of the whole lake surface for that hour is given by the expression

$$W = +0.088 \left(\frac{h^{2.4}}{100}\right) \Sigma\chi \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

in which W is expressed in 0.01 foot, h is the wind velocity at the gage in miles per hour as determined by observations at the nearest regular Weather Bureau station, and $\Sigma\chi$ is a quantity dependent upon the location of the gage with reference to the lake, the shape of the lake, the depth in every part of the lake, and the direction of the wind. Equation (18) is equation (69), page 63 of Publication No. 317. The meaning of $\Sigma\chi$ and the method of computing it for any gage for any wind direction, as well as the method of evaluating the constant 0.088 and the exponent 2.4 in equation (18) are fully stated in Publication No. 317.

The values of $\Sigma\chi$ for the various gages on Lakes Michigan-Huron and Superior for the various wind directions are given in Table 9.

The wind effect for any given wind velocity is proportional to the quantity $\Sigma\chi$. The maximum wind effect therefore, the wind velocity being the same, at any of the four gages shown in Table 9, occurs at the Milwaukee gage for an east or a west wind. The minimum wind effect at any of the four gages shown occurs at the Marquette gage for a north or south wind. The maximum wind effect at Mackinaw, the nearest gage to Marquette, occurs with a northeast or southwest wind ($\Sigma\chi = -0.64$ or $+0.64$, respectively). The maximum wind effect at Marquette occurs with an east or west wind ($\Sigma\chi = +0.40$ or -0.40, respectively). In the computations of wind effects by equation (18) at Mackinaw for the eight months June to September of each of 1910 and 1911, the maximum effect computed was 0.001 foot, and this occurred on only two days in the eight months stated. The average wind velocities at the two stations are not greatly different. It is 8 and 10 miles per hour at Sault Ste. Marie and Marquette, respectively, based upon 22 and 21 years of record, respectively. It is clear, therefore, that the wind effects at Marquette are less than 0.001 foot on practically all of the days. Accordingly the wind effects at that gage were neglected, and the observed elevation of the surface of Lake Superior at Marquette was corrected for barometric effects only in the computation of I_1.

TABLE 9.—*Values of $\Sigma\chi$ for use in formula (18) at the gages indicated*

Direction of wind	At Milwaukee	At Harbor Beach	At Mackinaw	At Marquette
NE	+0.16	−0.86	−0.64	+0.16
E	+ .95	− .77	− .41	+ .40
SE	+ .25	− .05	+ .15	+ .38
S	+ .36	+ .35	+ .44	− .02
SW	− .16	+ .86	+ .64	− .16
W	− .95	+ .77	+ .41	− .40
NW	− .25	+ .05	− .15	− .38
N	− .36	− .35	− .44	+ .02

Another picture of the character of the wind effects at Marquette may be had by an inspection of the nodal lines and depths for Lake Superior shown on Plate 3. From an inspection of this plate, together with the values of $\Sigma\chi$ for Marquette shown in Table 9, it may be seen that for all axes, *i. e.*, all wind direction except north and south, the nodal line comes near to Marquette; so near that the wind effect at the gage is practically a local effect within the bay or bight in which the gage is located for all except north and south winds. That the wind effects at Marquette are smaller than at any of the other gages on the Great Lakes is also to be expected because of the greater depth of Lake Superior, which is, roughly, 100 fathoms deep on an average—deeper than any of the other lakes. By the theory developed in this investigation (see Publication No. 317) the wind effect varies inversely as the cube of the depth.

A nodal line on the lake surface is a line which remains unchanged in elevation for a wind of any velocity, but constant direction. All parts of the lake surface to leeward of the nodal line are raised, and all parts to windward are lowered. (For full discussion see Publication No. 317, pages 48–54.)

The center of gravity of the area of Lake Superior, shown on Plate 3, is 359,510 feet north and 85,200 feet west of the Marquette gage.

The unit of time used in the observation equation, (1), is one day. To compute the wind effect at a gage for a day requires the computation of the effect for each hour for each wind direction by equation (18) by the proper substitution therein of h and $\Sigma\chi$ for that hour and wind direction, respectively, and the total effect for the 24 hours divided by 24 to get the mean effect for the day. To thus compute the daily wind effect at each gage for each day of the 1,500-odd days of observation at each gage used in this investigation would be an unnecessarily slow process. As a device for saving time in the computations of daily wind effects, a table was constructed for each gage, giving the values of $\frac{w}{24}$ in terms of the two arguments h and $\Sigma\chi$. With the Weather Bureau records of wind velocity and direction and such a table before one, it is a simple matter to take out the tabular values, one for each hour, to the nearest 0.001 foot, and enter them into the computations. (This computation becomes further simplified by virtue of the fact that for the gages under consideration a large percentage of the winds are too feeble to produce an effect as much as 0.001 foot.) The total of these values for any day gives the daily wind effects at the gage in question for that day.

In concluding these statements on the method of computing I_1 for use in equation (1), it may again be convenient to note its definition (page 9). For Lake Superior the only available, reliable gage being at Marquette, and the daily wind effects being so small as to be neglected, I_1 for any day is the mean observed elevation at Marquette corrected for barometric effects on the current day *minus* the mean observed elevation at Marquette corrected for barometric effects on the preceding day. In obtaining I_1 for any day for Lake Michigan-Huron, the procedure is as follows: (*a*) the observed elevation at each of the gages Milwaukee, Harbor-Beach and Mackinaw is corrected for wind and barometric effects in the manner described in the preceding pages; (*b*) the weights 1.9, 2.9 and 5.1 are assigned to the elevations corrected as stated in (*a*) at the three stations there mentioned, respectively, and the weighted mean elevation obtained of the whole lake surface corrected for winds and barometric effects; (*c*) the difference current minus preceding is taken of the values obtained in (*b*) as the I_1 for Lake Michigan-Huron for the current day.

The method of computation of I_1 will be made specific later by a numerical illustration. Note that a complete illustration of steps (*a*) and (*b*) in the computation of I_1 for Lake Michigan-Huron is given in Publication No. 317, of which the statements in these pages is an abstract, and is here included merely for convenience in reference and to make the publication a self-contained entity.

The weights assigned to the elevations corrected for wind and barometric effects, namely 1.9, 2.9 and 5.1, at the three gages Milwaukee, Harbor Beach and Mackinaw, respectively, were obtained by dividing the square of the probable error of the observed elevation at the gage by the square of the probable error of the corrected elevation (elevation corrected for wind and barometric effects), unit weight being assigned to the Buffalo gage on Lake Erie for which the probable error of a corrected elevation of the surface of Lake Erie was ±0.036 foot. (See Publication No. 317, Table 28, page 110.)

EXAMPLES OF COMPUTATION OF KNOWN QUANTITIES IN THE OBSERVATION EQUATION, EQUATION (1)

In the preceding pages (beginning with page 8), the final form of observation equation used in this investigation for determining the evaporation has been stated, equation (1), and the definitions of the various quantities and the methods of computing them have been set forth in general terms. In order to remove any ambiguity which might exist in those general definitions and statements of method, it is now proposed to give a specific example of the computations of each of the quantities e, w and I. The examples will be taken from the computations on Lake Michigan-Huron, inasmuch as the observations on that lake had by far the greater weight in fixing the final numerical values of the unknowns in the evaporation formula derived in this investigation.

EXAMPLE OF COMPUTATION OF e, LAKE MICHIGAN-HURON

The definition of e is given on page (8), and the meteorological stations at which the weather elements of vapor-pressure and temperature were observed are listed in Table 1 and shown on Plate 1. The computation of e for Lake Michigan-Huron for the dates September 1 to October 10 inclusive, 1910, is shown in Table 10. In this table, for each station in the order in which they appear in Table 1, there are three columns headed, respectively, "Mean Temp.", "Sat. Pres." and "Vap. Pres.". In the first of the three columns is recorded for each day the mean air temperature as observed at the station during the two days ending at midnight on the current day. In obtaining this mean, the mean temperature for each day was first computed by taking the mean of the maximum and minimum for that day. The mean of two such means, one for the preceding and one for the current day, was then taken as the mean temperature as recorded on the current day. In the second column is recorded the saturation vapor-pressure corresponding to the mean temperature for the two days ending at midnight on the current day. In the third column is recorded the vapor-pressure as observed at the station. The vapor-pressure on the current day was taken as the mean between the vapor-pressure as observed at 8 p. m. of the preceding day and 8 a. m. of the current day, 75th meridian time. The temperature is in degrees Fahrenheit, and the vapor-pressures are in units of 0.01 inch of mercury.

On the right-hand side of the table are shown the values of e. These values are obtained for each station by subtracting the observed vapor-pressure from the saturation vapor-pressure for that station, and are in 0.01 inch of mercury. The mean e for the lake computed from the stations on the United States side is shown in the last column but 4. The mean e computed from the two Canadian stations, Parry Sound and Saugeen, is shown in the last column but one. The final mean e for the lake shown in the last column is the arithmetic mean at all the ten stations. This is the e used in the observation equation, (1).

Note that the observation equation for any day covers a two-day time interval ending at midnight of the current day, or day to which the equation is assigned in listing the equations. All of the observed quantities in the equation, therefore,

TABLE 10.—*Computation of e for Lake Michigan-Huron*

Date	Chicago			Milwaukee			Grand Haven			Green Bay			Escanaba			Sault Ste. Marie			Alpena		
	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.
1910																					
Sept. 1.....	67	66	48	60	52	38	64	60	35	60	52	36	58	48	28	54	42	28	57	47	30
2.....	66	64	42	59	50	42	61	54	36	60	52	35	57	47	34	54	42	29	54	42	27
3.....	70	73	61	62	56	52	62	56	52	60	52	45	58	48	40	53	40	41	55	43	45
4.....	71	76	63	62	56	48	61	54	48	58	48	41	58	48	40	55	43	34	56	45	34
5.....	71	76	70	64	60	59	64	60	60	59	50	42	58	48	41	56	45	41	54	42	44
6.....	74	84	72	69	71	50	66	64	63	62	56	48	60	52	49	58	48	44	62	56	48
7.....	74	84	54	68	68	49	65	62	48	66	64	46	62	56	44	62	56	42	66	64	46
8.....	75	87	70	68	68	59	66	64	59	66	64	59	62	56	56	62	56	48	66	64	49
9.....	69	71	48	62	56	34	59	50	42	59	50	31	58	48	28	55	43	33	60	52	32
10.....	61	54	35	56	45	28	52	39	24	55	43	28	54	42	26	52	39	30	56	45	28
11.....	64	60	39	60	52	35	56	45	28	60	52	30	56	45	36	56	45	30	58	48	32
12.....	66	64	60	62	56	47	59	50	48	58	48	44	52	39	37	52	39	38	55	43	46
13.....	62	56	41	59	50	30	56	45	30	53	40	27	48	33	23	49	35	24	50	36	26
14.....	60	52	36	57	47	29	54	42	32	57	47	28	54	42	26	52	39	26	52	39	26
15.....	62	56	34	60	52	36	56	45	40	61	54	31	58	48	32	56	45	29	54	42	30
16.....	64	60	43	63	58	34	60	52	40	63	58	34	58	48	36	59	50	31	59	50	36
17.....	67	66	36	64	60	36	60	52	38	65	62	45	61	54	43	61	54	40	63	58	36
18.....	66	64	48	64	60	48	61	54	48	62	56	46	58	48	39	57	47	34	58	48	38
19.....	63	58	55	60	52	44	61	54	38	60	52	32	54	42	32	52	39	26	55	43	33
20.....	62	56	48	61	54	47	60	52	48	64	60	46	57	47	40	56	45	38	60	52	36
21.....	64	60	40	63	58	36	61	54	41	62	56	38	56	45	38	52	39	32	58	48	30
22.....	65	62	44	62	56	42	60	52	32	58	48	35	52	39	28	48	33	24	52	39	29
23.....	66	64	53	61	54	52	60	52	44	57	47	40	52	39	36	49	35	32	53	40	36
24.....	64	60	58	60	52	48	62	56	50	54	42	33	50	36	31	46	31	21	54	42	32
25.....	60	52	40	58	48	38	58	48	40	53	40	32	49	35	30	46	31	30	52	39	38
26.....	60	52	40	58	48	36	54	42	34	54	42	34	52	39	34	49	35	30	50	36	28
27.....	60	52	41	56	45	38	56	45	44	54	42	37	55	43	37	52	39	36	52	39	38
28.....	58	48	34	55	43	33	56	45	34	55	43	32	54	42	30	56	45	32	56	45	36
29.....	60	52	38	59	50	38	54	42	33	58	48	34	54	42	36	57	47	31	56	45	30
30.....	66	64	40	64	60	38	57	47	44	64	60	40	59	50	42	58	48	36	60	52	38
Oct. 1.....	69	71	46	65	62	40	58	48	44	64	60	34	58	48	34	52	39	39	58	48	39
2.....	66	64	28	62	56	30	54	42	30	57	47	28	51	37	28	42	27	18	48	33	22
3.....	69	71	44	66	64	44	60	52	36	61	54	40	54	42	36	46	31	29	52	39	34
4.....	71	76	60	68	68	52	66	64	59	62	56	46	57	47	43	56	45	46	60	52	54
5.....	69	71	64	63	58	55	64	60	57	58	48	46	56	45	44	58	48	50	62	56	54
6.....	64	60	41	57	47	27	57	47	36	57	47	26	54	42	30	52	39	26	54	42	36
7.....	56	45	32	54	42	20	48	33	21	53	40	24	50	36	24	46	31	22	44	29	20
8.....	56	45	30	58	48	26	50	36	28	57	47	28	56	45	28	50	36	30	48	33	23
9.....	57	47	32	56	45	28	50	36	35	56	45	26	54	42	26	48	33	26	48	33	29
10.....	56	45	33	54	42	28	48	33	26	53	40	26	50	36	26	47	32	24	48	33	22

TABLE 10—*Continued*

Date	Port Huron			Parry Sound			Saugeen			Values of *e*		
	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.	Mean temp.	Sat. pres.	Vap. pres.	Chicago	Milwaukee	Grand Haven
1910												
Sept. 1........	62	56	33	60	52	44	66	64	41	18	14	25
2........	62	56	42	58	48	31	60	52	38	22	8	18
3........	66	64	44	62	56	40	59	50	52	12	4	4
4........	65	62	36	62	56	46	60	52	39	13	8	6
5........	66	64	62	58	48	46	63	58	50	6	1	0
6........	70	73	64	60	52	48	68	68	74	12	21	1
7........	68	68	46	64	60	52	66	64	56	30	19	14
8........	68	68	63	64	60	46	65	62	50	17	9	5
9........	62	56	42	60	52	53	63	58	53	23	22	8
10........	54	42	28	56	45	32	56	45	29	19	17	15
11........	58	48	36	56	45	39	56	45	31	21	17	17
12........	64	60	44	59	50	41	58	48	47	4	9	2
13........	61	54	36	54	42	36	55	43	40	15	20	15
14........	54	42	28	48	33	25	51	37	30	16	18	10
15........	55	43	31	52	39	34	53	40	37	22	16	5
16........	58	48	32	56	45	38	56	45	41	17	24	12
17........	62	56	39	59	50	42	60	52	36	30	24	14
18........	62	56	42	58	48	38	60	52	52	16	12	6
19........	62	56	40	52	39	22	54	42	32	3	8	16
20........	64	60	46	53	40	39	54	42	45	8	7	4
21........	62	56	44	56	45	42	56	45	43	20	22	13
22........	58	48	30	49	35	28	53	40	28	18	14	20
23........	56	45	29	46	31	30	50	36	27	11	2	8
24........	60	52	48	47	32	31	52	39	35	2	4	6
25........	60	52	48	49	35	34	54	42	39	12	10	8
26........	56	45	36	54	42	35	54	42	36	12	12	8
27........	57	47	50	55	43	38	55	43	41	11	7	1
28........	56	45	32	56	45	36	56	45	37	14	10	11
29........	57	47	34	55	43	35	54	42	33	14	12	9
30........	60	52	39	56	45	38	56	45	37	24	22	3
Oct. 1........	59	50	39	56	45	43	60	52	46	25	22	4
2........	53	40	25	52	39	22	52	39	29	36	26	12
3........	56	45	34	55	43	25	51	36	26	27	20	16
4........	65	62	52	60	52	43	62	56	51	16	16	5
5........	67	66	58	63	58	56	68	68	58	7	3	3
6........	61	54	48	60	52	52	62	56	53	19	20	11
7........	51	37	25	48	33	24	50	36	26	13	22	12
8........	50	36	27	46	31	25	46	31	23	15	22	8
9........	50	36	30	56	45	31	50	36	32	15	17	1
10........	50	36	25	54	42	23	48	33	25	12	14	7

TABLE 10—*Continued*

Date	Values of e									
	Green Bay	Escanaba	Sault Ste. Marie	Alpena	Port Huron	Mean U. S.	Parry Sound	Saugeen	Mean Canada	Final Mean
1910										
Sept. 1	16	20	14	17	23	18	8	23	16	18
2	17	13	13	15	14	14	17	14	16	14
3	7	8	−1	−2	20	6	16	−2	7	6
4	7	8	9	11	26	11	10	13	12	11
5	8	7	4	−2	2	3	2	8	5	3
6	8	3	4	8	9	8	4	−6	−1	6
7	18	12	14	18	22	18	8	8	8	16
8	5	0	8	15	15	9	14	12	13	10
9	19	20	10	20	14	17	−1	5	2	14
10	15	16	9	17	14	15	13	16	14	15
11	22	9	15	16	12	16	6	14	10	15
12	4	2	1	−3	16	4	9	1	5	4
13	13	10	11	10	18	14	6	3	4	12
14	19	16	13	13	14	15	8	7	8	14
15	23	16	16	12	12	15	5	3	4	13
16	24	12	19	14	16	17	7	4	6	15
17	17	11	14	22	17	19	8	16	12	18
18	10	9	13	10	14	11	10	0	5	10
19	20	10	13	10	16	12	17	10	14	12
20	14	7	7	16	14	10	1	−3	−1	8
21	18	7	7	18	12	15	3	2	2	12
22	13	11	9	10	18	14	7	12	10	13
23	7	3	3	4	16	7	1	9	5	7
24	9	5	10	10	4	6	1	4	2	5
25	8	5	1	1	4	6	1	3	2	5
26	8	5	5	8	9	8	7	6	6	8
27	5	6	3	1	−3	4	5	2	4	4
28	11	12	13	9	13	12	9	8	8	11
29	14	6	16	15	13	12	8	9	8	11
30	20	8	12	14	13	14	7	8	8	13
Oct. 1	26	14	0	9	11	14	2	6	4	12
2	19	9	9	11	15	17	17	10	14	16
3	14	6	2	5	11	13	18	10	14	13
4	10	4	−1	−2	10	7	9	5	7	7
5	2	1	−2	2	8	3	2	10	6	4
6	21	12	13	6	6	14	0	3	2	12
7	16	12	9	9	12	13	9	10	10	12
8	19	17	6	10	9	13	6	8	7	12
9	19	16	7	4	6	11	14	4	9	11
10	14	10	8	11	11	11	19	8	14	12

should cover the same time interval. In the computation of e, the mean temperature and vapor-pressure on which it depends do cover this interval. But, in the computation of the vapor-pressure, the time interval covered is from 12:30 p. m. of the preceding day to 12:30 p. m. of the current day. The mean of the 8 p. m. value of the preceding day and the 8 a. m. value of the current day is approximately the mean for the 24-hour period stated in the preceding sentence, which overlaps, approximately equally, the last half and the first half, respectively, of the preceding and current days. A more exact method of computing the mean vapor-pressure for the two days might have been obtained by first computing separately the mean for the preceding and the mean for the current day from their 8 a. m. and 8 p. m. values, respectively, and then taking the mean of these two means. This would have doubled the labor, and it is believed that it would have produced only moderately greater accuracy than the method adopted.

The corresponding values of e for Lake Superior were obtained in the same manner, except that different meteorological stations were used as shown in Table 2 and on Plate 4.

EXAMPLE OF COMPUTATION OF $\left(\frac{w}{100} - x\right)$. LAKE MICHIGAN-HURON

The definition of x is given on page 9. It is an unknown to be determined indirectly from the least-square solutions by successive trials of various values, the final value adopted being the one which makes the sum of the squares of the residuals a minimum.

The final adopted value for x in this investigation is 2.6. The more detailed statement of the method of arrival at this value, and the evidence in support of it, will be reserved for a later place. This preliminary statement of its value is given in order to use that value in the numerical illustration following.

The definition of w is given on page 9, and the meteorological stations at which it was observed are shown in Table 1 and on Plate 1. The computation of $\left(\frac{w}{100} - 2.6\right)$ for Lake Michigan-Huron for the dates September 1 to October 10 inclusive, 1910, is shown in Table 11. The tabular values in this table are values of $\frac{w}{100}$ at the stations shown; w, in all columns of this table except the last, referring to any wind velocity, and is the total travel of the wind in miles each day as recorded by self-recording anemometers. The mean value of $\frac{w}{100}$ is shown in the twelfth column. In the next column is shown the mean value of $\frac{w}{100}$ from noon to noon, this being taken as the mean between the two midnight-to-midnight values in the column preceding, and assumed to be the mean $\frac{1}{100}$th part of the wind travel for the lake for the two days ending at midnight of the current day. In the last column are shown values of $\left(\frac{w}{100} - 2.6\right)$, these values being limited to positive ones only, for reasons to be explained later in the proper context. These are the values used in the observation equation (1).

The corresponding values for Lake Superior are computed in the same manner. The meteorological stations for this lake are shown in Table 2 and on Plate 4.

TABLE 11.—*Computation of* $\left(\frac{w}{100}-2.6\right)$, *Lake Michigan-Huron*

Date	Values of $\frac{w}{100}$, midnight to midnight, 90th meridian time										Mean		$\left(\frac{w}{100}-2.6\right)$
	Chicago	Milwaukee	Grand Haven	Green Bay	Escanaba	Sault Ste. Marie	Alpena	Port Huron	Parry Sound	Saugeen	Midnight to midnight	Noon to noon	
1910 Sept. 1	3.2	2.9	2.0	2.2	1.1	1.3	2.0	3.3	1.9	1.7	2.2	......	....
2	2.3	2.4	1.7	2.2	2.3	1.3	2.6	2.1	1.9	1.7	2.0	2.1	....
3	2.7	2.0	2.0	1.4	1.4	2.0	2.2	2.1	1.0	2.6	1.9	2.0	....
4	2.0	2.4	1.9	1.7	1.3	0.7	1.4	1.5	0.7	1.4	1.5	1.7	....
5	3.7	2.2	2.6	2.7	3.1	1.6	3.5	1.7	2.2	1.7	2.5	2.0	....
6	4.0	2.2	2.6	2.5	2.1	2.5	2.6	3.0	1.9	3.6	2.7	2.6	0
7	2.5	2.1	1.6	2.1	1.9	1.3	2.0	1.6	1.4	1.9	1.8	2.2	....
8	3.4	2.2	2.8	2.6	2.7	3.3	3.0	2.4	2.2	3.1	2.8	2.3	....
9	3.0	2.2	2.6	2.1	2.0	3.3	3.3	3.3	2.6	4.3	2.9	2.8	+0.2
10	2.0	2.4	1.9	2.1	3.0	1.4	2.1	1.7	1.2	1.7	2.0	2.4	....
11	4.0	3.3	3.3	2.9	3.3	1.5	2.8	1.8	1.2	1.4	2.6	2.3	....
12	3.7	3.9	3.4	3.8	3.7	2.7	3.0	3.3	1.7	2.6	3.2	2.9	+0.3
13	1.7	2.9	2.2	1.2	1.9	1.2	1.7	4.0	1.7	2.9	2.1	2.6	0.0
14	0.8	1.5	1.3	1.8	1.7	1.6	2.1	1.6	1.2	1.9	1.6	1.8	....
15	1.0	1.9	1.4	1.5	1.7	0.8	1.9	1.7	1.0	2.2	1.5	1.6	....
16	1.4	2.5	2.0	2.2	2.3	1.4	1.8	1.2	0.7	1.7	1.7	1.6	....
17	2.9	3.0	3.5	3.2	3.7	1.9	2.6	2.0	1.2	2.9	2.7	2.2	....
18	1.8	4.0	3.2	3.7	3.5	1.9	2.9	2.4	3.1	2.4	2.9	2.8	+0.2
19	0.7	1.7	1.6	1.3	2.1	1.2	2.7	1.5	1.4	1.7	1.6	2.2	....
20	1.0	1.5	1.7	1.5	1.5	1.8	1.9	1.3	1.4	1.2	1.5	1.6	....
21	2.7	2.7	2.3	2.0	3.1	2.0	2.4	2.4	1.7	1.9	2.3	1.9	....
22	3.3	2.7	2.0	1.9	2.7	2.4	3.8	2.2	1.4	1.7	2.4	2.4	....
23	2.0	3.5	2.1	4.7	4.4	2.2	2.8	1.0	1.7	2.2	2.7	2.6	0.0
24	4.5	3.7	4.2	3.5	4.2	3.1	5.4	2.7	2.9	2.9	3.7	3.2	+0.6
25	1.5	1.5	1.8	1.3	1.0	2.5	2.3	1.9	1.9	2.2	1.8	2.8	+0.2
26	3.1	3.5	2.3	2.0	2.4	1.7	2.9	1.8	1.2	0.7	2.2	2.0	....
27	3.2	2.1	4.2	3.2	2.5	3.8	3.3	2.9	3.8	4.1	3.3	2.8	+0.2
28	1.7	1.5	1.2	1.7	2.2	1.6	1.7	1.5	2.2	2.4	1.8	2.6	0.0
29	2.3	2.0	1.8	1.9	2.3	1.4	2.7	1.5	1.2	1.7	1.9	1.8	....
30	4.4	2.9	3.8	3.0	2.6	1.6	3.1	2.8	2.2	3.4	3.0	2.4	....
Oct. 1	3.0	2.3	3.7	2.9	3.4	5.0	4.6	3.9	4.6	6.7	4.0	3.5	+0.9
2	3.0	4.4	2.5	2.4	2.8	1.8	2.6	2.0	1.0	2.2	2.5	3.2	+0.6
3	4.7	3.3	4.1	3.1	3.5	3.0	4.4	3.2	3.1	3.6	3.6	3.0	+0.4
4	3.3	1.7	3.1	1.3	1.4	1.0	1.7	3.6	3.1	3.8	2.4	3.0	+0.4
5	2.6	2.0	3.0	2.8	2.5	2.7	3.1	2.8	2.2	3.1	2.7	2.6	0.0
6	2.1	2.3	2.6	2.3	1.9	2.8	3.0	4.2	2.2	4.1	2.8	2.8	+0.2
7	1.2	2.4	1.9	3.0	3.7	1.8	3.0	1.5	0.2	1.0	2.0	2.4	....
8	2.3	2.9	3.5	3.4	3.4	3.0	2.7	2.3	2.2	2.6	2.8	2.4	....
9	1.3	2.3	2.4	1.9	2.4	3.3	3.5	3.7	2.2	4.3	2.7	2.8	+0.2
10	1.2	1.8	1.9	2.3	2.8	1.6	2.2	2.2	1.2	2.2	1.9	2.3	....

EXAMPLE OF COMPUTATION OF I_3, LAKE MICHIGAN-HURON

The quantity, I_3, for Lake Michigan-Huron, is the computed rise of the surface of that lake due to the inflow into it from Lake Superior through St. Mary's River. This flow is given by equation (3) in 0.001 foot of depth on Lake Michigan-Huron in terms of the elevation of the water surface above mean sea-level, at the S. W. Pier (above locks), Sault Ste. Marie. In Table 12 is shown the computation of I_3 for the dates September 1 to October 10 inclusive, 1910. The first column shows the water levels as observed by the U. S. Lake Survey, and the second the discharge of the St. Mary's River to the nearest 0.001 foot of rise of Lake Michigan-Huron, computed by simple substitution in equation (3). Throughout the period May to

October in the years used in this investigation, 1909 to 1913 inclusive, I_3 was practically constant at 0.005 foot per day. Its extreme range was from 0.003 foot to 0.006 foot of depth on Lake Michigan-Huron.

Lake Superior being the highest lake in the series, there was no inflow into it in the sense here meant, the inflow from Lake Nipigon being considered as part of the run-off into the lake, and therefore included in the I_c-term of the observation equations for that lake.

Table 12.—*Computation of I_3, Lake Michigan-Huron*

Date	Water levels at Sault Ste. Marie (above locks) 600.+ft.	Discharge of St. Mary's River (I_3)
1910		
Sept. 1	1.50	5
2	1.28	5
3	1.63*	5
4	1.59	5
5	1.41	5
6	1.51	5
7	1.38	5
8	1.75	5
9	1.71	5
10	1.47	5
11	1.46	5
12	1.74	5
13	1.51	5
14	1.57	5
15	1.49	5
16	1.42	5
17	1.43	5
18	1.47	5
19	1.39	5
20	1.53	5
21	1.55	5
22	1.15	4
23	1.39	5
24	1.16	4
25	1.48*	5
26	1.16	4
27	1.50	5
28	1.38	5
29	1.23	4
30	1.30	5
Oct. 1	1.89	5
2	1.17	4
3	1.08	4
4	1.39	5
5	1.61	5
6	1.58	5
7	1.23	4
8	1.59	5
9	1.66	5
10	1.42	5

* Interpolated values, or partially uncertain values.

EXAMPLE OF COMPUTATION OF I_4, LAKE MICHIGAN-HURON

The quantity, I_4, for Lake Michigan-Huron is the computed fall of the lake surface due to the outflow from the lake into Lake St. Clair through the St. Clair River. This outflow is given by equation (6), in 0.001 foot of depth on Lake Michigan-Huron, as a function of the slope of the water surface between Harbor-Beach and the St. Clair Flats Canal. The first two columns in Table 13 show the elevations of the water surface at Harbor Beach and St. Clair Flats Canal, respectively, in feet above mean tide at New York. The third column shows I_4 computed from equation (6), by simple substitution, to the nearest 0.001 foot. The extreme range in I_4 for Lake Michigan-Huron for the days used in this investigation was from 0.011 to 0.014 foot, with the value 0.013 foot predominating in frequency.

TABLE 13.—*Computation of I_4, Lake Michigan-Huron*

Date	Water levels at Harbor Beach, H′ 579.+ft.	Water levels at St. Clair Flats Canal, h. 575.+ft.	Discharge of St. Clair River (I_4)
1910			
Sept. 1	....	0.40	12*
2	1.24	.45	12
3	1.19	.44	12
4	1.26	.47	13
5	1.15	.52	12
6	1.17	.50	12
7	....	.46	12*
8	1.34	.46	13
9	1.55	.42	13
10	1.34	.45	13
11	1.27	.44	13
12	1.30	.40	13
13	1.34	.38	13
14	1.31	.39	13
15	1.27	.38	13
16	1.23	.37	12
17	1.21	.37	12
18	1.41	.36	13
19	1.13	.35	12
20	1.21	.36	12
21	1.26	.34	13
22	1.02	.33	12
23	1.08	.30	12
24	.77	.36	12
25	.95	.28	12
26	1.10	.28	12
27	.96	.29	12
28	1.24	.25	13
29	1.08	.27	12
30	1.06	.28	12
Oct. 1	1.40	.26*	13
2	1.25	.25*	13
3	.86	.27*	12
4	1.01	.33	12
5	1.19	.29	12
6	1.39	.29	13
7	1.25	.36	12
8	1.13	.38	12
9	1.40	.34	13
10	1.13	.38	12

* Interpolated values or partially uncertain values due to a few hours of record on which daily mean depends being missing.

The computation of I_4 for Lake Superior is the same as that of the computation of I_3 for Lake Michigan-Huron, except that equation (4) was used instead of equation (3), to take account of the difference in lake area.

EXAMPLE OF COMPUTATION OF I_2, LAKE MICHIGAN-HURON

The definition of I_2 is given on page 9, and the factors for use in computing it for Lake Michigan-Huron are shown in Table 3. The rainfall stations listed in this table are shown on Plate 1. In Table 14 is shown in detail the computation of I_2

for each day from September 1 to October 10 inclusive, 1910. The stations shown at the column headings in Table 14 are those of Table 3 in the order given in that table. The figures in parentheses at each column heading in Table 14 are the factors from Table 3, and should be considered as repeated down the column. The other figures in Table 14, except in the last two columns, represent the daily rainfall at each station in inches from midnight to midnight, as observed by the regular U. S. Weather Bureau and Canadian Meteorological Service stations, and by co-operative observers. The product of the factor of a station by the observed rainfall at that station in inches converts the rainfall to units of 0.0001 foot rise of the whole surface of the lake. The sum of such products for any day for all of the stations available around and near the lake shore gives the total computed rise of the lake surface due to rainfall on it that day. These computed rises are shown in the last column but one. In the last column are shown the rainfall on the lake from noon of the preceding day to noon of the current day in units of 0.001 foot. This was computed from the preceding column by assuming that it is the mean between the rise, as computed from the rainfall from midnight to midnight of the preceding day, and the rise as computed from the rainfall from midnight to midnight of the current day.

The maximum value of the rainfall on Lake Michigan-Huron in one day shown in Table 14 is 0.040 foot on September 6, 1910. The maximum value computed in the 28 months used in this investigation was 0.060 foot on August 25, 1910.

For Lake Superior, I_2 was computed in the same manner as the above, except that the rainfall stations and factors shown in Table 4 were used.

In the computation of I_2 as shown in Table 14, whenever the rainfall at the station for a day was missing, the rainfall at the nearest station at which the rainfall had been observed that day was substituted for it, and the substitute station so used indicated by enclosing it in parentheses.

Rainfalls of less than 0.01 inch are indicated by a "T" in the Weather Bureau records. These T's were not neglected in the computation of I_2. The product of a factor and a T gives a rise of lake surface less than unity (in most cases) in the units used. It was found by experience that the T's were properly evaluated by counting 3 to 7 T's 1, 8 to 12 T's 2, and so on.

EXAMPLE OF COMPUTATION OF I_1, LAKE MICHIGAN-HURON

The definition of I_1 is given on page 9 and the method of computing it is briefly stated in general terms on pages 19 to 24. It is now proposed to give a numerical illustration of the computation of I_1 for Lake Michigan-Huron. There are five steps involved, namely, (*a*) the computation of the daily barometric correction at each of the gages Milwaukee, Harbor-Beach and Mackinaw, (*b*) the computation of the daily wind correction at each gage, (*c*) the application of the corrections computed in steps (*a*) and (*b*) to the observed level to get the corrected level at each gage, (*d*) the combination of the corrected levels at all three gages to get the weighted mean corrected level of the whole lake surface, and (*e*) taking the differences of the weighted mean corrected level of the whole lake surface from day to day.

EXAMPLE OF COMPUTATION OF DAILY BAROMETRIC CORRECTION

To illustrate the method of computing the daily barometric correction, as summarized on page 22, the computations pertaining to the Harbor-Beach gage will be used. The barometric pressures, as read from the weather forecast

TABLE 14.—*Computation of I_2, Lake Michigan-Huron*

Date	Chicago	Whiting	St. Joseph	South Haven	Ganges	Holland	Grand Haven	Muskegon	Montague (Muskegon)	Hart	Ludington	Manistee
1910												
Sept. 1......	(17.4) T	(9.4) T	(22.0) —	(9.2) —	(11.3) —	(2.6) —	(13.8) —	(3.5) —	(16.0) —	(7.4) —	(14.7) —	(11.4) —
2......	—	—	—	—	—	—	—	—	—	—	—	—
3......	0.55	—	1.20	—	T	—	0.12	0.15	0.15	—	—	—
4......	1.05	—	1.50	0.70	.12	0.02	.05	—	—	—	—	—
5......	.41	—	1.85	—	.21	.11	.03	T	T	1.00	0.92	1.25
6......	.17	—	.03	.60	—	.39	.20	T	T	.82	—	.28
7......	—	—	—	—	—	—	—	—	—	—	—	—
8......	.12	—	.06	.75	.25	.02	T	—	—	—	—	.03
9......	—	—	—	—	—	—	—	—	—	—	—	—
10......	—	—	—	—	—	—	—	—	—	—	—	—
11......	—	—	—	—	—	—	—	—	—	—	—	—
12......	1.28	.84	.12	.10	.58	.65	.67	.35	.35	.75	1.32	1.80
13......	—	—	.04	—	—	—	T	—	—	—	—	—
14......	—	—	—	—	—	—	—	—	—	—	—	—
15......	—	—	—	—	—	—	—	—	—	—	—	—
16......	—	—	—	—	—	—	—	—	—	—	—	—
17......	T	T	T	.70	.45	.44	.74	.40	.40	.80	1.62	.51
18......	—	—	.75	—	.04	—	—	T	T	—	—	—
19......	—	.12	T	—	—	—	—	—	—	—	—	—
20......	—	—	—	—	—	—	—	—	—	—	—	—
21......	—	—	—	—	—	—	—	—	—	—	—	—
22......	T	—	—	—	—	—	—	—	—	—	—	—
23......	.15	.10	.18	.45	.05	.02	.03	T	T	.38	—	—
24......	.11	—	.07	.20	.18	.46	.02	.30	.30	—	.12	—
25......	T	—	.02	—	.03	—	T	T	T	—	—	.35
26......	.06	.04	.03	.25	.14	.10	.12	.10	.10	.25	.05	—
27......	—	—	—	—	—	—	—	T	T	—	.04	.34
28......	—	—	—	—	—	—	—	—	—	—	—	—
29......	—	—	—	—	—	—	—	—	—	—	—	—
30......	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 1......	—	—	—	—	—	—	—	—	—	—	—	—
2......	—	—	—	—	—	—	—	—	—	—	—	T
3......	.96	T	.20	—	.05	.14	.55	—	—	—	.65	—
4......	.14	1.10	1.30	.40	1.39	1.48	.22	.65	.44	.80	.47	.37
5......	.16	1.00	.75	—	.05	—	T	T	—	—	—	—
6......	.23	.29	.11	.20	.22	.20	.08	—	—	.16	—	—
7......	—	—	—	—	—	—	—	—	—	—	—	—
8......	—	—	—	—	—	—	—	—	—	—	—	—
9......	—	—	—	—	—	—	—	—	—	—	—	—
10......	—	—	—	—	—	—	—	—	—	—	—	—

TABLE 14—*Continued.*

Date	Frankfort	Traverse City	Old Mission	Charlevoix	Petoskey	St. James	Mackinaw	Cheboygan	Alpena	Harrisville	East Tawas	Omer (East Tawas)
1910												
Sept. 1......	(29.3) —	(1.6) —	(15.3) —	(13.3) —	(2.2) —	(45.5) —	(4.6) —	(6.6) —	(57.6) —	(30.6) —	(6.3) —	(3.9) —
2......	—	—	—	—	0.10	—	—	—	—	—	—	—
3......	—	—	—	—	—	0.42	0.51	0.30	0.10	—	—	—
4......	—	—	—	—	—	—	—	—	—	—	—	—
5......	1.24	—	1.65	0.90	.60	.40	—	.15	1.28	0.85	0.60	.60
6......	—	—	.05	—	—	.10	.65	—	.21	.35	.10	.10
7......	—	—	—	—	—	—	.15	—	—	—	—	—
8......	.05	.12	.21	—	.21	.15	.05	.10	.50	.42	.50	.50
9......	—	—	—	—	—	—	—	—	—	—	—	—
10......	—	—	—	—	—	—	—	—	—	—	—	—
11......	—	.50	—	—	—	—	—	—	T	—	—	—
12......	1.53	—	1.22	1.06	—	1.10	.75	1.25	.89	.55	1.00	1.00
13......	—	—	—	—	—	—	—	—	—	—	—	—
14......	—	—	—	—	—	—	—	—	—	—	—	—
15......	—	—	—	—	—	—	—	—	—	—	—	—
16......	—	—	—	—	—	—	—	—	—	—	—	—
17......	—	—	.32	.10	.12	.10	T	.25	.15	.20	.10	.10
18......	—	—	—	—	—	—	—	—	—	.04	—	—
19......	—	—	—	—	—	—	—	—	—	—	—	—
20......	—	—	—	—	—	—	—	—	—	—	—	—
21......	—	—	—	—	—	—	—	—	—	—	—	—
22......	—	—	—	.05	—	—	—	—	—	—	—	—
23......	.60	—	.10	—	.12	—	.10	.10	.45	.37	.15	.15
24......	.15	.33	.05	—	—	1.25	.55	—	.26	.75	.75	.75
25......	—	.12	—	—	.10	.10	—	—	T	—	—	—
26......	—	.50	—	—	.08	.39	—	—	.15	.15	.15	.15
27......	—	—	—	—	.14	.40	.40	.25	.03	.20	—	—
28......	—	—	—	—	—	—	—	—	—	—	—	—
29......	—	—	—	—	—	—	—	—	—	—	—	—
30......	—	—	—	—	—	—	—	—	.04	—	—	—
Oct. 1......	—	—	—	.05	—	.60	—	.10	—	.10	—	—
2......	.30	.63*	.63	—	.12	.60	.30	.50	.30	.05	.20	.20
3......	—	.26*	.26	—	—	—	—	.95	.33	.20	.30	.30
4......	.60	.13*	.13	—	.10	.40	.95	1.62	.76	1.28	1.00	1.00
5......	—	.03*	.03	.42	.12	—	—	—	.01	—	—	—
6......	—	—	—	—	—	—	—	—	.17	.27	.30	.30
7......	—	—	—	—	—	—	—	—	—	—	—	—
8......	—	—	—	—	.24	—	T	T	.01	.02	—	—
9......	—	—	—	—	—	—	—	—	T	—	—	—
10......	—	—	—	—	—	—	—	—	—	—	—	—

* (Old mission.)

Table 14—*Continued*

Date	Bay City	Hayes	Pt. Austin (Hayes)	Harbor Beach	Jeddo	Sarnia (Port Huron)	Clinton	Point Clark	Lorne Park	North Bruce	Saugeen	Wiarton
1910 Sept. 1......	(4.2) —	(5.6) —	(16.9) —	(19.7) —	(9.3) —	(4.3) —	(12.3) —	(18.5) —	(12.8) —	(0.7) —	(19.5) —	(60.8)) —
2......	—	—	—	—	—	.01	—	—	—	—	—	—
3......	—	0.65	0.65	0.30	—	T	0.02	—	—	0.02	T	0.06
4......	—	—	—	.10	T	.42	—	—	—	.18	—	—
5......	.25	.42	.42	.95	.57	.75	1.63	.25	1.31	.53	.39	1.05
6......	—	.42	.42	—	.57	.05	.20	—	.68	.61	.99	.47
7......	—	—	—	—	—	—	—	—	—	—	—	—
8......	.20	.74	.74	—	T	.06	.45	—	—	.05	.01	.11
9......	—	—	—	.90	—	—	—	—	—	.13	.01	—
10......	—	—	—	—	—	—	—	—	—	—	—	—
11......	—	—	—	—	—	—	—	—	—	—	—	—
12......	.20	—	—	—	—	T	—	—	—	—	.03	—
13......	—	—	—	—	—	—	—	—	—	—	—	—
14......	—	—	—	—	—	—	—	—	—	—	—	—
15......	—	.01	.01	—	—	—	—	—	—	—	—	—
16......	—	—	—	—	—	—	—	—	—	—	—	—
17......	.15	T	T	—	—	.3	—	.20	—	—	.01	.26
18......	.10	—	—	.10	.11	.03	.14	—	—	.28	.57	.04
19......	.10	—	—	—	—	—	—	—	—	—	—	—
20......	—	—	—	—	—	—	—	—	—	—	T	—
21......	—	—	—	—	—	—	—	—	—	—	—	—
22......	—	—	—	—	—	—	—	—	—	—	—	—
23......	.10	—	—	—	.15	.31	.02	.10	—	—	.08	.07
24......	1.05	1.35	1.35	1.00	1.08	.41	.28	.45	.34	.08	.22	.11
25......	—	—	—	—	—	T	—	.45	—	.32	.03	.05
26......	—	—	—	—	.06	—	.01	—	—	—	—	—
27......	—	—	—	.05	—	.34	.15	—	.66	—	.13	.22
28......	—	—	—	—	—	—	—	—	—	.05	—	—
29......	—	—	—	—	—	—	—	—	—	—	—	—
30......	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 1......	—	—	—	—	—	—	—	—	—	.02	T	—
2......	—	—	—	—	—	—	.01	.35	—	—	T	—
3......	.25	—	—	—	—	—	—	.40	T	—	—	—
4......	1.25	—	—	.20	.46	.15	.53	—	.65	.47	.85	.83
5......	—	.25	.25	—	—	.01	.01	—	.90	.18	T	1.04
6......	.75	.73	.73	.15	.61	.43	.90	.50	.—	1.35	1.62	.18
7......	—	—	—	—	—	—	—	—	—	.23	—	—
8......	—	—	—	—	—	—	—	—	—	—	.01	—
9......	—	—	—	—	—	—	—	—	—	—	—	—
10......	—	—	—	—	—	—	—	—	—	—	—	—

TABLE 14—*Continued*

Date	Owen Sound (Meaford)	Meaford	Collingwood (Meaford)	Midland	Parry Sound	Bruce Mines	Detour (Bruce Mines)	Mackinac Is. (St. Ignace)	St. Ignace	Escanaba	Menominee	Plum Is.
1910												
Sept. 1......	(0.8) —	(6.8) —	(4.0) —	(7.2) —	(40.2) —	(22.1) —	(17.9) —	(4.5) —	(4.4) —	(15.0) —	(6.6) —	(42.7) —
2......	—	—	—	—	—	—	—	—	—	0.14	0.24	—
3......	—	—	—	0.10	T	0.50	0.50	0.06	0.06	T	—	0.26
4......	—	—	—	—	—	—	—	—	—	—	.32	—
5......	0.11	.11	.11	1.05	0.74	.15	.15	.82	.82	.18	.63	.32
6......	.09	.09	.09	.55	.87	.05	.05	.06	.06	.17	—	.52
7......	.05	.05	.05	—	—	—	—	—	—	—	—	—
8......	—	—	—	.27	.02	.21	.21	.04	.04	T	—	—
9......	.02	.02	.02	—	.02	—	—	—	—	—	—	—
10......	—	—	—	—	—	—	—	—	—	—	—	—
11......	—	—	—	—	—	—	—	—	—	.95	1.40	—
12......	—	—	—	—	.09	.57	.57	.95	.95	—	—	1.15
13......	.03	.03	.03	.07	—	—	—	—	—	—	—	—
14......	—	—	—	—	—	—	—	—	—	—	—	—
15......	—	—	—	—	—	—	—	—	—	—	—	—
16......	—	—	—	—	—	—	—	—	—	—	—	—
17......	—	—	—	—	—	.54	.54	—	—	.01	—	.02
18......	.03	.03	.03	—	—	—	—	—	—	—	—	T
19......	—	—	—	—	—	—	—	—	—	—	—	—
20......	—	—	—	—	—	—	—	—	—	—	—	—
21......	—	—	—	—	—	—	—	—	—	—	—	—
22......	—	—	—	—	—	—	—	—	—	.10	.08	—
23......	—	—	—	.02	.12	.29	.29	.16	.16	.01	—	.08
24......	.06	.06	.06	.03	.08	.46	.46	.19	.19	1.30	1.64	1.24
25......	.13	.13	.13	—	.19	.13	.13	—	—	—	—	—
26......	—	—	—	.17	—	—	—	—	—	.39	.16	.03
27......	.02	.02	.02	—	.17	.41	.41	.16	.16	.06	—	.23
28......	—	—	—	—	T	.01	.01	—	—	—	—	—
29......	—	—	—	—	—	—	—	—	—	—	—	—
30......	—	—	—	—	T	—	—	—	—	.08	—	—
Oct. 1......	—*	—*	—*	—	.20	.13	.13	.49	.49	—	—	—
2......	—	—	—	—	—	.05	.05	.93	.93	.80	.81	.70
3......	—	—	—	.02	.12	.27	.27	—	—	.14	.12	.30
4......	.38	.38	.38	1.10	1.21	1.20	1.20	.45	.45	—	—	.08
5......	—	—	—	.73	.05	.02	.02	T	T	—	—	—
6......	1.25	1.25	1.25	.30	.69	.04	.04	T	T	—	—	—
7......	—	—	—	—	—	—	—	—	—	—	—	—
8......	—	—	—	—	T	.11	.11	T	T	—	—	—
9......	—	—	—	—	.05	—	—	—	—	—	—	—
10......	—	—	—	—	—	—	—	—	—	—	—	—

* From Oct. 1–10, Owen Sound.

TABLE 14—*Continued*

Date	Sturgeon Bay	Oconto	Green Bay	Kewanee	Manitowac	Sheboygan	Port Washington	Milwaukee	Racine	Total rainfall on lake, midnight to midnight (0.0001 ft.)	Total rainfall on lake, noon to noon in 0.001 ft., (I_2)
1910											
Sept. 1	(16.9) —	(5.1) —	(1.8) —	(14.0) —	(10.9) —	(16.5) —	(14.5) —	(11.1) —	(30.2) —	0	. . .
2	—	—	T	—	—	—	—	—	—	4	0
3	—	—	T	—	—	T	0.04	T	—	129	7
4	—	0.33	0.09	—	—	—	—	0.35	T	72	10
5	0.61	.45	.70	2.11	1.38	1.13	.72	—	.48	585	33
6	.06	.03	.01	.06	.01	T	—	.27	.28	207	40
7	—	—	—	—	—	—	—	—	—	2	10
8	—	—	T	—	—	T	—	.01	—	114	6
9	—	—	—	—	—	—	—	—	—	20	7
10	—	—	—	—	—	—	—	—	—	0	1
11	.50	1.57	1.21	—	—	—	—	—	—	42	2
12	1.01	.95	.15	.82	.90	.80	.61	.83	.43	494	27
13	—	—	—	—	—	—	—	T	—	2	25
14	—	—	—	—	—	—	—	—	—	0	0
15	—	—	—	—	—	—	—	—	—	0	0
16	—	—	—	—	—	—	—	—	—	0	0
17	.63	T	1.76	1.10	1.03	.17	.03	.03	.14	188	9
18	—	—	—	—	—	T	—	—	—	36	11
19	—	—	T	—	—	—	—	—	T	1	2
20	—	—	—	—	—	—	—	—	—	0	0
21	—	—	—	—	—	—	—	—	—	0	0
22	—	.05	.03	—	—	—	—	.01	—	4	0
23	.21	1.02	.10	.30	.38	.30	.03	.15	.04	131	7
24	1.17	.54	1.85	1.10	1.18	.78	.45	.02	.05	402	27
25	—	—	—	—	—	—	—	—	—	37	22
26	.31	.04	.34	.23	.22	.18	.36	.04	.05	71	5
27	.04	—	—	—	—	.05	—	—	—	100	9
28	—	—	—	—	—	—	—	—	—	1	5
29	—	—	—	—	—	—	—	—	—	0	0
30	—	—	—	—	—	—	—	—	—	2	0
Oct. 1	—	—	—	—	—	—	—	—	—	47	2
2	.53	.72	.18	—	.04	.25	—	T	T	151	10
3	.81	.16	.17	.06	.10	.08	—	.26	.90	161	16
4	.05	—	—	.04	.04	.10	.40	.34	.48	460	31
5	—	—	—	—	—	T	—	.02	.01	123	29
6	—	—	—	—	.03	—	—	.11	.12	181	15
7	—	—	—	—	—	—	—	—	—	48	11
8	—	—	—	—	—	—	—	—	—	8	3
9	—	—	—	—	—	—	—	—	—	2	0
10	—	—	—	—	—	—	—	—	—	0	0

maps at points 3, 4, 5 and 6, at 8 a. m. and 8 p. m., 75th meridian time by interpolation between the isobars are shown in Table 15. In the last two columns of this table are shown the pressure differences, (3–6) and (4–5) in the units used in equations (10) to (12) and (14) to (16) inclusive, pages 20–21.

TABLE 15—*Barometric pressures around Lake Michigan-Huron*

Date	Hour	Barometric pressure in inches of mercury at point—				In units of 0.01 inch	
		3	4	5	6	Diff. (4–5)	Diff. (3–6)
1910							
Aug. 24	8 p.m.	29.75	29.68	29.87	29.81	−19	− 6
25	8 a.m.	29.69	29.60	29.80	29.74	−20	− 5
25	8 p.m.	29.76	30.01	29.64	30.00	+37	−24
26	8 a.m.	30.08	30.21	30.11	30.23	+10	−15
26	8 p.m.	30.02	30.11	30.15	30.21	− 4	−19
27	8 a.m.	30.05	30.12	30.16	30.24	− 4	−19
27	8 p.m.	30.10	30.08	30.13	30.14	− 5	− 4
28	8 a.m.	30.22	30.16	30.18	30.13	− 2	+ 9
28	8 p.m.	30.25	30.18	30.21	30.16	− 3	+ 9
29	8 a.m.	30.36	30.25	30.33	30.21	− 8	+15
29	8 p.m.	30.30	30.12	30.36	30.12	−24	+18
30	8 a.m.	30.00	29.92	30.21	30.05	−29	− 5
30	8 p.m.	29.90	29.98	29.99	30.01	− 1	−11
31	8 a.m.	30.23	30.20	30.20	30.12	0	+11
31	8 p.m.	30.27	30.20	30.17	30.13	+ 3	+14
Sept. 1	8 a.m.	30.32	30.30	30.19	30.20	+11	+12
1	8 p.m.	30.23	30.18	30.16	30.14	+ 2	+ 9
2	8 a.m.	30.19	30.13	30.23	30.16	−10	+ 3
2	8 p.m.	29.83	29.80	29.96	29.87	−16	− 4
3	8 a.m.	29.70	29.76	29.73	29.77	+ 3	− 7
3	8 p.m.	29.85	29.82	29.74	29.80	+ 8	+ 5
4	8 a.m.	29.90	29.85	29.95	29.85	−10	+ 5
4	8 p.m.	29.93	29.83	29.95	29.84	−12	+ 9
5	8 a.m.	29.92	29.77	29.91	29.77	−14	+15
5	8 p.m.	29.89	29.78	29.89	29.80	−11	+ 9
6	8 a.m.	29.70	29.78	29.80	29.88	− 2	−18
6	8 p.m.	29.93	30.03	29.90	30.04	+13	−11
7	8 a.m.	30.19	30.15	30.13	30.23	+ 2	− 4
7	8 p.m.	30.02	30.04	30.18	30.16	−14	−14
8	8 a.m.	29.90	29.94	30.11	30.09	−17	−19
8	8 p.m.	29.99	30.08	29.89	29.99	+19	0
9	8 a.m.	30.20	30.28	30.03	30.21	+25	− 1
9	8 p.m.	30.13	30.18	30.13	30.23	+ 5	−10
10	8 a.m.	30.12	30.18	30.24	30.26	− 6	−14
10	8 p.m.	30.03	29.99	30.14	30.09	−15	− 6
11	8 a.m.	30.02	30.02	30.20	30.12	−18	−10
11	8 p.m.	30.03	29.95	30.10	29.99	−15	+ 4
12	8 a.m.	30.21	30.22	30.12	30.04	+10	+17
12	8 p.m.	30.37	30.34	30.21	30.22	+13	+15
13	8 a.m.	30.45	30.44	30.34	30.37	+10	+ 8
13	8 p.m.	30.31	30.36	30.30	30.32	+ 6	− 1
14	8 a.m.	30.30	30.40	30.33	30.41	+ 7	−11
14	8 p.m.	30.28	30.31	30.29	30.35	+ 2	− 7
15	8 a.m.	30.30	30.35	30.36	30.40	− 1	−10
15	8 p.m.	30.21	30.26	30.34	30.31	− 8	−10

The computation of the hourly barometric effects for September 5, 1910, is shown in Table 16. The barometric pressures at 8 a. m. and 8 p. m. there shown in columns 2 to 5 inclusive are taken directly from Table 15. In columns 6 and 7, at 8 a. m. and 8 p. m. are recorded the differences in pressure between the actual values as taken from the charts. The other values in these columns, the figures enclosed in square brackets, are the hourly differences in pressure obtained by linear interpolation between the observed 8 a. m. and 8 p. m. differences, on the assumption of uniform change in pressure between those two times. Columns 8 and 9 are computed from equation (11), the "N-S Effect" and "E-W Effect" being, respectively, the products (3-6)(−0.44) and (4–5)(+6.94), proper attention being paid to the lag of 6 hours in each case. The total effect, E_1, for any hour, is the sum of the two separate effects for that hour. The mean effect for the day, −0.095 foot, is the amount by which the water at the gage at Harbor Beach was forced down by the existing barometric pressures for that day below the mean elevation of the whole lake surface for that day as determined from the Harbor Beach gage reading. Hence, to obtain the mean elevation of the whole lake surface for that day, in so far as it can be determined from the reading on this gage alone, corrected for only barometric effects, the observed elevation at the gage is corrected by +0.095 foot.

Table 16—*Example of computation of hourly barometric effects, Harbor Beach*

Date and hour	Barometric pressure at point—						N-S effect	E-W effect	E_1, total effect
	3	6	4	5	(3–6)	(4–5)			
1910	*in.*	*in.*	*in.*	*in.*	*in.*	*in.*	*ft.*	*ft.*	*ft.*
Sept. 4									
8 a.m.	29.90	29.85	29.85	29.95	+0.05	−0.10			
7 p.m.					[+ .09]	[− .12]			
8	29.93	29.84	29.83	29.95	+ .09	− .12			
9					[+ .10]	[− .12]			
10					[+ .10]	[− .12]			
11					[+ .10]	[− .12]			
Midnight					[+ .11]	[− .13]			
Sept. 5:									
1 a.m.					[+ .12]	[− .13]	0.00	−0.08	−0.08
2					[+ .12]	[− .13]	.00	− .08	− .08
3					[+ .12]	[− .13]	.00	− .08	− .08
4					[+ .13]	[− .13]	.00	− .08	− .08
5					[+ .14]	[− .14]	.00	− .08	− .08
6					[+ .14]	[− .14]	.00	− .09	− .09
7					[+ .14]	[− .14]	− .01	− .09	− .10
8	29.92	29.77	29.77	29.91	+ .15	− .14	− .01	− .09	− .10
9					[+ .14]	[− .14]	− .01	− .09	− .10
10					[+ .14]	[− .14]	− .01	− .09	− .10
11					[+ .14]	[− .13]	− .01	− .10	− .11
Noon					[+ .13]	[− .13]	− .01	− .10	− .11
1 p.m.					[+ .12]	[− .13]	− .01	− .10	− .11
2					[+ .12]	[− .12]	− .01	− .10	− .11
3					[+ .12]	[− .12]	− .01	− .10	− .11
4					[+ .11]	[− .12]	− .01	− .10	− .11
5					[+ .10]	[− .12]	− .01	− .09	− .10
6					[+ .10]	[− .12]	− .01	− .09	− .10
7					[+ .10]	[− .11]	− .01	− .09	− .10
8	29.89	29.80	29.78	29.89	+ .09	− .11	− .01	− .08	− .09
9							− .01	− .08	− .09
10							.00	− .08	− .08
11							.00	− .08	− .08
Midnight							.00	− .08	− .08
								Mean	− .095

Thus far, in the two preceding paragraphs, there has been illustrated step (1), page 22. The value +0.095 is shown in parentheses in the third column of Table

17. The other two values in parentheses in that column were computed in the same manner.

The change in barometric correction from day to day, shown in the second column of Table 17, was computed by equation (15), and is illustrated in Table 18. In this table the values in columns 2 to 9 inclusive which are not enclosed in parentheses are the values of b_{w0}, b_{w1}, . . . b_{ns} as defined on page 21, and as computed from the pressure differences shown in Table 15. The values in parentheses at the heads of these columns are the constants of equation (15). These should be considered as repeated down the column. The sum of the products of the constants into the values of b_{w0}, b_{w1}, b_{w2} . . . , . . . b_{ns} gives the change in barometric correction from the preceding to the current day shown in the last column of Table 18. Thus +0.150 foot is the change in the barometric correction from August 26 to August 27, 1910. This means that to the observed rise in the water surface at that gage between the dates stated, one must add 0.150 foot to get the true rise in the mean surface of the whole lake, in so far as it can be determined from readings on this gage alone, and corrected for only barometric effects.

TABLE 17—*Example of computation of daily barometric correction, Harbor Beach*

Date	Barometric correction				Elevation of water surface	
	Change in correction	Based directly on computed values	To conform to values computed by step (1)	Final	Observed	Corrected for barometric effect
1910	(*ft.*)	(*ft.*)	(*ft.*)	(*ft.*)	579+*ft.*	579+*ft.*
Aug. 26	....	(−0.124)	0.000	−0.12	....	....
27	+0.150	+ .026	− .003	+ .02	....	....
28	− .002	+ .024	− .005	+ .02	....	....
29	+ .043	+ .067	− .008	+ .06	....	....
30	+ .129	+ .196	− .010	+ .19	....	....
31	− .181	+ .015	− .013	.00	....	....
Sept. 1	− .027	− .012	− .016	− .03	....	....
2	+ .092	+ .080	− .018	+ .06	1.24	1.30
3	− .051	+ .029	− .021	+ .01	1.19	1.20
4	+ .019	+ .048	− .023	+ .02	1.26	1.28
5	+ .073	+ .121 (+0.095)	.000	+ .10	1.15	1.25
6	− .055	+ .040	+ .001	+ .04	1.17	1.21
7	− .056	− .016	+ .002	− .01	....	....
8	+ .099	+ .083	+ .003	+ .09	1.34	1.43
9	− .184	− .101	+ .004	− .10	1.55	1.45
10	+ .140	+ .039	+ .005	+ .04	1.34	1.38
11	+ .090	+ .129	+ .006	+ .14	1.27	1.41
12	− .135	− .006	+ .007	.00	1.30	1.30
13	− .050	+ .056	+ .008	− .05	1.34	1.29
14	+ .015	− .041	+ .009	− .03	1.31	1.28
15	+ .037	− .004 (+ .006)	.000	+ .01	1.27	1.28

In the preceding paragraph there has been illustrated step (2), page 22.

Referring to Table 17, step (3) consists in applying, one by one, the change in barometric correction from day to day shown in column 2, to the computed correction for the selected day, shown in parentheses in column 3. This gives the values in column 3 not in parentheses. Thus, beginning with the first selected day shown, August 26, the values +0.150, −0.002, . . . are applied successively to (−0.124), obtaining +0.026, +0.024 . . . . At the second selected day, Septem-

TABLE 18—*Example of computation of change in daily barometric corrections, Harbor Beach*

Date	(+2.81) b_{w0}	(+5.29) b_{w1}	(+5.10) b_{w2}	(+0.68) b_{w3}	(−1.00) b_{n0}	(−0.84) b_{n1}	(+0.49) b_{n2}	(+0.47) b_{n3}	Change in barometric correction (0.001 ft.)
1910									
Aug. 26	(+2.81) +1	(+5.29) −57	(+5.10) +27	(+0.68) +14	(−1.00) −1	(−0.84) +19	(+0.49) −9	(+0.47) +4	−168
27	+27	+14	0	+1	−9	+4	0	−15	+150
28	0	+1	−3	+1	0	−15	−13	0	−2
29	−3	+1	+5	+16	−13	0	−6	−3	+43
30	+5	+16	+5	−28	−6	−3	+23	+6	+129
31	+5	−28	−1	−3	+23	+6	−22	−3	−181
Sept. 1	−1	−3	−8	+9	−22	−3	+2	+3	−27
2	−8	+9	+12	+6	+2	+3	+6	+7	+92
3	+12	+6	−19	−5	+6	+7	+3	−12	−51
4	−19	−5	+18	+2	+3	−12	0	−4	+19
5	+18	+2	+2	−3	0	−4	−6	+6	+73
6	+2	−3	−9	−15	−6	+6	+27	−7	−55
7	−9	−15	+11	+16	+27	−7	−7	+10	−56
8	+11	+16	+3	−36	−7	+10	+5	−10	+99
9	+3	−36	−6	+20	+5	−19	+1	+9	−184
10	−6	+20	+11	+9	+1	+9	+4	−8	+140
11	+11	+9	+3	−3	+4	−8	+4	−14	+90
12	+3	−3	−25	−3	+4	−14	−13	+2	−135
13	−25	−3	+3	+4	−13	+2	+7	+9	−50
14	+3	+4	−1	+5	+7	+9	+10	−4	+15
15	−1	+5	+3	+7	+10	−4	+3	0	+37

ber 5, there is a discrepancy of 0.026 foot between the barometric correction as computed by steps (1), +0.095, and (3), +0.121, respectively. This discrepancy is distributed proportionally to time (as shown in column 4) between the first and second selected day in such a manner as to make the discrepancy disappear on the second selected day, and all intermediate values are corrected as shown in column (5). This constitutes step (4), page 22.

The values in column 5 are the final daily barometric corrections. They are applied to the observed elevation of the water surface, shown in column (6), to get the elevation corrected for barometric effects shown in column (7).

Beginning at the second selected day, September 5, the computation is carried forward in the manner illustrated to the second selected day, September 15, and so on.

The illustration just presented is for the Harbor Beach gage. In the same manner the mean daily observed elevations of the water surface at the other two gages on Lake Michigan-Huron, viz, Milwaukee and Mackinaw, and at the Marquette gage on Lake Superior, were corrected for barometric effects for each day of the 28 months of observation used in this investigation.

There has thus far been illustrated in the Computation of I_1, step (a), page 33.

EXAMPLE OF COMPUTATION OF WIND CORRECTION

In addition to the barometric correction for each day for each gage, the observed elevation of the water surface was corrected for wind effects. The necessary formula and method are stated on pages 22 to 24. In order to expedite the computation of the daily wind effect a table, like Table 19 for Harbor Beach, was constructed for each gage. In this table ΣH.B. is the value of Σ_{χ} for Harbor Beach shown in Table 9. The constant 0.0367 is the constant 0.088 of equation (18) multiplied by 10 to express the wind effects in units of 0.001 foot; and divided by 24, to convert the effect for each hour to the mean effect for the day. From this table it is seen that a wind from any direction of less than 22 miles per hour produces an effect on the *daily* mean water level of less than 0.001 foot, and it requires a wind of over 70

Table 19—*Table for computation of wind effects, Harbor Beach*

Wind direction	N. E.	E.	S. E.	S.	S. W.	W.	N. W.	N.
ΣH.B. ΣH.B.×0.0367	−0.86 − .0316	−0.77 − .0283	−0.05 − .00184	+0.35 + .0128	+0.86 + .0316	+0.77 + .0283	+0.05 + .00184	−0.35 − .0128
Wind effect in units of 0.001 ft.	Wind velocity in miles per hour							
0	0 to 21	0 to 22	0 to 70	0 to 31	0 to 21	0 to 22	0 to 70	0 to 31
1	21 to 34	23 to 35	....	32 to 50	22 to 34	23 to 35	....	32 to 50
2	35 to 42	36 to 44	....	51 to 61	35 to 42	36 to 44	....	51 to 61
3	43 to 48	45 to 50	....	....	43 to 48	45 to 50	....	....
4	49 to 53	51 to 56	....	....	49 to 53	51 to 56	....	....
5	54 to 59	57 to 61	....	....	54 to 59	57 to 61	....	....

Note: The wind effect is to raise the level of the water if the sign of ΣH.B. is +.

miles per hour from the southeast to lower the mean water surface for a day as much as 0.001 foot at Harbor Beach.

With such a table as No. 19 before one, together with the Weather Bureau records of hourly wind velocity and direction, it is a simple matter to evaluate the daily wind effects, particularly since they are less than 0.001 foot most of the time at the Michigan-Huron gages. For example in the eight months June to September inclusive each of 1910 and 1911, there were only 6 days on which the wind effect at Harbor Beach exceeded 0.001 foot. The maximum effect in that time interval was 0.020 foot on July 24, 1911.

The preceding two paragraphs serve to illustrate step (b), page 33, in the Computation of I_1.

ELEVATIONS AT MILWAUKEE, HARBOR BEACH AND MACKINAW FOR JUNE TO SEPTEMBER 1910, CORRECTED FOR WIND AND BAROMETRIC EFFECTS

In a manner similar to the numerical illustration given for August-September 1910 on pages 33 to 44, the observed elevation of the water surface at each of the gages Milwaukee, Harbor Beach and Mackinaw on Lake Michigan-Huron, and Marquette on Lake Superior, was corrected for wind and barometric effects for each day of the 28 months of observations. The observed elevations so corrected at Milwaukee, Harbor Beach and Mackinaw for June to September inclusive, 1910, are shown in Tables Nos. 20, 21 and 22, respectively. On days on which there was no record of observed elevation the missing value is indicated by a dash. The values of the corrected elevations which are in parentheses are values detected by the criterion developed from the least-square solutions which served to determine the barometric effects. This criterion, for each gage, is 3.5 times the probable error of a single observation as determined by the last barometric solution at that gage. For the four gages Milwaukee, Harbor Beach, Mackinaw and Marquette, the probable errors of a single observation of the corrected rise in the water surface (corrected for wind and barometric effects) as determined from the least-square solutions are, respectively, ±0.028 foot, ±0.025 foot, ±0.023 foot and ±0.020 foot. Hence the criterion by which to detect abnormal corrected elevations are, respectively, ±0.10 foot, ±0.09 foot, ±0.08 foot and ±0.07 foot. Thus, between August 22 and August 23, 1910, the corrected elevation is shown to have fallen fron 580.26 feet to 580.10 feet at the Milwaukee gage (Table 20). This is a change of 0.16 foot, which is greater than the criterion for that gage, ±0.10 foot, hence the corrected elevation for August 23, 1910, at that gage was not used in evaluating the elevation of the mean surface of the whole of Lake Michigan-Huron for that day. It so happened that on August 23, 1910, the record at the Harbor Beach gage was missing (Table 21). On that day also at the Mackinaw gage (Table 22), the indicated rise from the preceding day in the corrected elevation is 0.22 foot, which is larger than the limit ±0.08 foot for that gage established by the above criterion. Hence for August 23, 1910, the evidence shows that the lake surface was affected in such an abnormal way, possibly by seiches initiated by abnormal changes in barometric pressures, that the corrected elevations are unreliable and therefore that day could not be used in the investigation of the laws of evaporation.

In Tables 20 to 22, the sums of the barometric and wind corrections, added to the observed elevations, give the corrected elevations of the water surface. This illustrates Step (c), page 33.

TABLE 20—*Observed and corrected elevations of water surface at the Milwaukee gage on Lake Michigan-Huron*

Date	Barometric correction	Wind correction	Observed elevation 579.+	Corrected elevation 579.+
1910	(*ft.*)	(*ft.*)	(*ft.*)	(*ft.*)
June 1	+0.11	0.00	1.46	1.57
2	+ .05	.00	1.47	1.52
3	− .01	.00	1.59	1.58
4	− .07	.00	1.64	1.57
5	− .13	.00	1.66	1.53
6	.00	.00	1.51	1.51
7	+ .11	.00	1.45	1.56
8	+ .05	.00	...	...
9	− .09	.00	1.66	1.57
10	− .07	.00	1.65	1.58
11	− .04	.00	1.61	1.57
12	+ .15	.00	1.55	1.70
13	+ .18	.00	1.51	1.69
14	+ .08	.00	1.59	1.67
15	+ .05	.00	1.56	1.61
16	+ .06	.00	...	...
17	+ .07	.00	...	...
18	.00	.00	...	...
19	+ .03	.00	...	...
20	.00	.00	...	...
21	− .01	.00	...	...
22	+ .01	.00	...	...
23	− .04	.00	...	...
24	− .10	.00	...	...
25	− .06	.00	...	...
26	+ .04	.00	1.58	1.62
27	+ .07	.00	1.51	1.58
28	+ .04	.00	...	...
29	+ .06	.00	...	...
30	+ .06	.00	1.51	1.57
July 1	+ .11	.00	1.48	1.59
2	+ .08	.00	1.46	1.54
3	− .05	.00	1.58	1.53
4	− .10	.00	1.72	1.62
5	− .07	.00	1.61	1.54
6	− .01	.00	1.60	1.59
7	+ .06	.00	1.47	1.53
8	+ .08	.00	1.46	1.54
9	+ .04	.00	1.52	1.56
10	+ .06	.00	1.49	1.55
11	+ .09	.00	1.45	1.54
12	− .02	.00	1.58	1.56
13	+ .08	.00	1.40	1.48
14	+ .05	.00	1.49	1.54
15	− .01	.00	1.49	1.48
16	− .09	.00	1.63	1.54
17	− .12	.00	1.70	1.58
18	− .08	.00	1.62	1.54
19	+ .02	.00	1.54	1.56
20	+ .11	.00	1.43	1.54
21	+ .12	.00	1.39	1.51
22	+ .05	.00	1.46	1.51
23	− .05	.00	1.47	1.42
24	+ .03	.00	1.40	1.43
25	+ .11	.00	1.29	1.40
26	+ .06	.00	1.36	1.42
27	.00	.00	1.45	1.45
28	− .01	.00	1.44	1.43
29	+ .03	.00	1.42	1.45
30	+ .06	.00	1.37	1.43
31	+ .06	.00	1.37	1.43
Aug. 1	.00	.00	...	...
2	− .08	.00	...	...
3	.00	.00	...	...

Table 20—*Continued*

Date	Barometric correction	Wind correction	Observed elevation 579.+	Corrected elevation 579.+
1910	(*ft.*)	(*ft.*)	(*ft.*)	(*ft.*)
Aug. 4	+0.13	0.00	......	......
5	+ .20	.00	......	......
6	+ .09	.00	1.22	1.31
7	− .05	.00	1.37	1.32
8	− .02	.00	1.34	1.32
9	.00	.00	1.31	1.31
10	+ .02	.00	1.28	1.30
11	+ .01	.00	1.26	1.27
12	− .01	.00	1.26	1.25
13	.00	.00	1.26	1.26
14	− .07	.00	1.30	1.23
15	− .06	.00	1.35	1.29
16	− .08	.00	1.41	1.33
17	− .06	.00	1.41	1.35
18	− .06	.00	1.45	1.39
19	.00	.00	1.35	1.35
20	+ .04	.00	1.26	1.30
21	.00	.00	1.26	1.26
22	+ .01	.00	1.25	1.26
23	− .02	.00	1.12	(1.10)
24	− .03	.00	1.27	1.24
25	+ .04	.00	1.22	1.26
26	+ .24	.00	1.00	(1.24)
27	+ .21	.00	1.14	1.35
28	+ .02	.00	1.33	1.35
29	− .09	.00	1.34	1.25
30	− .05	.00	1.34	1.29
31	.00	.00	1.24	1.24
Sept. 1	− .03	.00	1.32	1.29
2	− .03	.00	......	......
3	+ .02	.00	......	......
4	− .07	.00	......	......
5	− .17	.00	1.25	1.08
6	+ .07	.00	1.07	1.14
7	+ .09	.00	1.00	1.09
8	+ .06	.00	1.23	1.29
9	+ .13	.00	1.25	1.38
10	+ .08	.00	1.25	1.33
11	− .03	.00	1.29	(1.26)
12	− .11	.00	1.51	1.40
13	− .01	.00	1.37	1.36
14	+ .09	.00	1.25	1.34
15	+ .07	.00	1.31	1.38
16	+ .05	.00	1.30	1.35
17	+ .05	.00	1.16	(1.21)
18	− .14	.00	1.44	1.30
19	− .13	.00	1.40	1.27
20	+ .04	.00	1.23	1.27
21	− .05	.00	1.34	1.29
22	− .10	.00	1.40	1.30
23	− .16	.00	1.51	1.35
24	− .28	.00	1.71	(1.43)
25	+ .07	.00	1.46	(1.53)
26	− .09	.00	1.37	1.28
27	+ .04	.00	1.28	1.32
28	+ .12	.00	1.14	1.26
29	+ .03	.00	1.24	1.27
30	+ .10	.00	1.12	1.22

Table 21—*Observed and corrected elevations of water surface at Harbor Beach gage on Lake Michigan-Huron*

Date	Barometric correction	Wind correction	Observed elevation 579.+	Corrected elevation 579.+
1910	(*ft.*)	(*ft.*)	(*ft.*)	(*ft.*)
June 1	−0.16	0.00	1.61	1.45
2	− .10	.00	1.59	1.49
3	− .08	.00	1.55	1.47
4	+ .04	.00	1.43	1.47
5	+ .09	.00	1.48	(1.57)
6	− .02	.00	1.55	1.53
7	− .10	.00	1.62	1.52
8	− .05	.00	1.57	1.52
9	+ .01	.00	1.49	1.50
10	− .01	.00	1.55	1.54
11	− .03	.00	1.61	1.58
12	− .07	.00	1.60	1.53
13	− .04	.00	1.55	1.51
14	− .03	.00	1.56	1.53
15	− .02	.00	1.57	1.55
16	+ .01	.00	1.53	1.54
17	+ .01	.00	1.48	1.49
18	− .02	.00	1.55	1.53
19	− .02	.00	1.51	1.49
20	− .06	.00	1.54	1.48
21	− .01	.00	1.54	1.53
22	+ .01	.00	1.51	1.52
23	− .02	.00	1.55	1.53
24	− .02	.00	1.52	1.50
25	.00	.00	1.48	1.48
26	.00	.00	1.48	1.48
27	− .08	.00	1.52	1.44
28	− .07	.00	1.54	1.47
29	− .05	.00	1.52	1.47
30	− .02	.00	1.50	1.48
July 1	− .05	.00	1.50	1.45
2	− .02	.00	1.45	1.43
3	− .07	.00	1.54	1.47
4	− .04	.00	1.46	1.42
5	+ .07	.00	1.38	1.45
6	+ .10	.00	1.34	1.44
7	− .03	.00	1.46	1.43
8	+ .06	.00	1.39	1.45
9	.00	.00	1.38	1.38
10	− .04	.00	1.40	1.36
11	− .02	.00	1.45	1.43
12	+ .07	.00	1.37	1.44
13	− .04	.00	1.58	(1.54)
14	+ .05	.00	1.39	1.44
15	+ .03	.00	1.41	1.44
16	− .03	.00	1.42	1.39
17	− .07	.00	1.45	1.38
18	− .08	.00	1.47	1.39
19	.00	.00	1.40	1.40
20	+ .05	.00	1.34	1.39
21	− .02	.00	1.42	1.40
22	− .04	.00	1.40	1.36
23	+ .04	.00	1.39	1.43
24	+ .14	.00	1.18	(1.32)
25	− .06	.00	1.48	1.42
26	− .04	.00	1.44	1.40
27	− .03	.00	1.44	1.41
28	− .04	.00	1.43	1.39
29	+ .02	.00	1.36	1.38
30	− .11	.00	1.53	1.42
31	− .04	.00	1.45	1.41
Aug. 1	+ .03	.00	1.37	1.40
2	+ .03	.00	1.39	1.42
3	+ .11	.00	1.26	1.37

TABLE 21—*Continued*

Date	Barometric correction	Wind correction	Observed elevation 579.+	Corrected elevation 579.+
1910	(*ft.*)	(*ft.*)	(*ft.*)	(*ft.*)
Aug. 4	−0.09	0.00	1.51	1.42
5	− .13	.00	1.65	(1.52)
6	− .12	.00	1.56	1.44
7	.00	.00	1.39	1.39
8	+ .05	.00	1.30	1.35
9	+ .08	.00	1.25	1.33
10	− .02	.00	1.39	1.37
11	− .02	.00	1.36	1.34
12	+ .03	.00	1.23	1.26
13	+ .03	.00	1.21	1.24
14	− .01	.00	1.30	1.29
15	− .01	.00	1.26	1.25
16	+ .04	.00	1.29	1.33
17	+ .02	.00	1.27	1.29
18	− .10	.00	1.40	1.30
19	− .03	.00	1.37	1.34
20	+ .07	.00	1.21	1.28
21	+ .12	.00	1.16	1.28
22	+ .12	.00	1.14	1.26
23	+ .07	.00	....	....
24	+ .10	.00	....	....
25	+ .04	− .01	....	....
26	− .12	.00	....	....
27	+ .02	.00	....	....
28	+ .02	.00	....	....
29	+ .06	.00	....	....
30	+ .19	.00	....	....
31	.00	.00	....	....
Sept. 1	− .03	.00	....	....
2	+ .06	.00	1.24	1.30
3	+ .01	.00	1.19	(1.20)
4	+ .02	.00	1.26	1.28
5	+ .10	.00	1.15	1.25
6	+ .04	.00	1.17	1.21
7	− .01	.00	....	....
8	+ .09	.00	1.34	1.43
9	− .10	.00	1.55	1.45
10	+ .04	.00	1.34	1.38
11	+ .14	.00	1.27	(1.41)
12	.00	.00	1.30	1.30
13	− .05	.00	1.34	1.29
14	− .03	.00	1.31	1.28
15	+ .01	.00	1.27	1.28
16	+ .04	.00	1.23	1.27
17	+ .10	.00	1.21	(1.31)
18	.00	.00	1.41	(1.41)
19	+ .05	.00	1.13	1.18
20	− .03	.00	1.21	1.18
21	− .06	.00	1.26	1.20
22	+ .09	.00	1.02	1.11
23	+ .04	.00	1.08	1.12
24	+ .12	.00	.77	(0.89)
25	.00	.00	.95	(0.95)
26	+ .06	.00	1.10	1.15
27	+ .06	.00	.96	(1.02)
28	− .06	.00	1.24	1.18
29	+ .06	.00	1.08	1.14
30	+ .11	.00	1.06	1.17

Table 22—*Observed and corrected elevations of water surface at Mackinaw gage on Lake Michigan-Huron*

Date	Barometric correction	Wind correction	Observed elevation 579.+	Corrected elevation 579.+
1910	(*ft.*)	(*ft.*)	(*ft.*)	(*ft.*)
June 1	+0.04	0.00	1.38	1.42
2	+ .06	.00	1.37	1.43
3	+ .05	.00	1.41	1.46
4	+ .01	.00	1.44	1.45
5	+ .04	.00	1.43	1.47
6	+ .04	.00	1.46	1.50
7	+ .02	.00	1.46	1.48
8	+ .01	.00	1.47	1.48
9	+ .04	.00	1.45	1.49
10	+ .04	.00	1.45	1.49
11	+ .04	.00	1.44	1.48
12	− .06	.09	1.53	1.47
13	− .07	.00	1.53	1.46
14	− .04	.00	1.50	1.46
15	− .03	.00	1.52	1.49
16	− .06	.00	1.52	1.46
17	− .06	.00	1.57	1.51
18	.00	.00	1.52	1.52
19	− .01	.00	1.52	1.51
20	+ .03	.00	1.50	1.53
21	− .01	.00	1.52	1.51
22	− .02	.00	1.53	1.51
23	+ .04	.00	1.50	1.54
24	+ .05	.00	1.43	1.48
25	+ .01	.00	1.48	1.49
26	− .04	.00	1.51	1.47
27	+ .01	.00	1.49	1.50
28	+ .02	.00	1.50	1.52
29	− .01	.00	1.50	1.49
30	− .01	.00	...	...
July 1	− .02	.00	...	...
2	− .02	.00	1.51	1.49
3	+ .10	.00	1.37	1.47
4	+ .08	.00	1.36	1.44
5	− .02	.00	...	...
6	− .04	.00	1.46	1.42
7	− .01	.00	1.46	1.45
8	− .08	.00	1.50	1.42
9	− .02	.00	1.40	1.38
10	− .01	.00	1.45	1.44
11	− .05	.00	1.48	1.43
12	− .02	.00	1.39	1.37
13	− .03	.00	1.47	1.44
14	− .06	.00	1.44	1.38
15	− .01	.00	1.46	1.45
16	+ .09	.00	1.32	1.41
17	+ .12	.00	1.27	1.39
18	+ .09	.00	1.33	1.42
19	− .04	.00	1.43	1.39
20	− .12	.00	1.49	1.37
21	− .06	.00	1.43	1.37
22	− .01	.00	...	...
23	− .03	.00	1.41	1.38
24	− .13	.00	...	...
25	− .01	.00	...	...
26	− .03	.00	1.38	1.35
27	+ .03	.00	1.34	1.37
28	+ .02	.00	1.34	1.36
29	− .02	.00	1.41	1.39
30	+ .06	.00	1.29	1.35
31	− .01	.00	1.34	1.33
Aug. 1	.00	.00	1.36	1.36
2	+ .02	.00	1.32	1.34
3	− .06	.00	1.44	1.38

TABLE 22—*Continued*

Date	Barometric correction	Wind correction	Observed elevation 579.+	Corrected elevation 579.+
1910	(*ft.*)	(*ft.*)	(*ft.*)	(*ft.*)
Aug. 4	+ 0.02	0.00	1.24	(1.26)
5	− .01	.00	1.31	1.30
6	+ .04	.00	1.23	1.27
7	+ .03	.00	1.23	1.26
8	− .04	.00	1.31	1.27
9	− .05	.00	1.33	1.28
10	+ .01	.00	1.26	1.27
11	+ .01	.00	1.40	(1.41)
12	− .01	.00	1.35	1.34
13	− .03	.00	1.30	1.27
14	+ .06	.00	1.21	1.27
15	+ .03	.00	1.19	1.22
16	+ .01	.00	1.25	1.26
17	+ .03	.00	1.21	1.24
18	+ .12	.00	1.18	1.30
19	.00	.00	1.24	1.24
20	− .08	.00	1.34	1.26
21	− .08	.00	1.32	1.24
22	− .08	.00	1.31	1.23
23	− .04	.00	1.49	(1.45)
24	− .08	.00	1.30	1.22
25	− .02	.00	1.32	1.30
26	− .06	.00	1.45	(1.39)
27	− .09	.00	1.40	1.31
28	+ .02	.00	1.26	1.28
29	+ .02	.00	1.25	1.27
30	− .08	.00	1.36	1.28
31	+ .05	.00	1.25	1.30
Sept. 1	+ .04	.00	1.21	1.25
2	− .04	.00	1.30	1.26
3	− .02	.00	1.23	1.21
4	.00	.00	1.20	1.20
5	+ .02	.00	1.18	1.20
6	− .07	.00	1.35	1.28
7	− .05	.00	1.34	1.29
8	− .05	.00	1.29	1.24
9	.00	.00	1.26	1.26
10	− .08	.00	1.35	1.27
11	− .07	.00	1.37	1.30
12	+ .09	.00	1.15	1.24
13	+ .04	.00	1.24	1.28
14	− .04	.00	1.31	1.27
15	− .05	.00	1.29	1.24
16	− .08	.00	1.31	1.23
17	− .10	.00	1.45	(1.35)
18	+ .08	.00	1.10	1.18
19	.00	.00	1.24	1.24
20	+ .01	.00	1.23	1.24
21	+ .07	.00	1.12	1.19
22	− .05	.00	1.25	1.20
23	+ .03	.00	1.10	1.13
24	+ .03	.00	1.10	1.13
25	− .02	.00	1.18	1.16
26	+ .02	.00	1.17	1.19
27	− .06	.00	1.26	1.20
28	− .03	.00	....	....
29	− .07	.00	....	....
30	− .14	.00	....	....

Applying the weights 1.9, 2.9 and 5.1 to the corrected elevations at the three gages, as shown in Tables Nos. 20 to 22, respectively, there is obtained the weighted mean elevation of the whole surface of Lake Michigan-Huron. This weighted mean elevation is shown in column 3 of Table 23, headed "Observed elevation corrected for wind and barometric effects." Usually the corrected elevations at all three gages were obtainable and usable in thus evaluating the elevation of the mean surface of the whole lake. At such times the sum of the weights is 9.9. On some days the criterion causes rejections to be made in the corrected elevation at one or more gages. In such instances the evaluation of the mean elevation of the whole lake surface for the day was made from the retained corrected elevations at the two or less gages. In the specific instance, August 23, 1910, referred to previously, the mean corrected elevation not being determinable, that fact is noted by a dash enclosed in parentheses.

In the preceding paragraph has been illustrated Step (d), page 33, in the computation of I_1. The final step, (e), in the computation, consists in taking the differences from day to day between the weighted mean elevation of the whole lake surface as computed for each day. This is shown in column 4, Table 23, headed "Rise of mean lake surface." Thus from August 24 to August 25, 1910, the mean surface of the whole lake rose 0.060 foot.

COMPUTATION OF ABSOLUTE TERM, *I*, IN OBSERVATION EQUATION (1), FOR LAKE MICHIGAN-HURON

The definition of I is given on page 9. Its computation for all of the days of observation on Lake Michigan-Huron used in this investigation is shown in the fourth to tenth columns, inclusive, of Table 23. In that table, beginning at the left, the second column shows the observed elevation of the lake surface in feet above mean sea-level at New York. This is taken as the arithmetic mean of the observed levels at Milwaukee, Harbor Beach and Mackinaw.

In the third column are listed the observed elevations corrected for wind and barometric effects. The fourth column shows the rise of the mean lake surface, the I_1 of equation (1). The fifth, sixth, seventh and eighth columns show the corrections for, respectively, inflow from Lake Superior ($-I_3$), outflow to Lake Erie ($+I_4$), rainfall on the lake ($-I_2$), and run-off into the lake ($-I_c$). The ninth column shows the sum of the separate corrections, i.e., $-I_3+I_4-I_2-I_c$. The tenth column shows the "net rise" of the lake surface, the I, or absolute term, of equation (1). This is the sum of the fourth and ninth columns. It will be observed that some of the values of I represent the combination of two or more days, and other values, marked with a star (*), are indicated as rejected.

The values indicated as rejected were detected by a definite numerical criterion, the final value of which gradually emerged from the series of least-square solutions which served to fix the laws of evaporation as established in this investigation. On lake Michigan-Huron, the last such solution but one, Solution V_4, had for a probable error of a single change in elevation ± 0.016 foot. Hence in making Solution V_5, every date which showed a residual larger than five times the probable error, or ($5 \times \pm 0.016 =$) ± 0.080 foot, was rejected. By this rule less than one equation per thousand would have been rejected if the causes of the abnormal residuals had been all accidental in character. In Solution V_5, the final solution on Lake Michigan-Huron, the total number of rejected equations was 39 out of a possible 849. This is at the rate of 46 per thousand. The large

TABLE 23—*Lake Michigan-Huron.* (*Unit=0.001 ft. except in second and third columns.*)

Date		Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 580.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
					Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_6$)				Net rise	Computed evaporation
1909		(*ft.*)	(*ft.*)										
July	1	2.10	1.09		−5	+14		−6					
	2	2.09	1.10	+ 10	−5	+14	−18	−6	−15	− 5	+ 5	− 5	+ 5
	3	2.12	1.10	0	−5	+14	−24	−6	−21	− 21	+ 8	− 26	+ 13
	4	2.10	1.09	− 10	−5	+14	− 5	−6	− 2	− 12	+ 4	− 38	+ 17
	5	2.11	1.11	+ 20	−5	+14	0	−6	+ 3	+ 23	+ 4	− 15	+ 21
	6	2.10	1.10	− 10	−5	+14	0	−6	+ 3	− 7	+ 4	− 22	+ 25
	7	2.07	1.12	+ 20	−5	+14	0	−6	+ 3	+ 23	+ 5	+ 1	+ 30
	8	2.03	1.09	− 30	−5	+14	0	−6	+ 3	− 27	+ 5	− 26	+ 35
	9	2.04	1.13	+ 40	−5	+14	− 1	−6	+ 2	+ 42	+ 6	+ 16	+ 41
	10	2.01	1.11	− 20	−5	+13	− 9	−6	− 7	− 27	+ 6	− 11	+ 47
	11	2.02	1.11	0	−5	+13	−15	−6	−13	− 13	+ 5	− 24	+ 52
	12	2.02	1.02	− 90	−5	+13	−11	−6	− 9				
	13	2.04	1.05	+ 30	−5	+14	− 5	−6	− 2	− 61	+13	− 85	+ 65
	14	2.07	1.06	+ 10	−5	+14	− 3	−6	0				
	15	2.05	1.06	0	−5	+14	− 8	−6	− 5	− 5	+ 5	− 90	+ 70
	16	2.06	1.07	+ 10	−5	+14	− 6	−6	− 3	+ 7	+ 5	− 83	+ 75
	17	2.07	1.08	+ 10	−5	+14	− 1	−6	+ 2	+ 12	+ 5	− 71	+ 80
	18	2.06	1.09	+ 10	−5	+14	− 3	−6	0	+ 10	+12	− 61	+ 92
	19	2.06	1.05	− 40	−5	+14	− 3	−6	0	− 40	+17	− 101	+ 109
	20	2.07	1.05	0	−5	+14	− 5	−6	− 2	− 2	+ 5	− 103	+ 114
	21	2.05	1.05	0	−5	+13	−12	−6	−10	− 10	+ 5	− 113	+ 119
	22	2.02	1.04	− 10	−5	+14	−19	−6	−16	− 26	+12	− 139	+ 131
	23	2.09	1.07	+ 30	−6	+14	−23	−6	−21	+ 9	+10	− 130	+ 141
	24	2.06	1.04	− 30	−5	+14	−14	−6	−11	− 41	+17	− 171	+ 158
	25	2.10	1.08	+ 40	−5	+14	− 2	−6	+ 1	+ 41	+ 4	− 130	+ 162
	26	2.09	1.06	− 20	−5	+14	0	−6	+ 3	− 17	+ 6	− 147	+ 168
	27	2.09	1.06	0	−5	+14	0	−6	+ 3	+ 3	+ 5	− 144	+ 173
	28	2.05	1.03	− 30	−5	+13	− 4	−6	− 2	− 32	+ 5	− 176	+ 178
	29	2.06	1.05	+ 20	−5	+14	−10	−6	− 7	+ 13	+ 4	− 163	+ 182
	30	2.07	1.06	+ 10	−5	+14	− 6	−6	− 3	+ 7	+ 5	− 156	+ 187
	31	2.08	1.05	− 10	−5	+14	− 1	−6	+ 2	− 8	+ 5	− 164	+ 192
Aug.	1	2.08	1.05	0	−5	+13	− 4	−6	− 2	− 2	+ 5	− 166	+ 197
	2	2.07	1.03	− 20	−5	+13	− 7	−6	− 5	− 25	+ 6	− 191	+ 203
	3	2.06	1.03	0	−5	+13	− 5	−6	− 3	− 3	+ 6	− 194	+ 209
	4	2.07	1.05	+ 20	−5	+13	− 9	−6	− 7	+ 13	+ 4	− 181	+ 213
	5	2.08	1.06	+ 10	−5	+13	− 9	−6	− 7	+ 3	+ 5	− 178	+ 218
	6	2.09	1.07	+ 10	−5	+13	− 2	−6	0	+ 10	+ 6	− 168	+ 224

7	2.06	1.04	− 30	−5	+13	0	−6	+ 2	− 28	+ 5	− 196	+ 229
8	2.05	1.04	0	−5	+13	0	−6	+ 2	+ 2	+ 6	− 194	+ 235
9	2.06	1.04	0	−5	+14	− 4	−6	− 1	− 1	+ 6	− 195	+ 241
10	2.09	1.04	0	−5	+14	− 3	−6	0	0	+ 6	− 195	+ 247
11	2.06	1.03	− 10	−5	+14	− 6	−6	− 3	− 13	+ 5	− 208	+ 252
12	2.02	.99	− 40	−5	+13	− 6	−6	− 4	− 44	+ 4	− 252	+ 256
13	2.05	.99	0	−5	+14	− 1	−6	+ 2	+ 2	+ 4	− 250	+ 260
14	2.05	1.03	+ 40	−5	+14	−11	−6	− 8	+ 32	+ 4	− 218	+ 264
15	2.00	1.02	− 10	−5	+14	−20	−6	−17	− 27	+ 3	− 245	+ 267
16	2.04	1.04	+ 20	−5	+14	−12	−6	− 9	+ 11	+ 3	− 234	+ 270
17	2.08	1.06	+ 20	−5	+14	− 3	−6	0	+ 20	+ 3	− 214	+ 273
18	2.07	1.05	− 10	−5	+14	− 1	−6	+ 2	− 8	+ 4	− 222	+ 277
19	2.05	1.04	− 10	−5	+14	0	−6	+ 3	− 7	+ 4	− 229	+ 281
20	2.07	1.12	+ 80	−5	+14	− 2	−6	+ 1	− 38	+10	− 267	+ 291
21	2.04	1.00	−120	−5	+14	− 2	−6	+ 1				
22	1.96	.95	− 50	−5	+14	0	−6	+ 3	− 47	+ 5	− 314	+ 296
23	1.96	.93	− 20	−5	+14	0	−6	+ 3	− 17	+ 6	− 331	+ 302
24	1.92	.92	− 10	−5	+13	− 1	−6	+ 1	− 9	+ 7	− 340	+ 309
25	1.90	.92	0	−5	+14	− 2	−6	+ 1	+ 1	+ 9	− 339	+ 318
26	1.91	.91	− 10	−5	+14	− 4	−6	− 1	− 11	+ 5	− 350	+ 323
27	1.94	.92	+ 10	−5	+13	−12	−6	−10	0	+ 5	− 350	+ 328
28	1.91	.94	+ 20	−5	+14	−18	−6	−15	− 52	+15	− 402	+ 343
29	1.94	.85	+ 10	−6	+14	−11	−6	− 9				
30	1.91	.89	+ 40	−5	+14	− 1	−6	+ 2				
31	1.85	.95	+ 60	−6	+13	− 9	−6	− 8	− 34	+21	− 436	+ 364
Sept. 1	1.89	.87	− 80	−5	+14	− 9	−6	− 6				
2	1.86	.85	− 20	−5	+14	− 1	−6	+ 2	− 18	+ 4	− 454	+ 368
3	1.86	.87	+ 20	−5	+14	− 6	−6	− 3	+ 17	+ 3	− 437	+ 371
4	1.86	.86	− 10	−5	+13	−10	−6	− 8	− 18	+ 2	− 455	+ 373
5	1.90	.87	+ 10	−5	+14	− 6	−6	− 3	− 61	+ 9	− 516	+ 382
6	1.76	.80	− 70	−5	+14	− 1	−6	+ 2				
7	1.82	.80	0	−5	+14	0	−6	+ 3	+ 3	+ 3	− 513	+ 385
8	1.79	.79	− 10	−5	+13	− 1	−6	+ 1	− 9	+ 4	− 522	+ 389
9	1.76	.75	− 40	−5	+13	− 5	−6	− 3	− 43	+ 3	− 565	+ 392
10	1.80	.76	+ 10	−5	+13	− 5	−6	− 3	+ 7	+ 2	− 558	+ 394
11	1.84	.78	+ 20	−5	+13	− 1	−6	+ 1	− 38	+ 5	− 596	+ 399
12	1.76	.72	− 60	−5	+13	− 1	−6	+ 1				
13	1.64	.71	− 10	−5	+13	− 3	−6	− 1	− 11	+ 3	− 607	+ 402
14	1.68	.73	+ 20	−5	+13	−11	−6	− 9	+ 11	+ 5	− 596	+ 407
15	1.74	.74	+ 10	−5	+13	−17	−6	−15	− 5	+ 3	− 601	+ 410
		579.+										
16	1.77	1.81	+ 70	−5	+14	−10	−6	− 7	− 36	+ 7	− 637	+ 417
17	1.75	1.71	−100	−5	+13	− 1	−6	+ 1				
18	1.80	1.77	+ 60	−5	+13	0	−6	+ 2	− 6	+ 7	− 643	+ 424
19	1.71	1.70	− 70	−5	+13	0	−6	+ 2				
20	1.70	1.69	− 10	−5	+13	0	−6	+ 2	− 8	+ 3	− 651	+ 427

TABLE 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 580.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_c$)				Net rise	Computed evaporation
1909	(*ft.*)	(*ft.*)										
Sept. 21	1.69	1.69	0	−5	+13	− 9	−6	− 7	− 7	+ 4	− 658	+ 431
22	1.70	1.78	+ 90	−5	+13	−29	−6	−27	− 17	+15	− 675	+ 446
23	1.78	1.73	− 50	−5	+13	−32	−6	−30				
24	1.77	1.76	+ 30	−5	+13	−15	−6	−13	+ 17	+ 3	− 658	+ 449
25	1.79	1.74	− 20	−5	+13	− 3	−6	− 1	− 21	+ 2	− 679	+ 451
26*	1.83	1.67	− 70	−5	+14	− 2	−6	+ 1	− 69*	....	....	....
27*	1.73	1.61	− 60	−5	+13	− 2	−6	0	− 60*	....	....	....
28	1.67	1.65	+ 40	−5	+13	− 2	−6	0	− 51	+ 4	− 730	+ 455
29	1.69	1.57	− 80	−5	+13	−13	−6	−11				
30	1.70	1.55	− 20	−6	+14	−19	−6	−17	− 37	+ 3	− 767	+ 458
Oct. 1	1.65	1.60	+ 50	−5	+13	− 8	−6	− 6	+ 44	+ 2	− 723	+ 460
2	1.62	1.58	− 20	−5	+13	0	−6	+ 2	− 18	+ 2	− 741	+ 462
3	1.59	1.56	− 20	−5	+13	0	−6	+ 2	− 18	+ 2	− 759	+ 464
4	1.58	1.55	− 10	−5	+13	0	−6	+ 2	− 8	+ 2	− 767	+ 466
5	1.56	1.54	− 10	−5	+13	0	−6	+ 2	− 8	+ 2	− 775	+ 468
6	1.53	1.52	− 20	−5	+13	0	−6	+ 2	− 18	+ 2	− 793	+ 470
7	1.53	1.52	0	−5	+13	0	−6	+ 2	+ 2	+ 2	− 791	+ 472
8	1.48	1.49	− 30	−5	+13	0	−6	+ 2	− 28	+ 3	− 819	+ 475
9	1.51	1.49	0	−5	+13	− 1	−6	+ 1	+ 1	+ 4	− 818	+ 479
10	1.47	1.48	− 10	−5	+13	− 7	−6	− 5	− 15	+ 3	− 833	+ 482
11	1.40	1.53	+ 50	−5	+13	−16	−6	−14	− 78	+47	− 911	+ 529
12	1.25	1.60	+ 70	−5	+12	−17	−6	−16				
13	1.13	1.44	−160	−6	+13	− 9	−6	− 8				
14	1.19	(—)	....	−6	+13	− 6	−6	− 5	− 11	+25	− 922	+ 554
15	1.26	1.44	0	−6	+13	− 7	−6	− 6				
16	1.31	1.42	− 20	−6	+13	− 6	−6	− 5	− 25	+10	− 947	+ 564
17	1.36	1.38	− 40	−5	+13	− 4	−6	− 2	− 42	+ 3	− 989	+ 567
18	1.38	1.36	− 20	−5	+13	− 2	−6	0	− 20	+ 1	−1009	+ 568
19	1.36	1.36	0	−5	+13	− 2	−6	0	0	+ 2	−1009	+ 570
20*	1.31	1.30	− 60	−5	+13	− 2	−6	0	− 60*	....	....	....
21	1.26	1.41	+110	−5	+12	−18	−6	−17	− 25	+ 5	−1034	+ 575
22	1.32	1.31	−100	−5	+13	−20	−6	−18				
23	1.37	1.30	− 10	−5	+13	− 6	−6	− 4	− 14	+ 6	−1048	+ 581
24	1.33	1.30	0	−5	+13	− 2	−6	0	0	+ 5	−1048	+ 586
25	1.25	1.28	− 20	−5	+12	− 2	−6	−1	− 21	+ 2	−1069	+ 588
26	1.24	1.26	− 20	−5	+13	− 3	−6	−1	− 21	+ 4	−1090	+ 592
27	1.22	1.25	− 10	−5	+13	− 2	−6	0	− 10	+10	−1100	+ 602

28	1.23	1.30	+ 50	−5	+13	− 1	−6	+1	− 39	+ 6	−1139	+ 608
29	1.23	1.21	− 90	−5	+13	− 2	−6	0				
30	1.18	1.18	− 30	−5	+12	− 2	−6	−1	− 31	+ 2	−1170	+ 610
31	1.20	1.18	0	−5	+13	− 1	−6	+1	+ 1	+ 5	−1169	+ 615
1910												
May 1	1.37	1.35		−5	+13							
2	1.42	1.38	+ 30	−5	+12	−18	−6	−18	+ 13	+ 2	+ 13	+ 2
3	1.42	1.39	+ 10	−5	+13	−18	−7	−17	− 7	+15	+ 6	+ 17
4	1.40	1.38	− 10	−5	+13	− 1	−6	+ 1	− 9	+ 9	− 3	+ 26
5	1.40	1.37	− 10	−5	+13	0	−7	+ 1	− 9	+ 3	− 12	+ 29
6	1.39	1.37	0	−5	+13	0	−6	+ 2	+ 2	+ 4	− 10	+ 33
7	1.40	1.37	0	−5	+13	0	−7	+ 1	+ 1	+ 4	− 9	+ 37
8	1.41	1.38	+ 10	−5	+12	− 2	−6	− 1	+ 9	+ 3	0	+ 40
9	1.37	1.36	− 20	−5	+12	− 2	−7	− 2	− 22	+ 2	− 22	+ 42
10	1.38	1.37	+ 10	−5	+13	0	−6	+ 2	+ 12	+ 9	− 10	+ 51
11	1.37	1.35	− 20	−5	+13	0	−7	+ 1	− 19	+ 9	− 29	+ 60
12	1.39	1.36	+ 10	−5	+13	0	−6	+ 2	+ 12	+ 6	− 17	+ 66
13	1.39	1.35	− 10	−5	+13	0	−7	+ 1	− 9	+ 3	− 26	+ 69
14	1.37	1.33	− 20	−4	+13	0	−6	+ 3	− 17	+ 2	− 43	+ 71
15	1.35	1.29	− 40	−4	+13	0	−7	+ 2	− 38	+ 3	− 81	+ 74
16	1.35	1.31	+ 20	−4	+12	0	−6	+ 2	+ 22	+ 3	− 59	+ 77
17	1.34	1.33	+ 20	−4	+12	−18	−7	−17	+ 3	+16	− 56	+ 93
18*	1.34	1.45	+120	−5	+13	−22	−6	−20	+100*			
19	1.38	1.46	+ 10	−4	+13	− 5	−7	− 3	+ 7	+22	− 49	+ 115
20*	1.39	1.37	− 90	−5	+12	− 4	−6	− 3	− 93*			
21	1.40	1.37	0	−5	+13	− 9	−7	− 8	− 8	+ 2	− 57	+ 117
22	1.43	1.40	+ 30	−5	+12	−18	−6	−17	+ 13	+ 3	− 44	+ 120
23	1.46	1.43	+ 30	−5	+13	−34	−7	−33	− 3	+ 1	− 47	+ 121
24	1.48	1.47	+ 40	−5	+13	−24	−6	−22	+ 18	+ 2	− 29	+ 123
25	1.44	1.43	− 40	−5	+13	− 3	−7	− 2	− 42	+ 3	− 71	+ 126
26	1.47	1.46	+ 30	−5	+13	− 1	−6	+1	+31	+ 2	− 40	+ 128
27	1.49	1.46	0	−5	+13	− 1	−7	0	0	+ 3	− 40	+ 131
28	1.47	1.44	− 20	−5	+13	− 2	−6	0	− 20	+ 4	− 60	+ 135
29	1.47	1.49	+ 50	−5	+13	−14	−7	−13	+ 37	+ 6	− 23	+ 141
30	1.49	1.48	− 10	−5	+13	−16	−6	−14	− 24	+14	− 47	+ 155
31	1.50	1.47	− 10	−5	+13	−10	−7	− 9	− 19	+ 5	− 66	+ 160
June 1	1.48	1.46	− 10	−5	+13	−13	−7	−12	− 22	+ 5	− 88	+ 165
2	1.48	1.46	0	−5	+13	−13	−6	−11	− 11	+ 3	− 99	+ 168
3	1.52	1.48	+ 20	−5	+13	− 6	−7	− 5	+ 15	+ 2	− 84	+ 170
4	1.50	1.48	0	−5	+13	− 1	−6	+ 1	+ 1	+ 2	− 83	+ 172
5	1.52	1.49	+ 10	−5	+13	−10	−7	− 9	+ 1	+ 2	− 82	+ 174
6	1.51	1.51	+ 20	−5	+13	−13	−6	−11	+ 9	+ 2	− 73	+ 176
7	1.51	1.51	0	−5	+13	− 4	−7	− 3	− 3	+ 3	− 76	+ 179
8	1.52	1.50	− 10	−5	+13	0	−6	+ 2	− 8	+ 4	− 84	+ 183
9	1.53	1.51	+ 10	−5	+13	0	−7	+ 1	+ 11	+ 5	− 73	+ 188
10	1.55	1.52	+ 10	−5	+13	0	−6	+ 2	+ 12	+ 5	− 61	+ 193

* Rejected.

Table 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 579.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_c$)				Net rise	Computed evaporation
1910	(*ft.*)	(*ft*)										
June 11	1.55	1.53	+ 10	−5	+13	− 7	−7	− 6	+ 4	+ 6	− 57	+ 199
12	1.56	1.53	0	−5	+13	−10	−6	− 8	− 8	+ 5	− 65	+ 204
13	1.53	1.52	− 10	−5	+13	− 3	−7	− 2	− 12	+ 6	− 77	+ 210
14	1.55	1.52	0	−5	+13	0	−6	+ 2	+ 2	+ 7	− 75	+ 217
15	1.55	1.53	+ 10	−5	+13	0	−7	+ 1	+ 11	+ 7	− 64	+ 224
16	1.52	1.49	− 40	−5	+13	0	−6	+ 2	− 38	+ 8	− 102	+ 232
17	1.52	1.50	+ 10	−5	+13	− 6	−7	− 5	+ 5	+ 7	− 97	+ 239
18	1.54	1.52	+ 20	−5	+13	− 8	−6	− 6	+ 14	+ 6	− 83	+ 245
19	1.52	1.50	− 20	−5	+13	− 1	−7	0	− 20	+ 8	− 103	+ 253
20	1.52	1.51	+ 10	−5	+13	0	−6	+ 2	+ 12	+ 6	− 91	+ 259
21	1.53	1.52	+ 10	−5	+13	0	−7	+ 1	+ 11	+ 6	− 80	+ 265
22	1.52	1.51	− 10	−5	+13	0	−6	+ 2	− 8	+ 7	− 88	+ 272
23	1.52	1.54	+ 30	−5	+13	0	−7	+ 1	+ 31	+ 6	− 57	+ 278
24	1.48	1.49	− 50	−5	+13	− 1	−6	+ 1	− 49	+ 6	− 106	+ 284
25	1.48	1.49	0	−5	+13	0	−7	+ 1	+ 1	+ 6	− 105	+ 290
26	1.52	1.50	+ 10	−5	+13	− 1	−6	+ 1	+ 11	+ 7	− 94	+ 297
27	1.51	1.50	0	−5	+13	− 3	−7	− 1	− 1	+ 5	− 95	+ 302
28	1.52	1.50	0	−5	+13	− 3	−6	− 1	− 1	+ 7	− 96	+ 309
29	1.51	1.48	− 20	−5	+13	0	−7	+ 1	− 19	+ 7	− 115	+ 316
30	1.50	1.52	+ 40	−5	+13	0	−6	+ 2	+ 42	+ 9	− 73	+ 325
July 1	1.49	1.51	− 10	−5	+12	0	−7	0	− 10	+10	− 83	+ 335
2	1.47	1.48	− 30	−5	+12	0	−6	+ 1	− 29	+11	− 112	+ 346
3	1.50	1.48	0	−5	+12	0	−7	0	0	+ 7	− 112	+ 353
4	1.51	1.47	− 10	−5	+12	0	−6	+ 1	− 9	+11	− 121	+ 364
5	1.50	1.49	+ 20	−5	+12	0	−7	0	+ 20	+ 6	− 101	+ 370
6	1.47	1.46	− 30	−5	+12	− 2	−6	− 1	− 31	+ 6	− 132	+ 376
7	1.46	1.46	0	−5	+12	− 3	−7	− 3	− 3	+ 5	− 135	+ 381
8	1.45	1.45	− 10	−5	+12	0	−6	+ 1	− 9	+ 9	− 144	+ 390
9	1.43	1.42	− 30	−5	+12	− 4	−7	− 4	− 34	+ 9	− 178	+ 399
10	1.45	1.44	+ 20	−5	+12	−15	−6	−14	+ 6	+ 6	− 172	+ 405
11	1.46	1.45	+ 10	−5	+12	−12	−7	−12	− 1	+ 7	− 173	+ 412
12	1.45	1.43	− 20	−5	+12	−15	−6	−14	− 34	+ 5	− 207	+ 417
13	1.48	1.46	+ 30	−5	+12	−15	−7	−15	+ 15	+ 5	− 192	+ 422
14	1.44	1.43	− 30	−5	+12	0	−6	+ 1	− 29	+ 6	− 221	+ 428
15	1.45	1.45	+ 20	−5	+12	− 2	−7	− 2	+ 18	+ 6	− 203	+ 434
16	1.46	1.43	− 20	−5	+12	− 7	−6	− 6	− 26	+ 4	− 229	+ 438
17	1.47	1.43	0	−5	+12	− 6	−7	− 6	− 6	+15	− 235	+ 453

18	1.47	1.43	0	−5	+12	0	−6	+ 1	+ 1	+11	− 234	+ 464
19	1.46	1.42	− 10	−5	+12	0	−7	0	− 10	+ 6	− 244	+ 470
20	1.42	1.41	− 10	−5	+12	− 1	−6	0	− 10	+ 7	− 254	+ 477
21	1.41	1.41	0	−5	+12	− 5	−7	− 5	− 5	+ 9	− 259	+ 486
22	1.43	1.42	+ 10	−5	+12	− 9	−6	− 8	+ 2	+ 7	− 257	+ 493
23	1.42	1.40	− 20	−4	+12	− 6	−7	− 5	− 25	+ 6	− 282	+ 499
24	1.29	1.43	+ 30	−5	+12	− 3	−6	− 2	+ 28	+10	− 254	+ 509
25	1.38	1.41	− 20	−5	+12	− 3	−7	− 3	− 23	+29	− 277	+ 538
26	1.39	1.38	− 30	−5	+12	− 4	−6	− 3	− 33	+ 7	− 310	+ 545
27	1.41	1.40	+ 20	−5	+12	− 3	−7	− 3	+ 17	+ 5	− 293	+ 550
28	1.40	1.38	− 20	−5	+12	0	−6	+ 1	− 19	+ 7	− 312	+ 557
29	1.40	1.40	+ 20	−5	+12	− 1	−7	− 1	+ 19	+ 6	− 293	+ 563
30	1.40	1.39	− 10	−5	+12	− 2	−6	− 1	− 11	+ 6	− 304	+ 569
31	1.39	1.37	− 20	−5	+12	− 1	−7	− 1	− 21	+ 5	− 325	+ 574
Aug. 1	1.36	1.37	0	−5	+13	− 4	−7	− 3	− 3	+ 5	− 328	+ 579
2	1.36	1.37	0	−5	+13	− 4	−6	− 2	− 2	+ 5	− 330	+ 584
3	1.35	1.38	+ 10	−5	+12	−12	−7	−12	− 2	+ 5	− 332	+ 589
4	1.38	1.42	+ 40	−5	+13	−14	−6	−12				
5	1.48	1.30	−120	−5	+13	− 4	−7	− 3	− 65	+51	− 397	+ 640
6	1.34	1.33	+ 30	−5	+13	− 2	−6	0				
7	1.33	1.31	− 20	−5	+13	0	−7	+ 1	− 19	+ 4	− 416	+ 644
8	1.32	1.31	0	−5	+13	− 2	−6	0	0	+ 4	− 416	+ 648
9	1.30	1.30	− 10	−5	+12	−14	−7	−14	− 24	+ 5	− 440	+ 653
10	1.31	1.30	0	−5	+13	−15	−6	−13	− 13	+ 5	− 453	+ 658
11	1.34	1.31	+ 10	−5	+13	− 2	−7	− 1	+ 9	+ 5	− 444	+ 663
12	1.28	1.30	− 10	−5	+12	− 1	−6	0	− 10	+ 5	− 454	+ 668
13	1.26	1.26	− 40	−5	+12	− 3	−7	− 3	− 43	+ 7	− 497	+ 675
14	1.27	1.27	+ 10	−5	+13	− 6	−6	− 4	+ 6	+ 6	− 491	+ 681
15	1.27	1.24	− 30	−5	+13	− 6	−7	− 5	− 35	+ 4	− 526	+ 685
16	1.32	1.29	+ 50	−5	+13	−10	−6	− 8	+ 42	+ 3	− 484	+ 688
17	1.30	1.28	− 10	−5	+13	−13	−7	−12	− 22	+ 3	− 506	+ 691
18	1.34	1.32	+ 40	−5	+13	− 6	−6	− 4	+ 36	+ 4	− 470	+ 695
19	1.32	1.29	− 30	−5	+13	− 2	−7	− 1	− 31	+ 6	− 501	+ 701
20	1.27	1.28	− 10	−5	+12	− 1	−6	0	− 10	+ 6	− 511	+ 707
21	1.25	1.26	− 20	−5	+12	− 1	−7	− 1	− 21	+ 6	− 532	+ 713
22	1.23	1.24	− 20	−5	+12	−16	−6	−15	− 35	+ 5	− 567	+ 718
23*	1.30	(—)	....	−5	+13	−37	−7	−36	....	....	....	....
24*	1.28	1.23	− 10	−5	+13	−24	−6	−22	− 32*	....	....	....
25	1.27	1.29	+ 60	−5	+13	−60	−7	−59	+ 1	0	− 566	+ 718
26	1.22	(—)	....	−5	+13	−59	−6	−57	− 26	+28	− 592	+ 746
27	1.27	1.32	+ 30	−5	+13	0	−7	+ 1				
28	1.30	1.30	− 20	−5	+13	0	−6	+ 2	− 18	+ 5	− 610	+ 751
29	1.30	1.26	− 40	−5	+13	0	−7	+ 1	− 39	+ 4	− 649	+ 755
30	1.35	1.28	+ 20	−5	+13	− 2	−6	0	+ 20	+ 5	− 629	+ 760
31	1.24	1.28	0	−5	+13	− 4	−7	− 3	− 3	+ 3	− 632	+ 763
Sept. 1	1.26	1.26	− 20	−5	+12	− 1	−7	− 1	− 21	+ 5	− 653	+ 768
2	1.27	1.26	0	−5	+12	0	−6	+ 1	+ 1	+ 4	− 652	+ 772

* Rejected.

TABLE 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 579.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_c$)				Net rise	Computed evaporation
1910	(*ft.*)	(*ft.*)										
Sept. 3	1.21	1.21	− 50	−5	+12	− 7	−7	− 7	− 57	+ 2	− 709	+ 774
4	1.23	1.23	+ 20	−5	+13	−10	−6	− 8	− 60	+ 5	− 769	+ 779
5	1.19	1.19	− 40	−5	+12	−33	−7	−33				
6	1.20	1.23	+ 40	−5	+12	−40	−6	−39				
7	1.17	1.24	+ 10	−5	+12	−10	−7	−10	0	+ 4	− 769	+ 783
8	1.29	1.31	+ 70	−5	+13	− 6	−6	− 4	+ 66	+ 3	− 703	+ 786
9	1.35	1.31	0	−5	+13	− 7	−7	− 1	− 6	+ 8	− 709	+ 794
10	1.31	1.31	0	−5	+13	− 1	−6	+ 1	+ 1	+ 4	− 708	+ 798
11	1.31	1.30	− 10	−5	+13	− 2	−7	− 1	− 11	+ 4	− 719	+ 802
12	1.32	1.29	− 10	−5	+13	−27	−6	−25	− 35	+ 2	− 754	+ 804
13	1.32	1.30	+ 10	−5	+13	−25	−7	−24	− 14	+ 3	− 768	+ 807
14	1.29	1.29	− 10	−5	+13	0	−6	+ 2	− 8	+ 4	− 776	+ 811
15	1.29	1.28	− 10	−5	+13	0	−7	+ 1	− 9	+ 3	− 785	+ 814
16	1.28	1.27	− 10	−5	+12	0	−6	+ 1	− 9	+ 4	− 794	+ 818
17	1.27	(—)	....	−5	+12	− 9	−7	− 9	− 60	+14	− 854	+ 832
18	1.32	1.21	− 60	−5	+13	−11	−6	− 9				
19	1.26	1.23	+ 20	−5	+12	− 2	−7	− 2				
20	1.22	1.23	0	−5	+12	0	−6	+ 1	+ 1	+ 2	− 853	+ 834
21	1.24	1.21	− 20	−5	+13	0	−7	+ 1	− 19	+ 3	− 872	+ 837
22	1.22	1.19	− 20	−4	+12	0	−6	+ 2	− 18	+ 3	− 890	+ 840
23	1.23	1.17	− 20	−5	+12	− 7	−7	− 7	− 27	+ 2	− 917	+ 842
24	1.19	1.13	− 40	−4	+12	−27	−6	−25	− 57	+ 8	− 974	+ 850
25	1.20	1.16	+ 30	−5	+12	−22	−7	−22				
26	1.21	1.20	+ 40	−4	+12	− 5	−6	− 3	+ 37	+ 2	− 937	+ 852
27	1.17	1.23	+ 30	−5	+12	− 9	−7	− 9	+ 21	+ 2	− 916	+ 854
28	1.19	1.21	− 20	−5	+13	− 5	−6	− 3	− 23	+ 3	− 939	+ 857
29	1.16	1.19	− 20	−4	+12	0	−7	+ 1	− 19	+ 3	− 958	+ 860
30	1.09	1.19	0	−5	+12	0	−6	+ 1	+ 1	+ 3	− 957	+ 863
Oct. 1	1.16	1.26	+ 70	−5	+13	− 2	−7	− 1	− 68	+34	−1025	+ 897
2	1.16	1.13	−130	−4	+13	−10	−6	− 7				
3	1.13	1.16	+ 30	−4	+12	−16	−7	−15	+ 15	+ 9	−1010	+ 906
4	1.15	1.17	+ 10	−5	+12	−31	−6	−30	− 20	+ 6	−1030	+ 912
5	1.19	1.21	+ 40	−5	+12	−29	−7	−29	+ 11	+ 1	−1019	+ 913
6	1.23	1.29	+ 80	−5	+13	−15	−6	−13	− 3	+ 9	−1022	+ 922
7	1.23	1.23	− 60	−4	+12	−11	−7	−10				
8	1.16	1.21	− 20	−5	+12	− 3	−6	− 2	− 22	+ 3	−1044	+ 925
9*	1.15	1.27	+ 60	−5	+13	0	−7	+ 1	+ 61*	....	....	....

10*	1.13	1.11	-160	-5	+12	0	-6	+1	-159*	......	......	......
11*	1.11	1.11	0	-5	+12	0	-7	0	0*	......	......	......
12*	1.19	1.23	+120	-5	+12	0	-6	+1	+121*	......	......	......
13*	1.08	1.11	-120	-4	+12	0	-7	+1	-119*	......	......	......
14	1.07	1.06	-50	-5	+12	0	-6	+1	-49	+1	-1093	+926
15	1.06	1.06	0	-5	+12	0	-7	0	0	+3	-1093	+929
16	1.06	1.04	-20	-5	+12	0	-6	+1	-19	+2	-1112	+931
17	1.04	1.02	-20	-4	+12	0	-7	+1	-19	+5	-1131	+936
18	1.06	1.04	+20	-5	+12	0	-6	+1	+21	+4	-1110	+940
19	1.03	1.04	0	-5	+12	-1	-7	-1	-1	+4	-1111	+944
20	1.12	1.01	-30	-5	+12	-6	-6	-5	-35	+7	-1146	+951
{21	1.07	1.14	+130	-4	+12	-23	-7	-22}	-74	+26	-1220	+977
22	1.00	.99	-150	-5	+12	-31	-6	-30				
23	1.05	1.05	+60	-5	+13	-13	-7	-12				
24}	1.02	1.00	-50	-5	+12	-1	-6	0}				
25	1.01	1.01	+10	-5	+12	0	-7	0	+10	+2	-1210	+979
26	1.03	1.00	-10	-5	+12	-9	-6	-8	-18	+3	-1228	+982
27*	.99	.96	-40	-5	+12	-23	-7	-23	-63*	......	......	......
28	1.02	.95	-10	-5	+13	-19	-6	-17	-27	+5	-1255	+987
29	1.03	.99	+40	-5	+12	-9	-7	-9	+31	+1	-1224	+988
30	.93	.98	-10	-5	+12	-4	-6	-3	-13	+5	-1237	+993
31*	.93	.90	-80	-5	+12	0	-7	0	-80*	......	......	......
1911												
May 1*	.59	.43	......	-4	+12	......	......	......	......	......	......	......
2	.62	.44	+10	-4	+12	-25	-6	-23	-13	+10	-13	+10
3*	.58	.60	+160	-4	+12	-1	-6	+1	+161*	......	......	......
4	.58	.59	-10	-4	+12	0	-6	+2	-8	+3	-21	+13
5	.59	.59	0	-4	+12	0	-6	+2	+2	+3	-19	+16
6	.60	.61	+20	-3	+12	0	-6	+3	+23	+3	+4	+19
7	.62	.59	-20	-3	+12	0	-6	+3	-17	+5	-13	+24
8	.62	.62	+30	-4	+12	-2	-6	0	+30	+5	+17	+29
9	.61	.61	-10	-4	+12	-3	-6	-1	-11	+3	+6	+32
10	.63	.61	0	-3	+12	-9	-6	-6	-6	+4	0	+36
11*	.52	(—)	......	-4	+11	-10	-6	-9	......	......	......	......
12*	.57	.63	+20	-4	+12	-12	-6	-10	+1*	......	......	......
13	.66	.64	+10	-4	+12	0	-6	+2	+12	+8	+12	+44
14	.64	.65	+10	-3	+12	-1	-6	+2	+12	+3	+24	+47
15	.65	.65	0	-4	+12	-12	-6	-10	-10	+4	+14	+51
16	.68	.66	+10	-4	+12	-31	-6	-29	-19	+3	-5	+54
17	.71	.69	+30	-4	+12	-31	-6	-29	+1	+3	-4	+57
18	.75	.72	+30	-4	+12	-18	-6	-16	+14	+3	+10	+60
19*	.77	.78	+60	-4	+12	-9	-6	-7	+53*	......	......	......
20	.77	.77	-10	-4	+12	-10	-6	-8	-18	+20	-8	+80
21	.79	.78	+10	-4	+12†	-22	-6	-20	-10	+5	-18	+85
22	.83	.80	+20	-4	+12†	-33	-6	-31	-11	+2	-29	+87
23	.87	.86	+60	-4	+12	-26	-6	-24	+36	+2	+7	+89
24	.88	.87	+10	-4	+12	-6	-6	-4	+6	+2	+13	+91
25	.87	.84	-30	-4	+12	0	-6	+2	-28	+4	-15	+95

* Rejected. † Values assumed.

TABLE 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 579.+	Rise of mean lake surface (I_1)	Correction for— Inflow from Superior ($-I_3$)	Correction for— Outflow to Erie ($+I_4$)	Correction for— Rainfall on lake ($-I_2$)	Correction for— Run-off from land ($-I_c$)	Total correction	Net rise (I)	Computed evaporation	Accumulated sum of— Net rise	Accumulated sum of— Computed evaporation
1911	(*ft.*)	(*ft.*)										
May 26	.89	.87	+ 30	−4	+12	0	−6	+ 2	+ 32	+ 5	+ 17	+ 100
27	.90	.89	+ 20	−4	+12	0	−6	+ 2	+ 22	+ 7	+ 39	+ 107
28	.88	.89	0	−4	+12	− 2	−6	0	0	+ 5	+ 39	+ 112
29	.90	.89	0	−4	+12	− 2	−6	0	0	+ 6	+ 39	+ 118
30	.89	.87	− 20	−4	+12	− 3	−6	− 1	− 21	+ 5	+ 18	+ 123
31	.87	.88	+ 10	−4	+12	−17	−6	−15	− 5	+ 3	+ 13	+ 126
June 1	.92	.93	+ 50	−4	+12	−16	−6	−14	+ 36	+ 5	+ 49	+ 131
2	.91	.92	− 10	−4	+12	− 5	−6	− 3	− 13	+ 3	+ 36	+ 134
3	.92	.90	− 20	−4	+12	−13	−6	−11	− 31	+ 4	+ 5	+ 138
4	.95	.92	+ 20	−4	+12	−23	−6	−21	− 1	+ 4	+ 4	+ 142
5	.96	.93	+ 10	−4	+12	−18	−6	−16	− 6	+ 2	− 2	+ 144
6	.97	.94	+ 10	−4	+12	−11	−6	− 9	+ 1	+ 3	− 1	+ 147
7	.96	.94	0	−4	+12	− 7	−6	− 5	− 5	+ 3	− 6	+ 150
8	.99	.96	+ 20	−4	+12	− 1	−6	+ 1	+ 21	+ 3	+ 15	+ 153
9	.98	.95	− 10	−4	+12	− 5	−6	− 3	− 13	+ 4	+ 2	+ 157
10	.98	.97	+ 20	−4	+12	− 7	−6	− 5	+ 15	+ 6	+ 17	+ 163
11	1.00	1.01	+ 40	−4	+12	− 9	−6	− 7	+ 33	+ 5	+ 50	+ 168
12	1.02	1.04	+ 30	−4	+12	−17	−5	−15	+ 15	+ 5	+ 65	+ 173
13	1.03	1.02	− 20	−4	+12	−12	−6	−10	− 30	+ 3	+ 35	+ 176
14	1.03	1.03	+ 10	−4	+12	− 2	−6	0	+ 10	+ 5	+ 45	+ 181
15	1.03	1.02	− 10	−4	+12	0	−6	+ 2	− 8	+ 6	+ 37	+ 187
16	1.04	1.04	+ 20	−4	+12	− 2	−6	0	+ 20	+ 5	+ 57	+ 192
17	1.06	1.06	+ 20	−4	+12	− 5	−6	− 3	+ 17	+ 3	+ 74	+ 195
18	1.04	1.03	− 30	−4	+12	− 3	−6	− 1	− 31	+ 4	+ 43	+ 199
19	1.03	1.04	+ 10	−4	+12	0	−6	+ 2	+ 12	+ 5	+ 55	+ 204
20	1.05	1.04	0	−4	+12	0	−6	+ 2	+ 2	+ 6	+ 57	+ 210
21	1.05	1.04	0	−4	+12	0	−6	+ 2	+ 2	+ 8	+ 59	+ 218
22	1.04	1.05	+ 10	−4	+12	− 1	−0	+ 1	+ 11	+ 7	+ 70	+ 225
23	1.01	1.06	+ 10	−4	+12	− 1	−6	+ 1	+ 11	+ 7	+ 81	+ 232
24	1.01	1.10	+ 40	−4	+12	− 6	−6	− 4	− 61	+10	+ 20	+ 242
25	1.03	1.01	− 90	−4	+12	− 9	−6	− 7				
26	1.05	1.02	+ 10	−4	+12	− 9	−6	− 7	+ 3	+ 3	+ 23	+ 245
27	1.00	1.00	− 20	−4	+12	− 7	−6	− 5	− 25	+ 2	− 2	+ 247
28	1.02	1.01	+ 10	−4	+12	− 2	−6	0	+ 10	+ 8	+ 8	+ 255
29	1.02	.99	− 20	−4	+12	0	−6	+ 2	− 18	+ 4	− 10	+ 259
30	1.01	.99	0	−4	+12	0	−6	+ 2	+ 2	+ 4	− 8	+ 263
July 1	1.00	1.00	+ 10	−4	+12	0	−6	+ 2	+ 12	+ 8	+ 4	+ 271

2	1.00	1.01	+10	−4	+12	0	−6	+2	+12	+12	+16	+283
3	1.01	1.01	0	−4	+12	0	−6	+2	+2	+11	+18	+294
4	1.00	.99	−20	−4	+12	−1	−6	+1	−19	+10	−1	+304
5	.99	.98	−10	−4	+12	−4	−6	−2	−12	+9	−13	+313
6	1.02	1.02	+40	−4	+12	−8	−6	−6	+34	+7	+21	+320
7	1.03	1.00	−20	−4	+12	−5	−6	−3	−23	+7	−2	+327
8	1.01	1.00	0	−4	+12	0	−6	+2	+2	+5	0	+332
9	.97	.99	−10	−4	+12	−2	−6	0	−10	+7	−10	+339
10	.98	.97	−20	−4	+12	−5	−6	−3	−23	+8	−33	+347
11	.96	1.00	+30	−5	+12	−4	−6	−3	+27	+8	−6	+355
12	.96	.99	−10	−5	+12	0	−6	+1	−9	+9	−15	+364
13	.95	.97	−20	−4	+12	−1	−6	+1	−19	+7	−34	+371
14	.96	.98	+10	−4	+12	−3	−6	−1	+9	+8	−25	+379
15	.94	.95	−30	−4	+12	−8	−6	−6	−36	+8	−61	+387
16	.94	.94	−10	−5	+12	−12	−6	−11	−21	+5	−82	+392
17	.94	.94	0	−4	+12	−8	−6	−6	−6	+5	−88	+397
18	.91	.91	−30	−4	+12	−4	−6	−2	−32	+6	−120	+403
19	.89	.90	−10	−4	+12	−13	−6	−11	−21	+5	−141	+408
20	.91	.93	+30	−4	+12	−11	−6	−9	+21	+5	−120	+413
21	.87	.90	−30	−5	+12	−2	−6	−1	−31	+5	−151	+418
22	.88	.89	−10	−5	+12	−1	−6	0	−10	+6	−161	+424
23	.88	.90	+10	−4	+12	−14	−6	−12	−2	+5	−163	+429
24	.78	.88	−20	−6	+12	−32	−6	−32	−52	+16	−215	+445
25	.85	.82	−60	−5	+13	−21	−6	−19	−21	+35	−236	+480
26	87	.88	+60	−5	+12	−3	−6	−2				
27	.86	.86	−20	−5	+12	0	−6	+1	−19	+5	−255	+485
28	.85	.86	0	−5	+12	−1	−6	0	0	+6	−255	+491
29	.85	.86	0	−5	+12	−2	−6	−1	−1	+5	−256	+496
30	.85	.86	0	−5	+12	−3	−6	−2	−2	+6	−258	+502
31	.86	.87	+10	−5	+12	−7	−6	−6	+4	+6	−254	+508
Aug. 1	.85	.86	−10	−5	+12	−16	−6	−15	−25	+10	−279	+518
2	.87	.87	+10	−5	+12	−13	−6	−12	−2	+5	−281	+523
3	.85	.84	−30	−5	+12	−3	−6	−2	−32	+5	−313	+528
4	.86	.85	+10	−5	+12	−5	−6	−4	+6	+3	−307	+531
5	.86	.87	+20	−5	+12	−5	−6	−4	+16	+5	−291	+536
6	.87	.87	0	−5	+12	−2	−6	−1	−1	+6	−292	+542
7	.87	.87	0	−5	+12	−10	−6	−9	−9	+5	−301	+547
8	.85	.86	−10	−5	+12	−13	−6	−12	−22	+10	−323	+557
9	.86	.87	+10	−5	+12	−6	−6	−5	+5	+6	−318	+563
10	.90	.88	+10	−5	+12	−7	−6	−6	+4	+6	−314	+569
11	.92	.89	+10	−5	+12	−11	−6	−10	0	+4	−314	+573
12	.89	.88	−10	−5	+12	−9	−6	−8	−18	+5	−332	+578
13	.89	.88	0	−5	+12	−6	−6	−5	−5	+4	−337	+582
14	.89	.89	+10	−5	+12	−7	−6	−6	+4	+3	−333	+585
15	.88	.89	0	−5	+12	−15	−6	−14	−14	+3	−347	+588
16	.90	.91	+20	−5	+12	−24	−6	−23	−3	+5	−350	+593
17	.91	.92	+10	−5	+12	−12	−6	−11	−1	+5	−351	+598

* Rejected.

TABLE 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 579.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_c$)				Net rise	Computed evaporation
1911	(*ft.*)	(*ft.*)										
Aug. 18	.90	.90	− 20	−5	+12	− 2	−6	− 1	− 21	+ 6	− 372	+ 604
19	.91	.90	0	−5	+12	− 1	−6	0	0	+ 6	− 372	+ 610
20	.89	.89	− 10	−5	+12	0	−6	+ 1	− 9	+ 5	− 381	+ 615
21	.85	.87	− 20	−5	+12	− 4	−6	− 3	− 23	+ 5	− 404	+ 620
22	.84	.89	+ 20	−5	+12	−13	−6	−12	+ 8	+14	− 396	+ 634
23	.84	.84	− 50	−5	+12	− 9	−6	− 8	− 58	+ 5	− 454	+ 639
24	.81	.80	− 40	−5	+12	− 1	−6	0	− 40	+ 4	− 494	+ 643
25	.81	.81	+ 10	−5	+12	− 1	−6	0	+ 10	+ 4	− 484	+ 647
26	.79	.79	− 20	−5	+12	0	−6	+ 1	− 19	+ 4	− 503	+ 651
27	.76	.77	− 20	−5	+12	− 1	−6	0	− 20	+ 5	− 523	+ 656
28	.73	.73	− 40	−5	+12	− 7	−6	− 6	− 46	+ 4	− 569	+ 660
29	.77	.70	− 30	−5	+12	− 6	−6	− 5	− 35	+ 5	− 604	+ 665
30	.69	.68	− 20	−5	+12	0	−6	+ 1	− 19	+ 4	− 623	+ 669
31	.69	.68	0	−5	+12	− 1	−6	0	0	+ 4	− 623	+ 673
Sept. 1	.68	.68	0	−5	+12	− 1	−6	0	0	+ 5	− 623	+ 678
2	.67	.68	0	−5	+12	− 6	−6	− 5	− 5	+ 5	− 628	+ 683
3	.72	.70	+ 20	−5	+12	− 6	−6	− 5	+ 15	+ 5	− 612	+ 688
4	.68	.66	− 40	−5	+12	0	−6	+ 1	− 39	+ 5	− 652	+ 693
5	.67	.64	− 20	−5	+12	−12	−6	−11	− 31	+ 3	− 683	+ 696
6	.70	.66	+ 20	−5	+12	−15	−6	−14	+ 6	+ 2	− 677	+ 698
7	.70	.64	− 20	−5	+12	− 9	−6	− 8	− 28	+ 6	− 705	+ 704
8	.67	.68	+ 40	−5	+11	− 5	−6	− 5	+ 35	+ 3	− 670	+ 707
9	.67	.66	− 20	−5	+12	− 0	−6	+ 1	− 19	+ 2	− 689	+ 709
10	.66	.65	− 10	−5	+12	− 4	−6	− 3	− 13	+ 3	− 702	+ 712
11	.64	.64	− 10	−5	+12	−16	−6	−15	− 25	+ 3	− 727	+ 715
12	.70	.63	− 10	−5	+12	−16	−6	−15	− 25	+ 2	− 752	+ 717
13	.66	.64	+ 10	−5	+12	− 4	−6	− 3	+ 7	+ 4	− 745	+ 721
14	.61	.62	− 20	−5	+11	−14	−6	−14	− 34	+ 4	− 779	+ 725
15	.59	.60	− 20	−5	+11	−18	−6	−18	− 38	+ 5	− 817	+ 730
16	.61	.61	+ 10	−5	+12	− 3	−6	− 2	+ 8	+ 6	− 809	+ 736
17	.66	.64	+ 30	−5	+12	− 1	−6	0	+ 30	+ 5	− 779	+ 741
18	.64	.63	− 10	−5	+11	−14	−6	−14	− 24	+ 3	− 803	+ 744
19	.62	.61	− 20	−5	+12	−14	−6	−13	− 33	+ 5	− 836	+ 749
20	.63	.61	0	−5	+12	− 1	−6	0	0	+ 4	− 836	+ 753
21	.61	.60	− 10	−5	+12	− 8	−6	− 7	− 17	+ 2	− 853	+ 755
22	.62	.60	0	−5	+12	− 8	−6	− 7	− 7	+ 3	− 860	+ 758
23	.61	.62	+ 20	−5	+12	− 1	−6	0	+ 20	+ 3	− 840	+ 761

24..........	.62	.59	−30	−5	+12	−8	−6	−7	−37	+2	−877	+763
25..........	.60	.62	+30	−5	+12	−12	−6	−11	+19	+2	−858	+765
26..........	.60	.57	−50	−5	+12	−8	−6	−7	−57	+3	−915	+768
27..........	.57	.56	−10	−5	+11	−14	−6	−14	−24	+1	−939	+769
28..........	.59	.55	−10	−5	+12	−20	−6	−19	−29	+9	−968	+778
29..........	.60	.52	−30	−5	+11	−22	−6	−22	−52	+1	−1020	+779
30..........	.58	.55	+30	−5	+12	−16	−6	−15	+15	+2	−1005	+781
Oct. 1..........	.67	.54	−10	−5	+11	−27	−6	−27	−37	+1	−1042	+782
{2..........	.68	.59	+50	−5	+12	−25	−6	−24}	−56	+16	−1098	+798
{3..........	.56	.53	−60	−5	+11	−22	−6	−22}				
4..........	.48	.54	+10	−6	+11	−31	−6	−32	−22	+11	−1120	+809
5..........	.63	.56	+20	−5	+12	−12	−6	−11	+9	+16	−1111	+825
{6..........	.65	(—)	....	−5	+12	−43	−6	−42}	−62	+15	−1173	+840
{7..........	.67	.53	....	−5	+12	−42	−6	−41}				
{8..........	.62	.58	+20	−5	+12	0	−6	+1}				
9..........	.62	.60	+20	−5	+12	0	−6	+1	+21	+2	−1152	+842
10..........	.62	.59	−10	−5	+11	−2	−6	−2	−12	+2	−1164	+844
11..........	.61	.60	+10	−5	+12	−3	−6	−2	+8	+2	−1156	+846
12..........	.63	.60	0	−5	+12	−2	−6	−1	−1	+2	−1157	+848
13..........	.62	.58	−20	−5	+12	0	−6	+1	−19	+3	−1176	+851
14..........	.61	.59	+10	−5	+12	−2	−6	−1	+9	+2	−1167	+853
15..........	.59	.56	−30	−5	+12	−2	−6	−1	−31	+2	−1198	+855
16..........	.58	.54	−20	−5	+11	−4	−6	−4	−24	+2	−1222	+857
17..........	.59	.59	+50	−5	+11	−23	−6	−23	+27	+4	−1195	+861
18..........	.62	.62	+30	−5	+12	−21	−6	−20	+10	+3	−1185	+864
19..........	.59	.60	−20	−5	+12	−4	−6	−3	−23	+3	−1208	+867
20..........	.59	.59	−10	−5	+12	−4	−6	−3	−13	+3	−1221	+870
21..........	.61	.60	+10	−5	+12	−4	−6	−3	+7	+2	−1214	+872
22..........	.60	.57	−30	−5	+12	−19	−6	−18	−48	+2	−1262	+874
{23..........	.54	.53	−40	−5	+11	−24	−6	−24}	+7	+13	−1255	+887
{24..........	.63	.61	+80	−5	+12	−10	−6	−9}				
25..........	.56	.57	−40	−5	+12	−8	−6	−7	−47	+2	−1302	+889
26..........	.58	.57	0	−5	+12	−8	−8	−7	−7	+2	−1309	+891
27..........	.58	.56	−10	−5	+12	−3	−6	−2	−12	+2	−1321	+893
28..........	.50	.58	+20	−5	+12	−1	−6	0	+20	+5	−1301	+898
29*..........	.43	(—)	....	−5	+12	0	−6	+1	....	....	....	....
30*..........	.51	(—)	....	−5	+12	−2	−6	−1	....	....	....	....
31*..........	.52	.48	−100	−5	+12	−4	−6	−3	....	....	....	....
1912												
May 1*..........	.62	.58	....	−4	+11	....	−6	....	....	....	....	....
2..........	.62	.60	+20	−5	+11	−1	−6	−1	+19	+1	+19	+1
3..........	.64	.59	−10	−4	+11	−4	−6	−3	−13	+3	+6	+4
4..........	.64	.58	−10	−5	+11	−6	−6	−6	−16	+5	−10	+9
5*..........	.66	.65	+70	−5	+11	−9	−6	−9	+61*	....	....	....
6..........	.68	.67	+20	−5	+11	−9	−6	−9	+11	+3	+1	+12
7..........	.68	.67	0	−5	+11	−5	−6	−5	−5	+3	−4	+15

* Rejected.

TABLE 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 579.+	Rise of mean lake surface (*I*₁)	Correction for—				Total correction	Net rise (*I*)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior (−*I*₃)	Outflow to Erie (+*I*₄)	Rainfall on lake (−*I*₂)	Run-off from land (−*I*₆)				Net rise	Computed evaporation
1912	(*ft.*)	(*ft.*)										
May 8	.67	.66	− 10	−5	+11	− 4	−6	− 4	− 14	+ 3	− 18	+ 18
9	.67	.67	+ 10	−5	+12	− 3	−6	− 2	+ 8	+ 6	− 10	+ 24
10	.69	.67	0	−5	+12	− 2	−6	− 1	− 1	+ 3	− 11	+ 27
11*	.75	.75	+ 80	−5	+11	−17	−6	−17	+ 63*	....	....	....
12	.85	.79	+ 40	−5	+12	−39	−6	−38	+ 2	+16	− 9	+ 43
13	.86	.81	+ 20	−5	+12	−36	−6	−35	− 15	+19	− 24	+ 62
14	.83	.80	− 10	−5	+12	−14	−6	−13	+ 34	+ 5	+ 10	+ 67
15	.85	.86	+ 60	−5	+12	− 4	−6	− 3				
16	.94	.88	+ 20	−5	+12	−15	−6	−14	+ 6	+ 5	+ 16	+ 72
17	.88	.87	− 10	−5	+12	−17	−6	−16	− 26	+11	− 10	+ 83
18	.86	.88	+ 10	−5	+12	− 6	−6	− 5	+ 5	+17	− 5	+ 100
19	.93	.90	+ 20	−5	+12	− 8	−6	− 7	+ 13	+ 6	+ 8	+ 106
20	1.00	.95	+ 50	−5	+12	−36	−6	−35	+ 15	+ 2	+ 23	+ 108
21	1.01	.97	+ 20	−5	+12	−45	−6	−44	− 24	+ 1	− 1	+ 109
22*	1.07	1.11	+140	−4	+12	−42	−6	−40	+100*	....	....	....
23*	1.13	1.11	0	−5	+12	−34	−6	−33	− 33*	....	....	....
24*	1.18	1.18	+ 70	−5†	+12	−20	−6	−19	+ 51*	....	....	....
25	1.21	1.33	+150	−5	+12	−12	−6	−11	+ 9	+ 8	+ 8	+ 117
26	1.19	1.20	−130	−5	+12	− 1	−6	0				
27*	1.26	1.30	+100	−5	+12	−18	−6	−17	+ 83*	....	....	....
28	1.32	1.33	+ 30	−5	+12	−41	−6	−40	− 10	+ 8	− 2	+ 125
29	1.37	1.36	+ 30	−5	+13	−31	−6	−29	+ 1	+ 6	− 1	+ 131
30	1.37	1.37	+ 10	−5	+13	− 8	−6	− 6	+ 4	+ 3	+ 3	+ 134
31	1.37	1.33	− 40	−5	+13	0	−6	+ 2	− 38	+ 3	− 35	+ 137
June 1	1.37	1.37	+ 40	−5	+13	− 2	−6	0	+ 40	+ 3	+ 5	+ 140
2	1.38	1.40	+ 30	−5	+13	−11	−6	− 9	+ 21	+ 3	+ 26	+ 143
3	1.40	1.43	+ 30	−5	+13	−12	−6	−10	+ 20	+ 3	+ 46	+ 146
4	1.38	1.37	− 60	−5	+13	− 6	−6	− 4	− 7	+20	+ 39	+ 166
5	1.41	1.43	+ 60	−5	+13	− 5	−6	− 3				
6	1.42	1.42	− 10	−5	+13	−11	−6	− 9	− 19	+ 2	+ 20	+ 168
7	1.43	1.44	+ 20	−5	+13	− 9	−6	− 7	+ 13	+ 3	+ 33	+ 171
8	1.44	1.44	0	−5	+13	0	−6	+ 2	+ 2	+ 3	+ 35	+ 174
9	1.45	1.44	0	−5	+13	0	−6	+ 2	+ 2	+ 3	+ 37	+ 177
10	1.43	1.42	− 20	−5	+13	0	−6	+ 2	− 18	+ 4	+ 19	+ 181
11	1.43	1.44	+ 20	−5	+13	− 2	−6	0	+ 20	+ 4	+ 39	+ 185
12	1.44	1.45	+ 10	−5	+13	− 6	−6	− 4	+ 6	+ 3	+ 45	+ 188
13	1.46	1.45	0	−5	+13	− 8	−6	− 6	− 6	+ 4	+ 39	+ 192

14	1.45	1.42	−30	−5	+13	−10	−6	−8	−38	+3	+1	+195
15	1.45	1.44	+20	−5	+12	−13	−6	−12	+8	+3	+9	+198
16	1.46	1.48	+40	−5	+13	−10	−6	−8	+32	+1	+41	+199
17	1.47	1.48	0	−5	+13	−2	−6	0	0	+4	+41	+203
18	1.47	1.48	0	−5	+13	0	−6	+2	+2	+3	+43	+206
19	1.47	1.47	−10	−5	+13	−2	−6	0	−10	+3	+33	+209
20	1.50	1.51	+40	−5	+13	−4	−6	−2	+38	+3	+71	+212
21	1.49	1.49	−20	−5	+13	−2	−6	0	−20	+3	+51	+215
22	1.49	1.48	−10	−5	+13	0	−6	+2	−8	+3	+43	+218
23	1.48	1.47	−10	−5	+13	0	−6	+2	−8	+4	+35	+222
24	1.48	1.48	+10	−5	+13	0	−6	+2	+12	+5	+47	+227
25	1.48	1.50	+20	−5	+13	−1	−6	+1	+21	+6	+68	+233
26	1.56	1.47	−30	−5	+13	−2	−6	0	−30	+17	+38	+250
27	1.52	1.48	+10	−5	+13	−1	−6	+1	+11	+15	+49	+265
28	1.47	1.47	−10	−5	+13	0	−6	+2	−8	+4	+41	+269
29	1.48	1.46	−10	−5	+13	0	−6	+2	−8	+4	+33	+273
30	1.53	1.46	0	−5	+13	0	−6	+2	+2	+6	+35	+279
July 1	1.48	1.44	−20	−5	+13	0	−6	+2	−18	+6	+17	+285
2	1.44	1.41	−30	−5	+13	0	−6	+2	−28	+5	−11	+290
3	1.40	1.38	−30	−5	+12	−5	−6	−4	−34	+4	−45	+294
4	1.41	1.40	+20	−5	+13	−15	−6	−13	+7	+5	−38	+299
5	1.44	1.43	+30	−5	+13	−15	−6	−13	+17	+6	−21	+305
6	1.43	1.42	−10	−5	+13	−9	−6	−7	−17	+6	−38	+311
7	1.43	1.43	+10	−5	+13	−10	−6	−8	+2	+6	−36	+317
8	1.45	1.44	+10	−5	+13	−11	−6	−9	+1	+5	−35	+322
9	1.45	1.46	+20	−5	+13	−12	−6	−10	+10	+5	−25	+327
10	1.43	1.45	−10	−5	+13	−12	−6	−10	−20	+5	−45	+332
11	1.48	1.45	0	−5	+13	−4	−6	−2	−2	+5	−47	+337
12	1.34	1.44	−10	−5	+13	−3	−6	−1	−11	+5	−58	+342
13	1.43	1.46	+20	−5	+13	−17	−6	−15	+5	+4	−53	+346
14	1.48	1.48	+20	−5	+13	−16	−6	−14	+6	+4	−47	+350
15	1.46	1.50	+20	−5	+13	−6	−6	−4	+16	+3	−31	+353
16	1.54	1.51	+10	−5†	+13	−5	−6	−3	+7	+12	−24	+365
17	1.48	1.47	−40	−5†	+13	−1	−6	+1	−39	+4	−63	+369
18	1.46	1.45	−20	−5†	+13	−4	−6	−2	−22	+3	−85	+372
19	1.46	1.46	+10	−5	+13	−4	−6	−2	+8	+4	−77	+376
20	1.44	1.45	−10	−5	+13	−16	−6	−14	−24	+3	−101	+379
21	1.48	1.43	−20	−5	+13	−27	−6	−25	−45	+3	−146	+382
22	1.48	1.44	+10	−5	+13	−12	−6	−10	0	+4	−146	+386
23	1.49	1.49	+50	−5	+13	−16	−6	−14	+36	+4	−110	+390
24	1.50	1.47	−20	−5	+13	−36	−6	−34	−54	+3	−164	+393
25	1.48	1.46	−10	−5	+13	−21	−6	−19	−29	+3	−193	+396
26	1.56	1.50	+40	−5	+13	−1	−6	+1	+41	+4	−152	+400
27	1.50	1.50	0	−5	+13	−1	−6	+1	+1	+5	−151	+405
28	1.52	1.54	+40	−5	+13	−8	−6	−6	+34	+4	−117	+409
29	1.55	1.55	+10	−5	+13	−13	−6	−11	−1	+3	−118	+412
30	1.59	1.55	0	−5	+13	−7	−6	−5	−5	+3	−123	+415

* Rejected. † Values assumed. Data missing.

TABLE 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 579.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_c$)				Net rise	Computed evaporation
1912	(*ft.*)	(*ft.*)										
July 31	1.58	1.54	− 10	−5	+13	− 2	−6	0	− 10	+ 3	− 133	+ 418
Aug. 1	1.56	1.54	0	−5	+13	− 2	−6	0	0	+ 3	− 133	+ 421
2	1.60	1.55	+ 10	−5	+13	− 6	−6	− 4	+ 6	+ 2	− 127	+ 423
3	1.56	1.52	− 30	−5	+13	− 5	−6	− 3	− 33	+ 3	− 160	+ 426
4	1.55	1.52	0	−5	+13	− 1	−6	+ 1	+ 1	+ 3	− 159	+ 429
5	1.52	1.50	− 20	−5	+13	− 1	−6	+ 1	− 19	+ 3	− 178	+ 432
6	1.50	1.48	− 20	−5	+13	− 3	−6	− 1	− 21	+ 3	− 199	+ 435
7	1.50	1.45	− 30	−5	+13	−15	−6	−13	− 43	+ 4	− 242	+ 439
8	1.49	1.46	+ 10	−5	+13	−22	−6	−20	− 10	+ 3	− 252	+ 442
9	1.50	1.49	+ 30	−5	+13	−42	−6	−40	− 10	+ 2	− 262	+ 444
10	1.56	1.53	+ 40	−5	+13	−42	−6	−40	0	+ 3	− 262	+ 447
11	1.59	1.56	+ 30	−5	+13	−14	−6	−12	+ 18	+ 3	− 244	+ 450
12	1.57	1.57	+ 10	−5	+13	− 5	−6	− 3	+ 7	+ 2	− 237	+ 452
13	1.57	1.57	0	−5	+13	0	−6	+ 2	+ 2	+ 3	− 235	+ 455
14	1.56	1.57	0	−5	+13	0	−6	+ 2	+ 2	+ 3	− 233	+ 458
15	1.60	1.58	+ 10	−5	+13	0	−6	+ 2	+ 12	+ 4	− 221	+ 462
16	1.59	1.56	− 20	−5	+13	0	−6	+ 2	− 18	+ 4	− 239	+ 466
17	1.56	1.54	− 20	−5	+13	−18	−6	−16	− 36	+ 3	− 275	+ 469
18	1.58	1.57	+ 30	−5	+13	−26	−6	−24	+ 6	+ 2	− 269	+ 471
19	1.59	1.58	+ 10	−5	+13	−13	−6	−11	− 1	+ 2	− 270	+ 473
20	1.60	1.59	+ 10	−5	+13	− 9	−6	− 7	+ 3	+ 3	− 267	+ 476
21	1.59	1.59	0	−5	+13	− 5	−6	− 3	− 3	+ 3	− 270	+ 479
22	1.57	1.59	0	−5	+13	− 7	−6	− 5	− 5	+ 3	− 275	+ 482
23	1.59	1.58	− 10	−5	+13	−10	−6	− 8	− 18	+ 3	− 293	+ 485
24	1.59	1.60	+ 20	−5	+13	−14	−6	−12	+ 8	+ 8	− 285	+ 493
25	1.59	1.58	− 20	−5	+13	−10	−6	− 8	− 28	+ 2	− 313	+ 495
26	1.58	1.56	− 20	−6	+13	− 2	−6	− 1	− 21	+ 4	− 334	+ 499
27	1.60	1.59	+ 30	−5	+13	− 2	−6	0	+ 30	+ 8	− 304	+ 507
28	1.55	1.56	− 30	−5	+13	−17	−6	−15	− 45	+ 3	− 349	+ 510
29	1.61	1.56	0	−5	+13	−17	−6	−15	− 15	+ 2	− 364	+ 512
30	1.57	1.55	− 10	−5	+13	− 6	−6	− 4	− 14	+ 3	− 378	+ 515
31	1.57	1.54	− 10	−5	+13	−13	−6	−11	− 21	+ 2	− 399	+ 517
Sept. 1	1.63	1.60	+ 60	−5	+13	−28	−6	−26	+ 34	+ 3	− 365	+ 520
2	1.64	1.63	+ 30	−5	+13	−37	−6	−35	− 5	+ 3	− 370	+ 523
3	1.66	1.64	+ 10	−5	+13	−17	−6	−15	− 5	+ 3	− 375	+ 526
4	1.69	1.66	+ 20	−5	+13	− 6	−6	− 4	+ 16	+ 3	− 359	+ 529
5	1.68	1.68	+ 20	−5	+13	−13	−6	−11	+ 9	+ 3	− 350	+ 532

6	1.71	1.72	+ 40	−5	+13	−10	−6	− 8	+ 32	+ 5	− 318	+ 537
7	1.70	1.70	− 20	−5	+13	− 2	−6	0	− 20	+ 5	− 338	+ 542
8	1.71	1.71	+ 10	−5	+13	− 4	−6	− 2	+ 8	+ 5	− 330	+ 547
9	1.71	1.70	− 10	−5	+13	− 7	−6	− 5	− 15	+ 4	− 345	+ 551
10	1.69	1.71	+ 10	−5	+13	− 9	−6	− 7	+ 3	+ 4	− 342	+ 555
11	1.73	1.71	0	−5	+14	− 6	−6	− 3	− 3	+ 4	− 345	+ 559
12	1.71	1.70	− 10	−5	+13	− 1	−6	0	− 9	+ 5	− 354	+ 564
13	1.70	1.70	0	−5	+13	− 1	−6	0	+ 1	+ 3	− 353	+ 567
14	1.69	1.69	− 10	−5	+13	− 1	−6	0	− 9	+ 3	− 362	+ 570
15	1.73	1.71	+ 20	−5	+14	−24	−6	−21	− 1	+ 2	− 363	+ 572
16	1.74	1.71	0	−5	+13	−25	−6	−23	− 23	+ 3	− 386	+ 575
17	1.71	1.69	− 20	−5	+13	−10	−6	− 8	− 28	+ 3	− 414	+ 578
18	1.73	1.68	− 10	−5	+13	−14	−6	−12	− 22	+ 1	− 436	+ 579
19	1.73	1.69	+ 10	−5	+13	−19	−6	−17	− 7	+ 1	− 443	+ 580
20	1.74	1.69	0	−5	+13	−17	−6	−15	− 15	+ 3	− 458	+ 583
21	1.73	1.78	+ 90	−5	+13	−13	−6	−11	− 9	+ 4	− 467	+ 587
22	1.72	1.71	− 70	−5	+13	−20	−6	−18				
23	1.71	1.68	− 30	−5	+13	−14	−6	−12	− 42	+ 3	− 509	+ 590
24	1.71	1.67	− 10	−5	+13	− 4	−6	− 2	− 12	+ 2	− 521	+ 592
25	1.67	1.59	− 80	−5	+13	− 6	−6	− 4	− 63	+24	− 584	+ 616
26	1.63	(—)	....	−6	+13	− 6	−6	− 5				
27	1.61	1.60	+ 10	−6	+14	− 2	−6	0				
28	1.62	1.62	+ 20	−5	+13	− 6	−6	− 4				
29	1.62	1.62	0	−6	+13	− 7	−6	− 6	− 6	+ 2	− 590	+ 618
30	1.66	1.60	− 20	−5	+13	− 5	−6	− 3	− 23	+ 3	− 613	+ 621
Oct. 1	1.64	1.57	− 30	−5	+13	− 6	−6	− 4	− 34	+ 3	− 647	+ 624
2	1.64	1.60	+ 30	−5	+13	− 3	−6	− 1	+ 29	+ 3	− 618	+ 627
3	1.54	1.61	+ 10	−5	+13	− 8	−6	− 6	+ 4	+ 4	− 614	+ 631
4	1.57	1.59	− 20	−5	+13	− 9	−6	− 7	− 27	+ 5	− 641	+ 636
5	1.58	1.60	+ 10	−5	+13	− 1	−6	+ 1	+ 11	+ 6	− 630	+ 642
6	1.57	1.58	− 20	−5	+13†	0	−6	+ 2	− 18	+ 5	− 648	+ 647
7	1.54	(—)	....	−6	+14	0	−6	+ 2	− 30	+15	− 678	+ 662
8	1.53	1.55	− 30	−5	+13	− 4	−6	− 2				
9	1.56	1.57	+ 20	−5	+13	−16	−6	−14	+ 6	+ 2	− 672	+ 664
10	1.55	1.55	− 20	−5	+13	−15	−6	−13	− 33	+ 1	− 705	+ 665
11	1.51	1.53	− 20	−5	+13	−16	−6	−14	− 34	+ 1	− 739	+ 666
12	1.49	1.57	+ 40	−6	+13	−30	−6	−29	+ 11	+ 5	− 728	+ 671
13	1.55	1.59	+ 20	−6	+13	−17	−6	−16	+ 4	+26	− 724	+ 697
14	1.52	1.58	− 10	−6	+13	0	−6	+ 1	− 9	+15	− 733	+ 712
15	1.54	1.57	− 10	−5	+13	0	−6	+ 2	− 8	+ 9	− 741	+ 721
16	1.53	1.57	0	−5	+13	0	−6	+ 2	+ 2	+ 3	− 739	+ 724
17	1.49	1.54	− 30	−5	+13	0	−6	+ 2	− 28	+ 9	− 767	+ 733
18	1.50	1.52	− 20	−5	+13	− 2	−6	0	− 20	+26	− 787	+ 759
19	1.44	1.48	− 40	−6	+13	− 3	−6	− 2	− 42	+15	− 829	+ 774
20	1.48	1.49	+ 10	−5	+13	− 1	−6	+ 1	+ 11	+ 8	− 818	+ 782
21	1.47	1.50	+ 10	−5	+13	− 5	−6	− 3	+ 7	+ 2	− 811	+ 784
22	1.43	1.50	0	−6	+13	−26	−6	−25	− 25	+ 2	− 836	+ 786

† Value assumed.   Data missing.

TABLE 23—*Continued*

Date	Observed elevation of lake surface 579.+	Observed elevation corrected for wind and barometric effects 579.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_e$)				Net rise	Computed evaporation
1912	(*ft.*)	(*ft.*)										
Oct. 23	1.46	1.47	− 30	−5	+13	−25	−6	−23	− 53	+10	− 889	+ 796
24	1.50	1.51	+ 40	−5	+13	− 4	−6	− 2	+ 38	+ 5	− 851	+ 801
25	1.48	1.51	0	−5	+13	− 1	−6	+ 1	+ 1	+ 2	− 850	+ 803
26	1.47	1.49	− 20	−5	+13	− 1	−6	+ 1	− 19	+ 3	− 869	+ 806
27*	1.41	1.42	− 70	−5	+13	0	−6	+ 2	− 68*			
28	1.41	1.40	− 20	−5	+13	0	−6	+ 2	− 18	+ 3	− 887	+ 809
29	1.47	1.37	− 30	−5	+12	− 3	−6	− 2	− 32	+10	− 919	+ 819
30	1.37	1.34	− 30	−6	+13	− 4	−6	− 3	+ 10	+23	− 909	+ 842
31	1.40	1.39	+ 50	−5	+13	− 9	−6	− 7				
1913	580.+ft.	580.+ft.										
May 1*	.85	.82		−4	+13							
2	.86	.85	+ 30	−4	+13	0	−6	+ 3	+ 33	+ 9	+ 33	+ 9
3	.88	.86	+ 10	−4	+13	0	−6	+ 3	+ 13	+22	+ 46	+ 31
4*	.91	.92	+ 60	−4	+13	− 5	−6	− 2	+ 58*			
5	.96	.93	+ 10	−4	+13	−20	−6	−17	− 7	+ 8	+ 39	+ 39
6*	1.00	1.02	+ 90	−5	+14	−20	−6	−17	+ 73*			
7	.99	1.00	− 20	−5	+13	− 5	−6	− 3	− 23	+ 4	+ 16	+ 43
8	.99	1.01	+ 10	−5	+13	− 4	−6	− 2	+ 8	+ 3	+ 24	+ 46
9	1.00	1.03	+ 20	−5	+13	− 4	−6	− 2	+ 18	+ 6	+ 42	+ 52
10	1.00	.99	− 40	−5	+13	0	−6	+ 2	− 38	+ 9	+ 4	+ 61
11	.92	.94	− 50	−4	+13	0	−6	+ 3	+ 5	+ 7	+ 9	+ 68
12	.95	.99	+ 50	−4	+13	− 1	−6	+ 2				
13	.92	.96	− 30	−5	+14	− 7	−6	− 4	− 34	+ 8	− 25	+ 76
14	1.02	.94	− 20	−5	+13	−10	−6	− 8	− 28	+11	− 53	+ 87
15	1.14	1.15	+210	−4	+13	−15	−6	−12	+ 6	+19	− 47	+ 106
16	1.03	.97	−180	−5	+13	−14	−6	−12				
17*	1.06	1.07	+100	−5	+13	− 5	−6	− 3	+ 97*			
18	1.04	1.05	− 20	−5	+14	− 5	−6	− 2	− 22	+ 6	− 69	+ 112
19	1.03	1.04	− 10	−5	+14	− 2	−6	+ 1	− 9	+ 5	− 78	+ 117
20	1.07	1.04	0	−5	+13	−10	−6	− 8	− 8	+ 3	− 86	+ 120
21	.95	1.05	+ 10	−5	+13	−30	−6	−28	+ 32	+ 4	− 54	+ 124
22	1.14	1.12	+ 70	−5	+14	−23	−6	−20				
23	1.14	1.14	+ 20	−5	+14	− 4	−6	− 1	+ 19	+ 2	− 35	+ 126
24	1.15	1.14	0	−5	+14	− 1	−6	+ 2	+ 2	+ 3	− 33	+ 129
25	1.17	1.14	0	−5	+14	− 1	−6	+ 2	+ 2	+ 2	− 31	+ 131
26	1.17	1.15	+ 10	−5	+14	− 9	−6	− 6	+ 4	+ 2	− 27	+ 133

27	1.18	1.16	+ 10	−5	+13	− 9	−6	− 7	+ 3	+ 3	− 24	+ 136
28	1.17	1.19	+ 30	−5	+13	− 2	−6	0	+ 30	+ 3	+ 6	+ 139
{29	1.10	1.13	− 60	−5	+13	− 9	−6	− 7}	− 24	+ 8	− 18	+ 147
{30	1.19	1.18	+ 50	−5	+13	− 9	−6	− 7}				
31	1.18	1.16	− 20	−5	+14	− 2	−6	+ 1	− 19	+ 3	− 37	+ 150
June 1	1.17	1.17	+ 10	−5	+13	− 2	−6	0	+ 10	+ 4	− 27	+ 154
2	1.16	1.15	− 20	−5	+14	− 1	−6	+ 2	− 18	+ 5	− 45	+ 159
{3	1.10	1.12	− 30	−5	+14	− 1	−6	+ 2}	+ 24	+ 9	− 21	+ 168
{4	1.17	1.17	+ 50	−5	+14	− 1	−6	+ 2}				
5	1.25	1.18	+ 10	−5	+13	− 2	−6	0	+ 10	+ 5	− 11	+ 173
6	1.14	1.15	− 30	−5	+13	− 8	−6	− 6	− 36	+ 8	− 47	+ 181
7	1.15	1.16	+ 10	−5	+14	− 9	−6	− 6	+ 4	+25	− 43	+ 206
{8	1.18	1.21	+ 50	−5	+14	− 3	−6	0}	− 7	+29	− 50	+ 235
{9	1.14	1.15	− 60	−5	+14	0	−6	+ 3}				
10	1.17	1.15	0	−5	+14	0	−6	+ 3	+ 3	+ 4	− 47	+ 239
11	1.16	1.13	− 20	−5	+14	0	−6	+ 3	− 17	+ 6	− 64	+ 245
12	1.20	1.13	0	−5	+14	0	−6	+ 3	+ 3	+ 7	− 61	+ 252
13	1.15	1.16	+ 30	−5	+14	0	−6	+ 3	+ 33	+ 9	− 28	+ 261
14	1.17	1.15	− 10	−5	+14	− 2	−6	+ 1	− 9	+ 8	− 37	+ 269
15	1.15	1.16	+ 10	−5	+14	− 5	−6	− 2	+ 8	+ 8	− 29	+ 277
16	1.17	1.18	+ 20	−5	+14	− 4	−6	− 1	+ 19	+ 7	− 10	+ 284
17	1.18	1.18	0	−5	+14	− 2	−6	+ 1	+ 1	+ 8	− 9	+ 292
18	1.18	1.17	− 10	−5	+13	− 2	−6	0	− 10	+ 7	− 19	+ 299
19	1.18	1.18	+ 10	−5	+13	−20	−6	−18	− 8	+ 6	− 27	+ 305
20	1.20	1.20	+ 20	−5	+14	−35	−6	−32	− 12	+ 4	− 39	+ 309
21	1.23	1.23	+ 30	−5	+14	−19	−6	−16	+ 14	+ 3	− 25	+ 312
22	1.18	1.21	− 20	−5	+14	− 3	−6	0	− 20	+ 5	− 45	+ 317
23	1.22	1.23	+ 20	−5	+14	0	−6	+ 3	+ 23	+ 5	− 22	+ 322
24	1.24	1.25	+ 20	−5	+14	0	−6	+ 3	+ 23	+ 6	+ 1	+ 328
25	1.24	1.24	− 10	−5	+14	−10	−6	− 7	− 17	+ 5	− 16	+ 333
26	1.25	1.26	+ 20	−5	+14	−25	−6	−22	− 2	+ 5	− 18	+ 338
27	1.25	1.25	− 10	−5	+14	−23	−6	−20	− 30	+ 6	− 48	+ 344
28	1.18	1.22	− 30	−5	+14	− 8	−6	− 5	− 35	+ 7	− 83	+ 351
29	1.29	1.26	+ 40	−5	+14	− 8	−6	− 5	+ 35	+ 6	− 48	+ 357
30	1.24	1.25	− 10	−5	+14	−10	−6	− 7	− 17	+ 8	− 65	+ 365
July 1	1.24	1.27	+ 20	−5	+14	− 5	−6	− 2	+ 18	+ 8	− 47	+ 373
2	1.25	1.26	− 10	−5	+14	− 3	−6	0	− 10	+ 8	− 57	+ 381
3	1.25	1.25	− 10	−5	+14	− 1	−6	+ 2	− 8	+ 7	− 65	+ 388
4	1.20	1.26	+ 10	−5	+14	−16	−6	−13	− 3	+ 6	− 68	+ 394
5	1.23	1.28	+ 20	−5	+14	−22	−6	−19	+ 1	+ 6	− 67	+ 400
6	1.32	1.26	− 20	−6	+14	− 5	−6	− 3	− 23	+ 9	− 90	+ 409
7	1.36	1.28	+ 20	−5	+14	0	−6	+ 3	+ 23	+24	− 67	+ 433
8	1.26	1.28	0	−5	+14	− 7	−6	− 4	− 4	+ 5	− 71	+ 438
9	1.30	1.30	+ 20	−5	+14	− 7	−6	− 4	+ 16	+ 8	− 55	+ 446
10	1.34	1.31	+ 10	−5	+14	0	−6	+ 3	+ 13	+10	− 42	+ 456
11	1.20	1.28	− 30	−5	+14	− 2	−6	+ 1	− 29	+ 5	− 71	+ 461
12	1.25	1.30	+ 20	−5	+13	−20	−6	−18	+ 2	+19	− 69	+ 480

* Rejected.

Table 23—*Continued*

Date	Observed elevation of lake surface 580.+	Observed elevation corrected for wind and barometric effects 580.+	Rise of mean lake surface (I_1)	Correction for— Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_c$)	Total correction	Net rise (I)	Computed evaporation	Accumulated sum of— Net rise	Computed evaporation
1913	(*ft.*)	(*ft.*)										
July 13	1.37	1.33	+ 30	−5	+14	−21	−6	−18	− 26	+57	− 95	+ 537
14	1.29	1.30	− 30	−5	+14	−11	−6	− 8				
15	1.29	1.30	0	−5	+14	−13	−6	−10	− 10	+ 4	− 105	+ 541
16	1.30	1.29	− 10	−5	+14	−13	−6	−10	− 20	+ 5	− 125	+ 546
17	1.30	1.30	+ 10	−5	+14	−10	−6	− 7	+ 3	+ 3	− 122	+ 549
18	1.31	1.30	0	−5	+14	− 3	−6	0	0	+ 4	− 122	+ 553
19	1.30	1.32	+ 20	−5	+14	− 4	−6	− 1	+ 19	+ 7	− 103	+ 560
20	1.32	1.33	+ 10	−6	+14	− 5	−6	− 3	+ 7	+ 6	− 96	+ 566
21	1.32	1.32	− 10	−5	+14	− 2	−6	+ 1	− 9	+ 6	− 105	+ 572
22	1.34	1.31	− 10	−5	+14	− 1	−6	+ 2	− 8	+ 6	− 113	+ 578
23	1.30	1.30	− 10	−5	+14	−13	−6	−10	− 20	+ 5	− 133	+ 583
24	1.32	1.31	+ 10	−6	+14	−12	−6	−10	0	0	− 133	+ 583
25	1.33	1.29	− 20	−5	+14	0	−6	+ 3	− 17	+ 6	− 150	+ 589
26	1.27	1.27	− 20	−5	+14	− 3	−6	0	− 20	+11	− 170	+ 600
27	1.26	1.27	0	−5	+14	−11	−6	− 8	− 8	+ 7	− 178	+ 607
28	1.28	1.29	+ 20	−5	+14	− 8	−6	− 5	+ 15	+ 6	− 163	+ 613
29	1.27	1.28	− 10	−5	+14	− 2	−6	+ 1	− 9	+ 7	− 172	+ 620
30	1.26	1.26	− 20	−5	+14	− 2	−6	+ 1	− 19	+ 7	− 191	+ 627
31	1.25	1.28	+ 20	−5	+14	−10	−6	− 7	+ 13	+ 6	− 178	+ 633
Aug. 1	1.28	1.27	− 10	−5	+14	−14	−6	−11	− 21	+ 8	− 199	+ 641
2	1.26	1.28	+ 10	−5	+14	− 4	−6	− 1	+ 9	+ 9	− 190	+ 650
3	1.27	1.29	+ 10	−6	+14	− 5	−6	− 3	+ 7	+ 8	− 183	+ 658
4	1.28	1.27	− 20	−5	+14	− 5	−6	− 2	− 22	+ 6	− 205	+ 664
5	1.25	1.25	− 20	−5	+14	0	−6	+ 3	− 17	+ 6	− 222	+ 670
6	1.24	1.25	0	−5	+14	− 2	−6	+ 1	+ 1	+ 4	− 221	+ 674
7	1.28	1.25	0	−5	+14	− 4	−6	− 1	− 1	+ 5	− 222	+ 679
8	1.19	1.22	− 30	−5	+14	−13	−6	−10	− 40	+ 5	− 262	+ 684
9	1.23	1.26	+ 40	−5	+14	−24	−6	−21	+ 19	+ 7	− 243	+ 691
10	1.29	1.28	+ 20	−6	+14	−17	−6	−15	+ 5	+ 3	− 238	+ 694
11	1.25	1.28	0	−5	+14	− 4	−6	− 1	− 1	+ 8	− 239	+ 702
12	1.24	1.24	− 40	−5	+14	0	−6	+ 3	− 37	+ 3	− 276	+ 705
13	1.23	1.24	0	−5	+14	0	−6	+ 3	+ 3	+ 4	− 273	+ 709
14	1.23	1.24	0	−5	+14	− 2	−6	+ 1	+ 1	+ 5	− 272	+ 714
15	1.21	1.23	− 10	−5	+14	− 4	−6	− 1	− 11	+ 6	− 283	+ 720
16	1.22	1.26	+ 30	−5	+14	− 9	−6	− 6	+ 24	+ 6	− 259	+ 726
17	1.24	1.26	0	−6	+14	− 9	−6	− 7	− 7	+ 7	− 266	+ 733
18	1.29	1.29	+ 30	−5	+14	− 3	−6	0	+ 30	+ 7	− 236	+ 740

19	1.25	1.24	- 50	-5	+14	- 1	-6	+ 2	- 48	+ 6	- 284	+ 746
20	1.22	1.24	0	-5	+13	- 1	-6	+ 1	+ 1	+ 6	- 283	+ 752
21	1.19	1.31	+ 70	-5	+13	- 1	-6	+ 1	+ 31	+10	- 252	+ 762
22	1.22	1.28	- 30	-6	+14	-12	-6	-10				
23	1.21	1.23	- 50	-5	+14	-11	-6	- 8	- 58	+ 5	- 310	+ 767
24	1.21	1.21	- 20	-6	+14	- 2	-6	0	- 20	+ 3	- 330	+ 770
25	1.19	1.15	- 60	-5	+14	- 2	-6	+ 1	- 59	+ 5	- 389	+ 775
26	1.13	1.18	+ 30	-6	+14	- 6	-6	- 4	+ 26	+ 5	- 363	+ 780
27	1.16	1.18	0	-5	+14	- 5	-6	- 2	- 2	+ 6	- 365	+ 786
28	1.09	1.15	- 30	-5	+13	- 4	-6	- 2	- 32	+ 4	- 397	+ 790
29	1.07	1.15	0	-6	+14	- 7	-6	- 5	- 5	+14	- 402	+ 804
30	1.08	1.13	- 20	-6	+14	- 4	-6	- 2	- 22	+ 5	- 424	+ 809
31	1.11	1.16	+ 30	-5	+14	- 2	-6	+ 1	+ 31	+ 4	- 393	+ 813
Sept. 1	1.06	1.12	- 40	-5	+14	- 2	-6	+ 1	- 39	+ 6	- 432	+ 819
2	1.05	1.13	+ 10	-5	+14	- 3	-6	0	+ 10	+ 9	- 422	+ 828
3	1.11	1.15	+ 20	-6	+14	-12	-6	-10	+ 10	+ 6	- 412	+ 834
4	1.11	1.12	- 30	-5	+14	-10	-6	- 7	- 37	+ 5	- 449	+ 839
5	1.06	1.10	- 20	-5	+14	0	-6	+ 3	- 17	+ 4	- 466	+ 843
6	1.06	1.11	+ 10	-5	+14	0	-6	+ 3	+ 13	+ 5	- 453	+ 848
7	1.05	1.10	- 10	-5	+14	- 2	-6	+ 1	- 9	+ 6	- 462	+ 854
8*	1.10	1.11	+ 10	-6	+14	- 5	-6	- 3	+ 7*	....	....	....
9*	1.06	1.01	-100	-5	+13	- 3	-6	- 1	-101*	....	....	....
10	.91	1.01	0	-5	+13	0	-6	+ 2	+ 2	+ 5	- 460	+ 859
11	.96	.99	- 20	-6	+13	- 1	-6	0	- 20	+ 4	- 480	+ 863
12	.94	.95	- 40	-6	+14	- 2	-6	0	- 40	+ 4	- 520	+ 867
13	1.00	.91	- 40	-6	+14	- 1	-6	+ 1	- 39	+10	- 559	+ 877
14	.92	.92	+ 10	-5	+14	0	-6	+ 3	+ 13	+ 4	- 546	+ 881
15	.87	.89	- 30	-5	+13	- 1	-6	+ 1	- 29	+ 4	- 575	+ 885
16	.86	.94	+ 50	-5	+13	-18	-6	-16	- 49	+ 7	- 624	+ 892
17	.88	.89	- 50	-6	+13	-34	-6	-33				
18	.92	.93	+ 40	-5	+14	-16	-6	-13	- 31	+ 7	- 655	+ 899
19	.83	.87	- 60	-5	+13	0	-6	+ 2				
20	.78	.87	0	-5	+13	-15	-6	-13	- 13	+ 1	- 668	+ 900
21	.81	.94	+ 70	-6	+13	-22	-6	-21	- 33	+14	- 701	+ 914
22	.80	.87	- 70	-6	+13	-13	-6	-12				
23*	.81	.81	- 60	-6	+13	- 8	-6	- 7	- 67*	....	....	....
24	.74	.79	- 20	-5	+13	- 2	-6	0	- 20	+ 3	- 721	+ 917
25	.78	.81	+ 20	-6	+13	- 3	-6	- 2	+ 18	+ 6	- 703	+ 923
26	.78	.81	0	-6	+14	- 5	-6	- 3	- 3	+ 3	- 706	+ 926
27	.77	.78	- 30	-6	+14	- 3	-6	- 1	- 31	+ 4	- 737	+ 930
28	.75	.79	+ 10	-6	+13	0	-6	+ 1	+ 11	+ 3	- 726	+ 933
29	.74	.78	- 10	-6	+13	- 1	-6	0	- 10	+ 3	- 736	+ 936
Oct. 30	.67	.72	- 60	-5	+13	- 4	-6	- 2	- 5	+ 5	- 741	+ 941
1	.77	.78	+ 60	-6	+13	- 4	-6	- 3				
2	.80	.76	- 20	-6	+14	- 9	-6	- 7	- 27	+ 2	- 768	+ 943
3	.80	.71	- 50	-5	+13	- 8	-6	- 6	- 5	+ 7	- 773	+ 950
4	.74	.76	+ 50	-5	+13	- 1	-6	+ 1				

* Rejected.

TABLE 23—*Continued*

Date	Observed elevation of lake surface 580.+	Observed elevation corrected for wind and barometric effects 580.+	Rise of mean lake surface (I_1)	Correction for—				Total correction	Net rise (I)	Computed evaporation	Accumulated sum of—	
				Inflow from Superior ($-I_3$)	Outflow to Erie ($+I_4$)	Rainfall on lake ($-I_2$)	Run-off from land ($-I_c$)				Net rise	Computed evaporation
1913	(*ft.*)	(*ft.*)										
Oct. 5	.71	.77	+ 10	−5	+13	− 6	−6	− 4	+ 6	+ 7	− 767	+ 957
6	.70	.74	− 30	−6	+13	− 6	−6	− 5	− 35	+ 2	− 802	+ 959
7	.76	.79	+ 50	−6	+13	− 4	−6	− 3	− 20	+ 6	− 822	+ 965
8	.73	.73	− 60	−6	+13	− 8	−6	− 7				
9	.78	.76	+ 30	−6	+13	− 6	−6	− 5	+ 25	+ 2	− 797	+ 967
10	.76	.79	+ 30	−5	+13	−12	−6	−10	+ 20	+ 3	− 777	+ 970
11	.74	.78	− 10	−6	+13	−24	−6	−23	− 33	+16	− 810	+ 986
12	.71	.78	0	−6	+14	−15	−6	−13	− 13	+ 7	− 823	+ 993
13	.70	.74	− 40	−6	+13	− 3	−6	− 2	− 42	+12	− 865	+1005
14	.71	.74	0	−5	+13	− 1	−6	+ 1	+ 1	+ 9	− 864	+1014
15	.68	.69	− 50	−6	+13	− 1	−6	0	− 50	+ 6	− 914	+1020
16	.73	.69	0	−6	+14	− 3	−6	− 1	− 1	+ 5	− 915	+1025
17	.72	.68	− 10	−6	+13	−25	−6	−24	− 34	+ 4	− 949	+1029
18	.72	.61	− 70	−6	+13	−28	−6	−27	− 46	+ 8	− 995	+1037
19	.75	.73	+120	−6	+14	− 6	−6	− 4				
20	.70	.67	− 60	−6	+14	− 7	−6	− 5				
21	.70	.64	− 30	−6	+14	− 9	−6	− 7	− 37	+ 7	−1032	+1044
22	.63	.59	− 50	−6	+13	− 6	−6	− 6	− 55	+ 2	−1087	+1046
23	.66	.63	+ 40	−6	+13	−19	−6	−18	+ 22	+ 2	−1065	+1048
24	.65	.57	− 60	−6	+14	−19	−6	−17	− 29	+ 5	−1094	+1053
25	.65	.62	+ 50	−6	+13	− 3	−6	− 2				
26	.61	.60	− 20	−6	+13	− 4	−6	− 3	− 23	+ 2	−1117	+1055
27*	.60	.55	− 50	−6	+13	−10	−6	− 9	− 59*	....	....	....
28*	.52	.50	− 50	−6	+13	−13	−6	−12	− 62*	....	....	....
29	.55	.57	+ 70	−6	+13	−11	−6	−10	− 6	+ 2	−1123	+1057
30	.55	.51	− 60	−6	+13	− 7	−6	− 6				
31	.58	.55	+ 40	−6	+13	− 5	−6	− 4	+ 36	+ 2	−1087	+1059

* Rejected.

residuals were found to be associated, as a rule, with seiches produced by sudden wind effects or sudden changes in barometric gradient. Or they were found to exist during the passage of a well-defined storm center over the lake, during which the conditions probably depart widely from the approximate theory postulated in the study of wind and barometric effects.

The rule adopted in combining two or more days of observation into a single observation equation covering the whole interval was as follows:

> Whenever any observation equation has a residual larger than 3.5 times the probable error of a single observation, and the residual for an equation immediately preceding or immediately following in time is of the opposite sign, the two observation equations shall be combined to form one equation, provided the residual for the new combined observation equation will be less than 3.5 times the probable error of a single observation.

Such a procedure rejects the elevation of the water surface on one day and treats a two-day interval as the basis of observation instead of a one-day interval, so far as the one combined equation is concerned. It retains all of the observed facts as to changes in barometric gradients, wind effects, inflow, outflow, run-off and rainfall on the lake, and uses them in the combined equation.

The combined equations were formed by adding the two or more separate equations term by term.

In Solution V_5, there was a total of 51 combined equations out of a possible 849, which is at the rate of 60 per thousand. If the errors involved in the large residuals were all of the accidental character, only 18 residuals per thousand would be outside the limit, 3.5 times the probable error, and less than that number would occur immediately preceded or followed by a residual of the opposite sign, large enough to make the combined residual less than 3.5 times the probable error. The combinations, as in the case of the rejections, were justified on the basis of abnormalities not taken into account by the approximate theory used in the investigation.

LIST OF OBSERVATION EQUATIONS OF THE FORM OF EQUATION (1), LAKE MICHIGAN-HURON

The form of the observation equation used in the last least-square solution on each of the two Lakes, Michigan-Huron and Superior, has thus far been presented in general terms, following which, specific examples taken from the Lake Michigan-Huron computations have been given, illustrating the methods of computation of the various quantities involved in it.

The complete list of daily observation equations for the 28 months, July to October inclusive, 1909, and May to October inclusive, of 1910, 1911, 1912 and 1913 is shown in Table 24. These are the observation equations from Solution V_5, Lake Michigan-Huron. In each column, the symbols (E_1) and (E_2) should be considered as repeated down the column.

The definitions of the quantities in the observation equations have already been given on pages 8 and 9. With the complete list of equations before one, it may be convenient to consider the known quantities further.

The coefficient of E_1, e, the vapor-pressure potential (or sometimes called the "saturation deficit") is sometimes zero for individual stations near the Great Lakes, and even negative, but the mean e for the whole lake, as computed from the 10 stations around it, is rarely zero. Only two such cases are shown in the list of equations for Lake Michigan-Huron, viz, August 25, 1910, and July 24, 1913.

TABLE 24—*Observation equations, Solution V_5, Lake Michigan-Huron.*

Date	Terms of Equation		
	$+e$ (E_1)	$+\left[e\left(\frac{w}{100}-x\right)\right]$ (E_2)	$+I=v$
1909			
July 2	+19 (E_1)	.... (E_2)	− 5=v
3	+10	+ 4	−21=v
4	+16	....	−12=v
5	+16	....	+23=v
6	+16	....	− 7=v
7	+18	....	+23=v
8	+20	....	−27=v
9	+24	....	+42=v
10	+22	....	−27=v
11	+18	....	−13=v
12–14	+49	....	−61=v
15	+17	....	− 5=v
16	+18	....	+ 7=v
17	+20	....	+12=v
18	+16	+ 6	+10=v
19	+22	+ 9	−40=v
20	+19	....	− 2=v
21	+17	....	−10=v
22	+15	+ 6	−26=v
23	+ 8	+ 6	+ 9=v
24	+16	+10	−41=v
25	+16	....	+41=v
26	+21	....	−17=v
27	+20	....	+ 3=v
28	+18	....	−32=v
29	+14	....	+13=v
30	+18	....	+ 7=v
31	+20	....	− 8=v
Aug. 1	+19	....	− 2=v
2	+21	....	−25=v
3	+22	....	− 3=v
4	+16	....	+13=v
5	+18	....	+ 3=v
6	+21	....	+10=v
7	+19	....	−28=v
8	+22	....	+ 2=v
9	+24	....	− 1=v
10	+24	....	0=v
11	+20	....	−13=v
12	+14	....	−44=v
13	+16	....	+ 2=v
14	+16	....	+32=v
15	+10	....	−27=v
16	+13	....	+11=v
17	+13	....	+20=v
18	+15	....	− 8=v
19	+14	....	− 7=v
20–21	+37	....	−38=v
22	+20	....	−47=v
23	+21	....	−17=v
24	+26	....	− 9=v
25	+33	....	+ 1=v
26	+18	....	−11=v
27	+18	....	0=v
28–30	+41	+ 3	−52=v
31–Sept. 1	+21	+12	−34=v
2	+14	....	−18=v
3	+11	....	+17=v
4	+ 7	....	−18=v
5–6	+22	+ 2	−61=v
7	+12	....	+ 3=v
8	+16	....	− 9=v
9	+10	....	−43=v
1909			
Sept. 10	+ 9 (E_1)	.... (E_2)	+ 7=v
11–12	+19	....	−38=v
13	+13	....	−11=v
14	+17	....	+11=v
15	+12	....	− 5=v
16–17	+27	....	−36=v
18–19	+26	....	− 6=v
20	+10	....	− 8=v
21	+14	....	− 7=v
22–23	+22	+ 7	−17=v
24	+11	....	+17=v
25	+ 8	....	−21=v
28–29	+14	....	−51=v
30	+ 6	+ 1	−37=v
Oct. 1	+ 6	....	+44=v
2	+ 8	....	−18=v
3	+ 6	....	−18=v
4	+ 8	....	− 8=v
5	+ 8	....	− 8=v
6	+ 7	....	−18=v
7	+ 9	....	+ 2=v
8	+12	....	−28=v
9	+15	....	+ 1=v
10	+13	....	−15=v
11–13	+20	+33	−78=v
14–15	+11	+17	−11=v
16	+ 8	+ 6	−25=v
17	+ 6	+ 1	−42=v
18	+ 6	....	−20=v
19	+ 6	....	0=v
21–22	+ 5	+ 3	−25=v
23	+ 5	+ 4	−14=v
24	+ 6	+ 2	0=v
25	+ 6	....	−21=v
26	+ 4	+ 2	−21=v
27	+ 6	+ 6	−10=v
28–29	+12	+ 2	−39=v
30	+ 4	+ 1	−31=v
31	+ 8	+ 2	+ 1=v
1910			
May 2	+ 4	+ 1	+13=v
3	+ 8	+10	− 7=v
4	+10	+ 5	− 9=v
5	+12	....	− 9=v
6	+14	....	+ 2=v
7	+14	....	+ 1=v
8	+11	....	+ 9=v
9	+ 9	....	−22=v
10	+13	+ 5	+12=v
11	+12	+ 5	−19=v
12	+11	+ 2	+12=v
13	+12	....	− 9=v
14	+ 9	....	−17=v
15	+10	....	−38=v
16	+12	....	+22=v
17	+12	+10	+ 3=v
19	+19	+13	+ 7=v
21	+ 7	....	− 8=v
22	+10	....	+13=v
23	+ 4	....	− 3=v
24	+ 7	....	+18=v
25	+10	....	−42=v

TABLE 24—*Continued*

Date	Terms of Equation			Date	Terms of Equation		
	$+e$ (E_1)	$+\left[e\left(\frac{w}{100}-x\right)\right]$ (E_2)	$+I=$v		$+e$ (E_1)	$+\left[e\left(\frac{w}{100}-x\right)\right]$ (E_2)	$+I=$v
1910				1910			
May 26	+ 8 (E_1)	.... (E_2)	+31 = v	July 28	+25 (E_1)	.... (E_2)	−19 = v
27	+11	....	0 = v	29	+21	....	+19 = v
28	+16	....	−20 = v	30	+22	....	−11 = v
29	+ 9	+ 3	+37 = v	31	+20	....	−21 = v
30	+11	+ 9	−24 = v	Aug. 1	+20	....	− 3 = v
31	+ 6	+ 2	−19 = v	2	+17	....	− 2 = v
June 1	+ 6	+ 2	−22 = v	3	+18	....	− 2 = v
2	+ 6	+ 1	−11 = v	4–6	+57	+28	−65 = v
3	+ 8	....	+15 = v	7	+14	....	−19 = v
4	+ 8	....	+ 1 = v	8	+16	....	0 = v
5	+ 7	....	+ 1 = v	9	+17	....	−24 = v
6	+ 7	....	+ 9 = v	10	+18	....	−13 = v
7	+11	....	− 3 = v	11	+17	....	+ 9 = v
8	+15	....	− 8 = v	12	+20	....	−10 = v
9	+17	....	+11 = v	13	+25	....	−43 = v
10	+19	....	+12 = v	14	+22	....	+ 6 = v
11	+22	....	+ 4 = v	15	+14	....	−35 = v
12	+17	....	− 8 = v	16	+12	....	+42 = v
13	+22	....	−12 = v	17	+12	....	−22 = v
14	+25	....	+ 2 = v	18	+14	....	+36 = v
15	+28	....	+11 = v	19	+23	....	−31 = v
16	+29	....	−38 = v	20	+21	....	−10 = v
17	+28	....	+ 5 = v	21	+21	....	−21 = v
18	+23	....	+14 = v	22	+20	....	−35 = v
19	+29	....	−20 = v	25	0	0	+ 1 = v
20	+23	....	+12 = v	26–27	+36	+14	−26 = v
21	+22	....	+11 = v	28	+18	....	−18 = v
22	+26	....	− 8 = v	29	+14	....	−39 = v
23	+24	....	+31 = v	30	+18	....	+20 = v
24	+24	....	−49 = v	31	+11	....	− 3 = v
25	+22	....	+ 1 = v	Sept. 1	+18	....	−21 = v
26	+25	....	+11 = v	2	+14	....	+ 1 = v
27	+20	....	− 1 = v	3	+ 6	....	−57 = v
28	+25	....	− 1 = v	4–6	+20	....	−60 = v
29	+27	....	−19 = v	7	+16	....	0 = v
30	+35	....	+42 = v	8	+10	....	+66 = v
July 1	+39	....	−10 = v	9	+14	+ 3	− 6 = v
2	+41	....	−29 = v	10	+15	....	+ 1 = v
3	+26	....	0 = v	11	+15	....	−11 = v
4	+22	+ 4	− 9 = v	12	+ 4	+ 1	−35 = v
5	+22	....	+20 = v	13	+12	....	−14 = v
6	+21	....	−31 = v	14	+14	....	− 8 = v
7	+20	....	− 3 = v	15	+13	....	− 9 = v
8	+35	....	− 9 = v	16	+15	....	− 9 = v
9	+35	....	−34 = v	17–19	+40	+ 2	−60 = v
10	+24	....	+ 6 = v	20	+ 8	....	+ 1 = v
11	+25	....	− 1 = v	21	+12	....	−19 = v
12	+20	....	−34 = v	22	+13	....	−18 = v
13	+17	....	+15 = v	23	+ 7	....	−27 = v
14	+23	....	−29 = v	24–25	+10	+ 4	−57 = v
15	+21	....	+18 = v	26	+ 8	....	+37 = v
16	+14	....	−26 = v	27	+ 4	+ 1	+21 = v
17	+20	+ 8	− 6 = v	28	+11	....	−23 = v
18	+22	+ 4	+ 1 = v	29	+11	....	−19 = v
19	+23	....	−10 = v	30	+13	....	+ 1 = v
20	+25	....	−10 = v	Oct. 1–2	+28	+21	−68 = v
21	+33	....	− 5 = v	3	+13	+ 5	+15 = v
22	+26	....	+ 2 = v	4	+ 7	+ 3	−20 = v
23	+22	....	−25 = v	5	+ 4	....	+11 = v
24	+19	+ 4	+28 = v	6–7	+24	+ 2	− 3 = v
25	+28	+17	−23 = v	8	+12	....	−22 = v
26	+25	....	−33 = v	14	+ 4	....	−49 = v
27	+18	....	+17 = v	15	+11	....	0 = v

TABLE 24—*Continued*

Date	Terms of Equation		
	$+e\ (E_1)$	$+\left[e\left(\frac{w}{100}-x\right)\right](E_2)$	$+I=\mathrm{v}$
1910			
Oct. 16	+ 8 (E_1)	.... (E_2)	−19=v
17	+18	....	−19=v
18	+16	....	+21=v
19	+15	....	− 1=v
20	+ 9	+ 4	−35=v
21–24	+29	+14	−74=v
25	+ 9	....	+10=v
26	+10	....	−18=v
28	+ 5	+ 3	−27=v
29	+ 5	....	+31=v
30	+ 8	+ 2	−13=v
1911			
May 2	+ 6	+ 6	−13=v
4	+10	....	− 8=v
5	+12	....	+ 2=v
6	+13	....	+23=v
7	+18	....	−17=v
8	+20	....	+30=v
9	+13	....	−11=v
10	+14	....	− 6=v
13	+14	+ 3	+12=v
14	+10	....	+12=v
15	+14	....	−10=v
16	+10	....	−19=v
17	+10	....	+ 1=v
18	+12	....	+14=v
20	+25	+10	−18=v
21	+18	....	−10=v
22	+ 7	....	−11=v
23	+ 8	....	+36=v
24	+ 9	....	+ 6=v
25	+14	....	−28=v
26	+18	....	+32=v
27	+28	....	+22=v
28	+20	....	0=v
29	+22	....	0=v
30	+17	....	−21=v
31	+10	....	− 5=v
June 1	+18	....	+36=v
2	+13	....	−13=v
3	+14	....	−31=v
4	+15	....	− 1=v
5	+ 9	....	− 6=v
6	+10	....	+ 1=v
7	+10	....	− 5=v
8	+11	....	+21=v
9	+15	....	−13=v
10	+24	....	+15=v
11	+20	....	+33=v
12	+17	....	+15=v
13	+12	....	−30=v
14	+17	....	+10=v
15	+21	....	− 8=v
16	+17	....	+20=v
17	+13	....	+17=v
18	+14	....	−31=v
19	+20	....	+12=v
20	+24	....	+ 2=v
21	+30	......	+ 2=v
22	+28	....	+11=v
23	+28	....	+11=v
24–25	+36	....	−61=v
26	+13	....	+ 3=v
1911			
June 27	+ 6 (E_1)	.... (E_2)	−25=v
28	+15	+ 3	+10=v
29	+14	....	−18=v
30	+16	....	+ 2=v
July 1	+29	....	+12=v
2	+44	....	+12=v
3	+41	....	+ 2=v
4	+38	....	−19=v
5	+35	....	−12=v
6	+27	....	+34=v
7	+26	....	−23=v
8	+20	....	+ 2=v
9	+27	....	−10=v
10	+30	....	−23=v
11	+31	....	+27=v
12	+34	....	− 9=v
13	+25	....	−19=v
14	+29	....	+ 9=v
15	+30	....	−36=v
16	+18	....	−21=v
17	+19	....	− 6=v
18	+21	....	−32=v
19	+18	....	−21=v
20	+17	....	+21=v
21	+19	....	−31=v
22	+24	....	−10=v
23	+19	....	− 2=v
24	+ 8	+11	−52=v
25–26	+28	+22	−21=v
27	+20	....	−19=v
28	+23	....	0=v
29	+19	....	− 1=v
30	+22	....	− 2=v
31	+24	....	+ 4=v
Aug. 1	+19	....	−25=v
2	+17	....	− 2=v
3	+18	....	−32=v
4	+13	....	+ 6=v
5	+18	....	+16=v
6	+21	....	− 1=v
7	+20	....	− 9=v
8	+18	+ 4	−22=v
9	+24	....	+ 5=v
10	+21	....	+ 4=v
11	+15	....	0=v
12	+19	....	−18=v
13	+15	....	− 5=v
14	+13	....	+ 4=v
15	+11	....	−14=v
16	+18	....	− 3=v
17	+20	....	− 1=v
18	+21	....	−21=v
19	+22	....	0=v
20	+18	....	− 9=v
21	+20	....	−23=v
22	+18	+ 7	+ 8=v
23	+19	....	−58=v
24	+16	....	−40=v
25	+14	....	+10=v
26	+14	....	−19=v
27	+19	....	−20=v
28	+14	....	−46=v
29	+17	....	−35=v
30	+14	....	−19=v

TABLE 24—*Continued*

Date	Terms of Equation			Date	Terms of Equation		
	$+e\ (E_1)$	$+\left[e\left(\frac{w}{100}-x\right)\right](E_2)$	$+I=\mathrm{v}$		$+e\ (E_1)$	$+\left[e\left(\frac{w}{100}-x\right)\right](E_2)$	$+I=\mathrm{v}$
1911				1912			
Aug. 31	+16 (E_1)	.... (E_2)	0=v	May 10	+12 (E_1)	.... (E_2)	−1=v
Sept. 1	+20	....	0=v	12	+6	+11	+2=v
2	+19	....	−5=v	13	+7	+13	−15=v
3	+20	....	+15=v	14–15	+14	+1	+34=v
4	+19	....	−39=v	16	+8	+2	+6=v
5	+13	....	−31=v	17	+10	+6	−26=v
6	+8	....	+6=v	18	+14	+10	+5=v
7	+10	+2	−28=v	19	+8	+3	+13=v
8	+10	....	+35=v	20	+6	....	+15=v
9	+8	....	−19=v	21	+4	....	−24=v
10	+11	....	−13=v	25–26	+29	....	+9=v
11	+12	....	−25=v	28	+6	+5	−10=v
12	+9	....	−25=v	29	+8	+3	+1=v
13	+15	....	+7=v	30	+12	....	+4=v
14	+13	+1	−34=v	31	+12	....	−38=v
15	+8	+2	−38=v	June 1	+10	....	+40=v
16	+23	....	+8=v	2	+10	....	+21=v
17	+19	....	+30=v	3	+12	....	+20=v
18	+10	....	−24=v	4–5	+25	+10	−7=v
19	+17	....	−33=v	6	+8	....	−19=v
20	+15	....	0=v	7	+12	....	+13=v
21	+8	....	−17=v	8	+11	....	+2=v
22	+10	....	−7=v	9	+11	....	+2=v
23	+12	....	+20=v	10	+15	....	−18=v
24	+7	....	−37=v	11	+16	....	+20=v
25	+7	....	+19=v	12	+11	....	+6=v
26	+10	....	−57=v	13	+15	....	−6=v
27	+4	0	−24=v	14	+13	....	−38=v
28	+13	+5	−29=v	15	+13	....	+8=v
29	+2	0	−52=v	16	+3	....	+32=v
30	+8	....	+15=v	17	+14	....	0=v
Oct. 1	+5	....	−37=v	18	+12	....	+2=v
2–3	+20	+9	−56=v	19	+12	....	−10=v
4	+4	+8	−22=v	20	+11	....	+38=v
5	+13	+12	+9=v	21	+13	....	−20=v
6–8	+23	+7	−62=v	22	+12	....	−8=v
9	+9	....	+21=v	23	+14	....	−8=v
10	+8	....	−12=v	24	+18	....	+12=v
11	+6	....	+8=v	25	+22	....	+21=v
12	+8	....	−1=v	26	+22	+9	−30=v
13	+10	....	−19=v	27	+22	+7	+11=v
14	+7	....	+9=v	28	+14	....	−8=v
15	+6	....	−31=v	29	+16	....	−8=v
16	+8	....	−24=v	30	+24	....	+2=v
17	+4	+2	+27=v	July 1	+22	....	−18=v
18	+13	....	+10=v	2	+20	....	−28=v
19	+10	....	−23=v	3	+16	....	−34=v
20	+10	....	−13=v	4	+20	....	+7=v
21	+8	....	+7=v	5	+22	....	+17=v
22	+6	....	−48=v	6	+22	....	−17=v
22–24	+14	+7	+7=v	7	+21	....	+2=v
25	+6	....	−47=v	8	+17	....	+1=v
26	+8	....	−7=v	9	+18	....	+10=v
27	+6	....	−12=v	10	+18	....	−20=v
28	+9	+2	+20=v	11	+20	....	−2=v
				12	+18	....	−11=v
1912				13	+14	....	+5=v
May 2	+5	....	+19=v	14	+14	....	+6=v
3	+12	....	−13=v	15	+9	+1	+16=v
4	+19	....	−16=v	16	+24	+5	+7=v
6	+12	....	+11=v	17	+14	....	−39=v
7	+12	....	−5=v	18	+10	....	−22=v
8	+12	....	−14=v	19	+16	....	+8=v
9	+12	+2	+8=v	20	+12	....	−24=v

TABLE 24—*Continued*

Date	Terms of Equation			Date	Terms of Equation		
	$+e\ (E_1)$	$+\left[e\left(\frac{w}{100}-x\right)\right](E_2)$	$+I=\text{v}$		$+e\ (E_1)$	$+\left[e\left(\frac{w}{100}-x\right)\right](E_2)$	$+I=\text{v}$
1912				1912			
July 21	+10 (E_1)	.... (E_2)	−45=v	Sept. 24	+7 (E_1)	.... (E_2)	−12=v
22	+14	....	0=v	25–28	+34	+12	−63=v
23	+16	....	+36=v	29	+8	....	−6=v
24	+12	....	−54=v	30	+11	....	−23=v
25	+13	....	−29=v	Oct. 1	+11	....	−34=v
26	+15	....	+41=v	2	+11	....	+29=v
27	+17	....	+1=v	3	+11	+1	+4=v
28	+14	....	+34=v	4	+17	....	−27=v
29	+12	....	−1=v	5	+21	....	+11=v
30	+12	....	−5=v	6	+18	....	−18=v
31	+12	....	−10=v	7–8	+23	+7	−30=v
Aug. 1	+12	....	0=v	9	+6	....	+6=v
2	+8	....	+6=v	10	+5	....	−33=v
3	+10	....	−33=v	11	+4	....	−34=v
4	+12	....	+1=v	12	+5	+3	+11=v
5	+10	....	−19=v	13	+14	+17	+4=v
6	+12	....	−21=v	14	+15	+9	−9=v
7	+14	....	−43=v	15	+10	+5	−8=v
8	+11	....	−10=v	16	+11	....	+2=v
9	+6	....	−10=v	17	+13	+5	−28=v
10	+10	....	0=v	18	+14	+17	−20=v
11	+10	....	+18=v	19	+9	+10	−42=v
12	+8	....	+7=v	20	+12	+4	+11=v
13	+13	....	+2=v	21	+8	....	+7=v
14	+13	....	+2=v	22	+2	+1	−25=v
15	+14	....	+12=v	23	+7	+6	−53=v
16	+16	....	−18=v	24	+8	+2	+38=v
17	+11	....	−36=v	25	+8	....	+1=v
18	+8	....	+6=v	26	+10	....	−19=v
19	+7	....	−1=v	28	+10	....	−18=v
20	+10	....	+3=v	29	+7	+6	−32=v
21	+12	....	−3=v	30–31	+19	+14	+10=v
22	+12	....	−5=v				
23	+12	....	−18=v	1913			
24	+14	+3	+8=v	May 2	+33	....	+33=v
25	+9	....	−28=v	3	+33	+10	+13=v
26	+10	+1	−21=v	5	+18	+2	−7=v
27	+16	+3	+30=v	7	+14	....	−23=v
28	+10	....	−45=v	8	+12	....	+8=v
29	+8	....	−15=v	9	+10	+2	+18=v
30	+11	....	−14=v	10	+12	+5	−38=v
31	+8	....	−21=v	11–12	+26	....	+5=v
Sept. 1	+12	....	+34=v	13	+10	+4	−34=v
2	+10	....	−5=v	14	+18	+5	−28=v
3	+10	....	−5=v	15–16	+21	+10	+6=v
4	+12	....	+16=v	18	+12	+2	−22=v
5	+10	....	+9=v	19	+19	....	−9=v
6	+19	....	+32=v	20	+12	....	−8=v
7	+19	....	−20=v	21–22	+14	....	+32=v
8	+19	....	+8=v	23	+7	....	+19=v
9	+14	....	−15=v	24	+11	....	+2=v
10	+16	....	+3=v	25	+9	....	+2=v
11	+16	....	−3=v	26	+8	....	+4=v
12	+17	....	−9=v	27	+10	....	+3=v
13	+10	....	+1=v	28	+13	....	+30=v
14	+12	....	−9=v	29–30	+29	....	−24=v
15	+6	....	−1=v	31	+13	....	−19=v
16	+10	....	−23=v	June 1	+16	....	+10=v
17	+10	....	−28=v	2	+19	....	−18=v
18	+4	....	−22=v	3–4	+32	....	+24=v
19	+4	....	−7=v	5	+17	....	+10=v
20	+10	....	−15=v	6	+15	+3	−36=v
21–22	+14	....	−9=v	7	+12	+17	+4=v
23	+10	....	−42=v	8–9	+27	+17	−7=v

TABLE 24—*Continued*

Date	Terms of Equation		
	$+c\ (E_1)$	$+\left[e\left(\frac{w}{100}-x\right)\right](E_2)$	$+I=$ v
1913			
June 10	+15 (E_1)	.... (E_2)	+ 3=v
11	+22	....	−17=v
12	+28	....	+ 3=v
13	+33	....	+33=v
14	+31	....	− 9=v
15	+29	....	+ 8=v
16	+26	....	+19=v
17	+31	....	+ 1=v
18	+26	....	−10=v
19	+21	....	− 8=v
20	+15	....	−12=v
21	+11	....	+14=v
22	+20	....	−20=v
23	+18	....	+23=v
24	+21	....	+23=v
25	+20	....	−17=v
26	+18	....	− 2=v
27	+24	....	−30=v
28	+25	....	−35=v
29	+24	....	+35=v
30	+29	....	−17=v
July 1	+29	....	+18=v
2	+29	....	−10=v
3	+26	....	− 8=v
4	+23	....	− 3=v
5	+22	....	+ 1=v
6	+22	+ 2	−23=v
7	+24	+14	+23=v
8	+20	....	− 4=v
9	+16	+ 3	+16=v
10	+20	+ 4	+13=v
11	+19	....	−29=v
12	+18	+11	+ 2=v
13–14	+51	+34	−26=v
15	+15	....	−10=v
16	+17	....	−20=v
17	+13	....	+ 3=v
18	+16	....	0=v
19	+26	....	+19=v
20	+21	....	+ 7=v
21	+22	....	− 9=v
22	+23	....	− 8=v
23	+19	....	−20=v
24	0	....	0=v
25	+23	....	−17=v
26	+40	....	−20=v
27	+26	....	− 8=v
28	+22	....	+15=v
29	+28	....	− 9=v
30	+26	....	−19=v
31	+22	....	+13=v
Aug. 1	+29	....	−21=v
2	+32	....	+ 9=v
3	+30	....	+ 7=v
4	+24	....	−22=v
5	+21	....	−17=v
6	+15	....	+ 1=v
7	+18	....	− 1=v
8	+19	....	−40=v
9	+25	....	+19=v
10	+12	....	+ 5=v
11	+18	+ 2	− 1=v
12	+13	....	−37=v
13	+15	....	+ 3=v
1913			
Aug. 14	+17 (E_1)	.... (E_2)	+ 1=v
15	+23	....	−11=v
16	+24	....	+24=v
17	+26	....	− 7=v
18	+26	....	+30=v
19	+24	....	−48=v
20	+24	....	+ 1=v
21–22	+37	....	+31=v
23	+19	....	−58=v
24	+12	....	−20=v
25	+20	....	−59=v
26	+18	....	+26=v
27	+23	....	− 2=v
28	+14	....	−32=v
29	+18	+ 7	− 5=v
30	+19	....	−22=v
31	+16	....	+31=v
Sept. 1	+24	....	−39=v
2	+32	....	+10=v
3	+24	....	+10=v
4	+19	....	−37=v
5	+16	....	−17=v
6	+19	....	+13=v
7	+24	....	− 9=v
10	+17	....	+ 2=v
11	+16	....	−20=v
12	+14	....	−40=v
13	+18	+ 4	−39=v
14	+15	....	+13=v
15	+15	....	−29=v
16–17	+14	+ 2	−49=v
18–19	+25	....	−31=v
20	+ 2	0	−13=v
21–22	+16	+ 8	−33=v
24	+10	....	−20=v
25	+10	+ 2	+18=v
26	+11	....	− 3=v
27	+14	....	−31=v
28	+13	....	+11=v
29	+13	....	−10=v
30–Oct. 1	+20	....	− 5=v
2	+ 7	....	−27=v
3–4	+28	....	− 5=v
5	+25	....	+ 6=v
6	+ 9	....	−35=v
7–8	+23	....	−20=v
9	+ 9	....	+25=v
10	+10	....	+20=v
11	+11	+10	−33=v
12	+16	+ 2	−13=v
13	+16	+ 6	−42=v
14	+17	+ 3	+ 1=v
15	+12	+ 2	−50=v
16	+11	+ 2	− 1=v
17	+ 4	+ 2	−34=v
18–20	+24	+ 2	−46=v
21	+ 7	+ 4	−37=v
22	+ 6	....	−55=v
23	+ 6	....	+22=v
24–25	+19	....	−29=v
26	+ 7	....	−23=v
29–30	+ 6	....	− 6=v
31	+ 6	....	+36=v

The coefficient of E_2, the product of e and $\left(\frac{w}{100}-2.6\right)$, enters only rarely. By actual count there are 145 in the 755 equations given. As some of these are for combined dates, it may be said that the mean wind velocity over two-day periods exceeds 10.8 miles per hour for somewhat more than 20 per cent of the time $\left(\frac{145}{755}=0.192\right)$.

The absolute term, in units of 0.001 foot, is that part of the directly observed rise in the lake surface, which is—as far as has been possible in this investigation to render it so—due only to evaporation from the lake surface. The absolute term for any day represents the rise from the preceding day to the day to which the observation equation refers.

If it had been possible to eliminate all of the influences except evaporation upon the change in elevation of the mean surface of the whole lake from each day to the one next following, the absolute terms would all be negative; that is, they would all be negative rises—or falls—of the lake surface from each day to the next following. The influences which were eliminated as far as possible were wind effects, barometric effects, inflow, outflow, rainfall on the lake surface, and the run-off into the lake, which last was taken as constant. There remain two principal influences which were not eliminated, and which are primarily responsible for the fact that the absolute terms are sometimes plus and sometimes minus instead of consistently minus. These two influences are the effects of seiches, and the effects of the variable part of the run-off into the lake.

Seiche effects are largely accidental in character. They could probably not be eliminated by any process. Because of their accidental character, they probably affect the final results but little.

By assuming the run-off into the lake to be constant, an error of a systematic character is introduced into the equations, the effect of which is to increase the probable error of a single observation and of the derived values of E_1 and E_2. If the constant part of the run-off into the lake is correctly evaluated, however, the actual error in the E_1 and E_2 will be small. The adopted value of 0.006 foot per day on the surface of Lake Michigan-Huron for the constant part of the run-off is probably very near the truth. The variable part of the run-off was not taken into account in this investigation because the only feasible way devised for evaluating it failed. This last will be discussed more fully later on.

The residuals, or v's, represent the discrepancies between the observed rise of the mean lake surface corrected for wind and barometric effects, inflow, outflow, rainfall on the lake and run-off into the lake on the one hand (the I of equation (1)), and the computed fall of the lake surface due to evaporation, $+eE_1+e\left(\frac{w}{100}-2.6\right)E_2$, on the other hand. The closer the agreement between the observed facts and the approximate theory, the smaller the v's tend to be, and the more perfectly they approach the law of distribution of accidental errors.

NORMAL EQUATIONS AND VALUES OF THE UNKNOWNS FROM THE TWO FINAL LEAST SQUARE SOLUTIONS FOR DETERMINING E_1 AND E_2

In this investigation, made by methods stated on pages 6 and 7 of Publication No. 317, a total of 24 least-square solutions was made in the investigation of the laws of evaporation proper, and of the run-off into the lake, but the 74 least-square solutions made prior to Publication No. 317 also gave information on these two subjects, and gradually served to point the way to the best form for the evaporation formula. On the subject of evaporation only, 7 least-square solutions were made, each having its observation equation of the form of equation (1), or slight modifications thereof, and each being based upon 28 months of daily observations on one or the other of Lakes Michigan-Huron or Superior. The two final solutions of the 7 in the evaporation series, which served to fix the values of the unknowns E_1 and E_2, and to establish the constant 2.6 in equation (1), were designated as Solution V_6 on Lake Michigan-Huron, and Solution BB_4 on Lake Superior.

The normal equations for Solution V_6, formed from the list of 755 observation equations shown in Table 24, are as follows:

$$\left.\begin{aligned}+240{,}627\,E_1+17{,}336\,E_2-85{,}953=0\\+\;17{,}336\,E_1+10{,}681\,E_2-18{,}176=0\end{aligned}\right\}\dots\dots\dots\dots(19)$$

The solution of the final normal equations for Solution V_6, (19), gives the following values for the unknowns:

$$E_1=+0.266\pm0.031 \qquad E_2=+1.27\pm0.148$$

The probable errors shown were computed rigorously from the normal equations, (19), and from the residuals, v, of the 755 observation equations of Solution V_6.

The observation equations for Solution BB_4, Lake Superior, similar to those for Solution V_6, Lake Michigan-Huron, will not be presented for want of space, and because they contributed very little, as will be seen, to the final results.

The normal equations for Solution BB_4, formed from the 745 observation equations in that solution, are as follows:

$$\left.\begin{aligned}+193{,}312\,E_1+15{,}122\,E_2+12{,}182=0\\+\;15{,}122\,E_1+\;9{,}500\,E_2-\;8{,}736=0\end{aligned}\right\}\dots\dots\dots\dots(20)$$

The solution of the final normal equations for Solution BB_4, (20), gives the following values for the unknowns:

$$E_1=-0.154\pm0.052 \qquad E_2=+1.16\pm0.234$$

The probable errors shown were computed rigorously from the normal equations, (20), and from the residuals of the 754 observation equations.

PRINCIPAL FACTS FROM THE TWO FINAL EVAPORATION SOLUTIONS

The principal facts from Solutions V_6 and BB_4 are summarized in Table 25.

TABLE 25—*Principal facts from the two final evaporation solutions*

	Solution V_6	Solution BB_4
Lake on which observations were made	Michigan-Huron	Superior
No. of days of observation used in Solution	818	847
No. of observation equations	755	754
Computed value of E_1	+0.266	−0.154
Computed probable error of E_1	±0.031	±0.052
Computed value of E_2	+1.27	+1.16
Computed probable error of E_2	±0.148	±0.234
Assumed value of x	2.6	2.6
Probable error of single observation of rise of water surface in one day	±0.014 ft.	±0.021 ft.

The probable error of a single observation as shown above is the probable error, computed rigorously from the normal equations and the residuals, of the rise of the mean surface of the whole lake from one day to the next. From this it appears that the elevation of the mean surface of the whole of Lake Michigan-Huron on any day may be determined with a probable error of ±0.010 foot.

Table 26 shows the best values of E_1 and E_2 determinable from the observations on Lakes Michigan-Huron and Superior. The values of E_1 and E_2 and their probable errors are the same as in Table 25. From these are computed the weights assigned to each unknown, the weighted-mean value and its probable error, and the residuals of each value from its own weighted mean. The assigned weights are inversely proportional to the squares of the respective probable errors, corresponding to the assumption that all errors in the computed values of these constants are of the accidental character. Unit weight corresponds to a probable error squared of 0.1.

TABLE 26—*Best values of E_1 and E_2*

	Probable error	Assigned weight	Value of—	Residual	Sum of weights	Weighted mean	Probable error of weighted mean
Best value of E_1:			E_1				
Solution V_5, Lake M-H.....	±0.031	104	+0.266	−0.110	....	....	....
Solution BB_4, Lake S........	± .052	37	− .154	+ .311	141	+0.157	±0.027
Best value of E_2:			E_2				
Solution V_5, Lake M-H.....	±0.148	4.57	+1.272	−0.03	....	....	....
Solution BB_4, Lake S.......	± .234	1.82	+1.163	+ .08	6.39	+1.24	±0.125

The residual of the value of E_1 in Solution BB_4, viz, +0.311, is over five times the probable error of the weighted mean E_1 from both solutions ($5\times\pm0.027=\pm0.135$), therefore that value of E_1, -0.154 ± 0.052, is rejected.

With the rejection stated in the preceding paragraph, the following are adopted as the best values of E_1, E_2 and x obtainable in this investigation.

$$\left.\begin{aligned} E_1&=+0.266\pm0.031\\ E_2&=+1.24\ \ \pm0.125\\ x&=2.6 \end{aligned}\right\}\ldots\ldots\ldots\ldots(21)$$

Using the adopted values, (21), equation (1), expressing the rate of natural evaporation from the surface of Lakes Michigan-Huron or Superior in units of 0.001 foot of depth per day, may be written.

$$E_v=+0.266e+1.24\left[e\left(\frac{w}{100}-2.6\right)\right]\ldots\ldots\ldots\ldots(22)$$

Expressed in units of 0.01 inch of depth per day, the unit used in Part II of this publication.

$$\underset{(\pm0.037)}{E_v=+0.319e}\ \underset{(\pm0.150)}{+1.49}\left[e\left(\frac{w}{100}-2.6\right)\right]\ldots\ldots\ldots\ldots(23)$$

The probable error of each constant in the last equation is shown in parentheses immediately beneath the constant, and is obtained by multiplying the probable error in (21) by the same conversion factor used in converting (22) into (23), viz, 6/5.

Equation (22) (or (23)) is the final evaporation formula as determined in this investigation of the laws of evaporation. It is based upon a total of 1,509 observations on two independent lakes, Michigan-Huron and Superior. It is based upon the facts drawn from the large scale of nature, and under natural conditions, as contrasted with the numerous evaporation equations based upon observations on evaporation pans of a few square inches or a few square feet in area. This equation should enable one to compute the evaporation from any free, open surface of water—from any reservoir, lake, river, bay, gulf—anywhere in the world from observations of the meteorological elements of air temperature, vapor-pressure and wind velocity as observed at the regular U. S. Weather Bureau Stations. The uncertainty which accompanies the use of the evaporation-pan formulas in estimating the evaporation on the large scale of nature should not exist with the use of equation (22), because it is based directly upon the large scale of nature and under natural conditions. *The evaporation pan has, in this case, been the lake itself.* The evaluation of the fluctuation of the elevation of the mean surface of the whole lake has been made with sufficient accuracy to segregate that part of the outgo which is evaporation, and the quantitative expression by which the evaporation can be evaluated has been obtained with a fair degree of accuracy. The probable error of E_1 is 1/9th of itself, $\left(\frac{0.031}{0.266}=\frac{1}{8.6}\right)$ and of E_2, 1/10th of itself $\left(\frac{0.125}{1.24}=\frac{1}{9.9}\right)$.

In developing equation (22), a two-day interval was involved in each observation equation (not counting the combined equations); that is, a change in the elevation of the water surface from one day to the next was used. In using the equation to estimate the evaporation from a water surface on any day, the quantities e and w to be substituted therein should be the mean values for the day only for which the evaporation is wanted. Hence the specific definitions of e and w for use in equations (22) and (23) are as follows:

e is the mean difference for the lake (or reservoir, river, etc.) between (1) the saturation vapor-pressure corresponding to the mean air temperature for the day for which the evaporation is wanted, and (2) the actual, observed mean vapor-pressure for the lake (or reservoir, river, etc.) for that day. The unit in which e is expressed is 0.01 inch of mercury.

w is the average travel of the wind, in miles per 24-hours, over the whole lake (or reservoir, river, etc.) surface for the day for which the evaporation is wanted, the mean being taken without any regard to the direction of the wind.

E_w is the total evaporation from the water surface for the day, expressed in units of 0.001 foot in equation (22), and in units of 0.01 inch in equation (23).

The second term of the right-hand member of equations (22) and (23), viz, 1.24 $\left[e\left(\frac{w}{100}-2.6\right)\right]$ and 1.49 $\left[e\left(\frac{w}{100}-2.6\right)\right]$, respectively, enters only for winds greater than 260 miles per day or 10.8 miles per hour; that is, only plus values of that term are to be used.

Since the quantities e and w used in deriving equation (22) were obtained from meteorological observations as ordinarily made by the U. S. Weather Bureau stations surrounding the Great Lakes, in using the formula to evaluate the evaporation in any locality it is to be understood that observations similarly made are to be substituted therein. This applies with special reference to the wind velocity, *w*, which is ordinarily observed at an elevation of approximately 100 feet above the ground surface, and is not the wind velocity at or very near the ground or water surface.[1]

COMPARISON OF COMPUTED EVAPORATION FROM LAKE MICHIGAN-HURON WITH DIRECTLY OBSERVED FALL OF LAKE SURFACE, CORRECTED FOR THE VARIOUS INFLUENCES

By substitution of the derived values of E_1 and E_2 from Solution V_5, shown in Table 25, into the observation equations for that solution shown in Table 24, the evaporation from the lake surface was computed for each day, or group of days combined into one equation, and also the residuals, v, were evaluated (shown in Table 28). The evaporation from the lake surface so computed is shown in the eleventh column of Table 23 marked "computed evaporation." The algebraic sum of this computed evaporation for any day and the "net rise," (*I*), is the residual, v, for that day. An examination of the computed evaporation in Table 23 shows that:

(*a*) The maximum evaporation computed on any day was 0.029 foot on July 25, 1910;

(*b*) The minimum evaporation computed on any day was 0.000 foot on August 25, 1910, and on July 24, 1913.

(*c*) The total and mean evaporation, computed, is as follows, expressed in feet of depth on Lake Michigan-Huron:

	1909	1910	1911	1912	1913	Mean monthly	Mean daily
May	....	(0.177)	(0.150)	(0.177)	(0.172)	0.169	0.0055
June	....	.165	.137	.142	.215	.165	.0055
July	(0.198)	.249	.245	.139	.268	.220	.0071
August	.151	.202	.160	.099	.180	.158	.0051
September	(.123)	.100	.108	.104	(.137)	.114	.0038
October	(.162)	.168	(.132)	(.228)	(.133)	.165	.0053
Total	.634	1.061	.932	.889	1.105	....	....
Mean monthly	.158	.177	.155	.148	.184	....	....
Mean daily	.0052	.0058	.0051	.0048	.0060	....	....

The values enclosed in parentheses are the values from the table corrected for the rejected days.

(*e*) The average evaporation for the whole period of observation used in this investigation, July to October 1909, and May to October of 1910, 1911, 1912 and 1913, was 0.0054 foot of depth per day on Lake Michigan-Huron.

(*f*) The maximum variation in evaporation from one day to the next was 0.022 foot from July 25 to 26, 1910. This represents a variation of over four times the average daily evaporation for the summer months, 0.0054 foot. There are numerous instances of variation in evaporation from one day to another of 2 and 3 times the average daily evaporation.

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[1] For a discussion of its accuracy and a comparison of this evaporation formula with others, see Appendix to Part I, page 121.

In order better to compare the computed evaporation and the absolute term, I, the accumulated sums of these quantities have been obtained, and are shown in the last two columns of Table 23. This accumulated sum begins anew each year; thus on October 31, 1913, the accumulated sum of the computed evaporation since May 2, 1913, amounted to 1.059 foot of depth on Lake Michigan-Huron. The corresponding accumulated sum of the absolute terms, I, or net rises, was −1.087 foot, which is the directly observed fall—that is, negative rise—of the mean lake surface in that time interval, corrected for the various influences. On Plate 5 is shown the comparison of the accumulated sums by months for the five years, and for the mean of the five years 1909–1913.

On Plate 5 it may be observed that in each year the computed rate of evaporation is larger in May and June, and smaller in September and October, than the actual rate of net fall of the lake surface, whereas in July and August the two rates are about the same. It is believed that this divergence of the two accumulation curves at the beginning of the season, and their subsequent convergences at the end of it, is due to principally the variable part of the run-off into the lake, attention to which has been called previously, with the statement that the only method devised for its evaluation failed. By reference to Table 23, it is easy to see that if the run-off had been taken larger than 6 (i.e., 0.006 foot of depth per day) in May and June, and smaller than that in September and October, the accumulation curves would have approached each other more closely throughout their extent. It is known qualitatively that the run-off is larger in May and June than it is in September and October, but it is not known, and could not be determined, quantitatively, how much. The value 0.006 foot of depth per day for the run-off is about right for the constant part of the run-off into the lake, or the run-off when the ground water is at its average level and (rainfall on land) minus (evaporation from land) minus (run-off from land) has been zero for a long time. This was obtained by revising the first approximate value of 0.008 foot, obtained as described on pages 16 to 19, by least-square computations in a manner yet to be described. Hence, it is reasonably certain that if the variable part of the run-off could have been properly taken into account, the accumulation curves on Plate 5 would have been closer together throughout their lengths, and therefore their differences can not all be charged against systematic errors in the computed evaporation, but must be sought, rather, in the variable part of the run-off. This will become clearer from an analysis of the residuals, which follows.

RESIDUALS FROM SOLUTION V_5

The comparison between the computed evaporation and the directly observed fall in the lake surface will now be given in the form of differences; that is, in the form of the residuals, v, a positive residual meaning that the computed fall of the lake surface due to evaporation is larger than the directly observed fall thereof, corrected for the various influences.

The mean residuals shown in Table 27 were computed from the list in Table 28. Each mean residual is the algebraic sum of the residuals divided by the number of residuals in the period involved, except in the last column but one, where they are the mean of the mean residuals in the 5 preceding columns, and except in the case of the mean annual v, which is the mean of the mean monthly v's in the 6 lines above it. The probable error of each mean v, monthly, annual, or for the whole period of observation, 1909 to 1913, was computed from the relation $r_0 = \frac{r}{\sqrt{n}}$, in which r_0 is the probable error of the mean v, r the probable error of a single observation of Solution V_6, viz, ±14.4, and n is the number of residuals involved.

The tabulation indicates that there is a strong systematic distribution of the residuals with reference to the *season*, positive residuals occurring from May to July and negative ones from August to September. Moreover, the probable errors of the mean monthly residuals are small in comparison with the mean v's themselves for the first two and last two months, especially. This is a very strong indication—proof—of a systematic error, and corresponds with the evidence shown in the accumulated sums in Table 23 and as plotted on Plate 5. That is, the run-off is probably greater than 0.006 foot per day in May and June, and less than that in September and October.

While the evidence of systematic error is indisputable in as far as the season is concerned, note that there is little or no evidence of systematic error for the solution as a whole. For the whole period 1909 to 1913 the mean residual is −2±0.53. The probable error is nearly ¼ the value of the residual in this case. But note that the omission of the + residuals, which would have occurred in May and June 1909 if observations for those months had been used, would have reduced this mean v. If one consider only the years 1910 to 1913, thereby eliminating the effect of omitting May and June 1909, the mean v is −1±0.6.

TABLE 27—*Mean residuals from Solution V_6, Lake Michigan-Huron*

Month	1909	1910	1911	1912	1913	Mean monthly v	No. of v's
May		+3	+5	+5	+ 5	+4±1	99
June		+5	+4	+7	+ 7	+6±1	116
July	+1	0	0	−1	+ 5	+1±1	150
August	− 3	−5	−7	−5	− 1	−4±1	145
September	−11	−9	−9	−4	−10	−9±1	126
October	−10	−8	−7	−3	−10	−8±1	119
Mean annual v	−5±1	−2±1	−2±1	0	0		
No. of v's	102	159	170	167	157		

Mean residual, v, for whole period 1909–1913 = −2±0.53.
Mean residual, v, for 1910–1913 inclusive = −1±0.6.

The effect of omitting the observations for May and June 1909 may also be seen by examining the mean annual residuals. For 1909 it is −5±1, strong indication of systematic error. For the other four years there is no indication of systematic error.

The comparison between the actual and theoretical distribution of the residuals is shown in Table 29. If the evaporation from the lake surface is fully expressed by the form of equation developed and the evaluated constants in it, on the one hand, and if all of the influences except evaporation, which affect the mean elevation of the whole lake surface have been properly evaluated, leaving nothing in such evaluations but accidental errors due to seiche effects, on the other hand, then the actual distribution of residuals will be the same as the theoretical distribution of accidental errors, neglecting the small difference which will result from using a finite number of observations.

Table 28—*Residuals of Solution V_8, Lake Michigan-Huron.* (*Unit=0.001 foot.*)

Date	Residual v	Date	Residual v	Date	Residual v	Date	Residual v	Date	Residual v
1909		1909		1910		1910		1910	
July 2	0	Sept. 10	+ 9	May 27	+ 3	July 30	− 5	Oct. 19	+35
3	−13	11–12	−33	28	−16	31	−16	20	−28
4	− 8	13	− 8	29	+43	Aug. 1	+ 2	21–24	−48
5	+27	14	+16	30	−10	2	+ 3	25	+12
6	− 3	15	− 2	31	−14	3	+ 3	26	−15
7	+28	16–17	−29	June 1	−17	4–6	−14	28	−22
8	−22	18–19	+ 1	2	− 8	7	−15	29	+32
9	+48	20	− 5	3	+17	8	+ 4	30	− 8
10	−21	21	− 3	4	+ 3	9	−19		
11	− 8	22–23	− 2	5	+ 3	10	− 8	1911	
12–14	−48	24	+20	6	+11	11	+14	May 2	− 3
15	0	25	−19	7	0	12	− 5	4	− 5
16	+12	28–29	−47	8	− 4	13	−36	5	+ 5
17	+17	30	−34	9	+16	14	+12	6	+26
18	+22	Oct. 1	+46	10	+17	15	−31	7	−12
19	−23	2	−16	11	+10	16	+45	8	+35
20	+ 3	3	−16	12	− 3	17	−19	9	− 8
21	− 5	4	− 6	13	− 6	18	+40	10	− 2
22	−14	5	− 6	14	+ 9	19	−25	13	+20
23	+19	6	−16	15	+18	20	− 4	14	+15
24	−24	7	+ 4	16	−30	21	−15	15	− 6
25	+45	8	−25	17	+12	22	−30	16	−16
26	−11	9	+ 5	18	+20	25	+ 1	17	+ 4
27	+ 8	10	−12	19	−12	26–27	+ 2	18	+17
28	−27	11–13	−31	20	+18	28	−13	20	+ 2
29	+17	14–15	+14	21	+17	29	−35	21	− 5
30	+12	16	−15	22	− 1	30	+25	22	− 9
31	− 3	17	−39	23	+37	31	0	23	+38
Aug. 1	+ 3	18	−19	24	−43	Sept. 1	−16	24	+ 8
2	−19	19	+ 2	25	+ 7	2	+ 5	25	−24
3	+ 3	21–22	−20	26	+18	3	−55	26	+37
4	+17	23	− 9	27	+ 4	4–6	−55	27	+29
5	+ 8	24	+ 5	28	+ 6	7	+ 4	28	+ 5
6	+16	25	−19	29	−12	8	+69	29	+ 6
7	−23	26	−17	30	+51	9	+ 2	30	−16
8	+ 8	27	0	July 1	0	10	+ 5	31	− 2
9	+ 5	28–29	−33	2	−18	11	− 7	June 1	+41
10	+ 6	30	−29	3	+ 7	12	−33	2	−10
11	− 8	31	+ 6	4	+ 2	13	−11	3	−27
12	−40			5	+26	14	− 4	4	+ 3
13	+ 6	1910		6	−25	15	− 6	5	− 4
14	+36	May 2	+15	7	+ 2	16	− 5	6	+ 4
15	−24	3	+ 9	8	0	17–19	−46	7	− 2
16	+14	4	0	9	−25	20	+ 3	8	+24
17	+23	5	− 6	10	+12	21	−16	9	− 9
18	− 4	6	+ 6	11	+ 6	22	−15	10	+21
19	− 3	7	+ 5	12	−29	23	−25	11	+38
20–21	−28	8	+12	13	+20	24–25	−49	12	+20
22	−42	9	−20	14	−23	26	+39	13	−27
23	−11	10	+21	15	+24	27	+23	14	+15
24	− 2	11	−10	16	−22	28	−20	15	− 2
25	+10	12	+18	17	+ 9	29	−16	16	+25
26	− 6	13	− 6	18	+12	30	+ 4	17	+20
27	+ 5	14	−15	19	− 4	Oct. 1–2	−34	18	−27
28–30	−37	15	−35	20	− 3	3	+24	19	+17
Aug. 31–		16	+25	21	+ 4	4	−14	20	+ 8
Sept. 1	−13	17	+19	22	+ 9	5	+12	21	+10
2	−14	19	+29	23	−19	6–7	+ 6	22	+18
3	+20	21	− 6	24	+38	8	−19	23	+18
4	−16	22	+16	25	+ 6	14	−48	24–25	−51
5–6	−52	23	− 2	26	−26	15	+ 3	26	+ 6
7	+ 6	24	+20	27	+22	16	−17	27	−23
8	− 5	25	−39	28	−12	17	−14	28	+18
9	−40	26	+33	29	+25	18	+25	29	−14

TABLE 28—*Continued*

Date	Residual v	Date	Residual v	Date	Residual v	Date	Residual v	Date	Residual v
1911		1911		1912		1912		1912	
June 30	+6	Sept. 3	+20	May 14–15	+39	July 24	−51	Sept. 30	−20
July 1	+20	4	−34	16	+11	25	−26	Oct. 1	−31
2	+24	5	−28	17	−15	26	+45	2	+32
3	+13	6	+8	18	+22	27	+6	3	+8
4	−9	7	−22	19	+19	28	+38	4	−22
5	−3	8	+38	20	+17	29	+2	5	+17
6	+41	9	−17	21	−23	30	−2	6	−13
7	−16	10	−10	25–26	+17	31	−7	7–8	−15
8	+7	11	−22	28	−2	Aug. 1	+3	9	+8
9	−3	12	−23	29	+7	2	+8	10	−32
10	−15	13	+11	30	+7	3	−30	11	−33
11	+35	14	−30	31	−35	4	+4	12	+16
12	0	15	−33	June 1	+43	5	−16	13	+30
13	−12	16	+14	2	+24	6	−18	14	+6
14	+17	17	+35	3	+23	7	−39	15	+1
15	−28	18	−21	4–5	+13	8	−7	16	+5
16	−16	19	−28	6	−17	9	−8	17	−19
17	−1	20	+4	7	+16	10	+3	18	+6
18	−26	21	−15	8	+5	11	+21	19	−27
19	−16	22	−4	9	+5	12	+9	20	+19
20	+26	23	+23	10	−14	13	+5	21	+9
21	−26	24	−35	11	+24	14	+5	22	−23
22	−4	25	+21	12	+9	15	+16	23	−43
23	+3	26	−54	13	−2	16	−14	24	+43
24	−36	27	−23	14	−35	17	−33	25	+3
25–26	+14	28	−20	15	+11	18	+8	26	−16
27	−14	29	−51	16	+33	19	+1	28	−15
28	+6	30	+17	17	+4	20	+6	29	−22
29	+4	Oct. 1	−36	18	+5	21	0	30–31	+33
30	+4	2–3	−40	19	−7	22	−2		
31	+10	4	−11	20	+41	23	−15	1913	
Aug. 1	−20	5	+27	21	−17	24	+16	May 2	+42
2	+3	6–8	−47	22	−5	25	−26	3	+35
3	−27	9	+23	23	−4	26	−17	5	+1
4	+9	10	−10	24	+17	27	+38	7	−19
5	+21	11	+10	25	+27	28	−42	8	+11
6	+5	12	+1	26	−13	29	−13	9	+24
7	−4	13	−16	27	+26	30	−11	10	−29
8	−12	14	+11	28	−4	31	−19	11–12	+12
9	+11	15	−29	29	−4	Sept. 1	+37	13	−26
10	+10	16	−22	30	+8	2	−2	14	−17
11	+4	17	+31	July 1	−12	3	−2	15–16	+25
12	−13	18	+13	2	−23	4	+19	18	−16
13	−1	19	−20	3	−30	5	+12	19	−4
14	+7	20	−10	4	+12	6	+37	20	−5
15	−11	21	+9	5	+23	7	−15	21–22	+36
16	+2	22	−46	6	−11	8	+13	23	+21
17	+4	23–24	+20	7	+8	9	−11	24	+5
18	−15	25	−45	8	+6	10	+7	25	+4
19	+6	26	−5	9	+15	11	+1	26	+6
20	−4	27	−10	10	−15	12	−4	27	+6
21	−18	28	+25	11	+3	13	+4	28	+33
22	+22			12	−6	14	−6	29–30	−16
23	−53	1912		13	+9	15	+1	31	−16
24	−36	May 2	+20	14	+10	16	−20	June 1	+14
25	+14	3	−10	15	+19	17	−25	2	−13
26	−15	4	−11	16	+19	18	−21	3–4	+33
27	−15	6	+14	17	−35	19	−6	5	+15
28	−42	7	−2	18	−19	20	−12	6	−28
29	−30	8	−11	19	+12	21–22	−5	7	+29
30	−15	9	+14	20	−21	23	−39	8–9	+22
31	+4	10	+2	21	−42	24	−10	10	+7
Sept. 1	+5	12	+18	22	+4	25–28	−39	11	−11
2	0	13	+4	23	+40	29	−4	12	+10

TABLE 28—*Continued*

Date	Residual v	Date	Residual v	Date	Residual v	Date	Residual v	Date	Residual v
1913		1913		1913		1913		1913	
June 13	+42	July 8	+ 1	Aug. 3	+15	Aug. 29	+ 9	Sept. 29	− 7
14	− 1	9	+24	4	−16	30	−17	Sept. 30-	
15	+16	10	+23	5	−11	31	+36	Oct. 1	0
16	+26	11	−24	6	+ 5	Sept. 1	−33	2	−25
17	+ 9	12	+21	7	+ 4	2	+19	3–4	+ 2
18	− 3	13–14	+31	8	−35	3	+16	5	+13
19	− 2	15	− 6	9	+26	4	−32	6	−33
20	− 8	16	−15	10	+ 8	5	−13	7–8	−14
21	+17	17	+ 6	11	+ 7	6	+18	9	+27
22	−15	18	+ 4	12	−34	7	− 3	10	+23
23	+28	19	+26	13	+ 7	10	+ 7	11	−17
24	+29	20	+13	14	+ 6	11	−16	12	− 6
25	−12	21	− 3	15	− 5	12	−36	13	−30
26	+ 3	22	− 2	16	+30	13	−29	14	+10
27	−24	23	−15	17	0	14	+17	15	−44
28	−28	24	0	18	+37	15	−25	16	+ 5
29	+41	25	−11	19	−42	16–17	−42	17	−30
30	− 9	26	− 9	20	+ 7	18–19	−24	18–20	−37
July 1	+26	27	− 1	21–22	+41	20	−12	21	−30
2	− 2	28	+21	23	−53	21–22	−19	22	−53
3	− 1	29	− 2	24	−17	24	−17	23	+24
4	+ 3	30	−12	25	−54	25	+24	24–25	−24
5	+ 7	31	+19	26	+31	26	0	26	−21
6	−14	Aug. 1	−13	27	+ 4	27	−27	29–30	− 4
7	+47	2	+18	28	−28	28	+14	31	+38

TABLE 29—*Comparison of actual with theoretical distribution of residuals of Solution V_5*

Residual limits (0.001 ft.)	Distribution of residuals	
	Theoretical	Actual
+∞ to +61	1.8	1
+60 to +51	5.4	1
+50 to +41	15.6	16
+40 to +31	37.6	33
+30 to +21	71	58
+20 to +11	110	99
+10 to 0	136	161
−10 to 0	136	119
−20 to −11	110	126
−30 to −21	71	73
−40 to −31	37.6	39
−50 to −41	15.6	18
−60 to −51	5.4	11
−∞ to −61	1.8	0
	754.8	755
No. of negative residuals greater than 14.4	188.8	220
No. of negative residuals less than 14.4	188.8	166
No. of positive residuals greater than 14.4	188.8	172
No. of positive residuals less than 14.4	188.8	197
	755.2	755
No. of negative residuals	377.5	386
No. of positive residuals	377.5	369
No. of residuals greater than 14.4	377.5	392
No. of residuals less than 14.4	377.5	363

No. of residuals in solution = 775. Probable error of a single observation = ±14.4.

The agreement between the actual and theoretical distribution of residuals is reasonably good. The evidence indicates that the residuals *on the whole* are largely accidental in character, a characteristic common to all good observations. The computation of the probable error of E_1 and E_2 on the assumption that the residuals obey the law of accidental error is therefore justified, and those probable errors should be increased but little to allow for the discrepancy between the actual and theoretical distribution of the residuals.

RESULTS FROM SOLUTION BB_4. LAKE SUPERIOR

The observations on Lake Superior contributed very little to the final evaporation formula. They served to verify the value of $x=2.6$, and modified very slightly the value of E_2. The value of E_2 from Solution V_6 is $+1.27$. The final adopted value of E_2 is $+1.24$, differing by only 0.03, or 2.4 per cent from the value 1.27. The negative value of E_1 in Solution BB_4, -0.154, is obviously absurd. It is only 3 times its own probable error, however. The probable error of E_2 in Solution BB_4 is nearly 5 times its own probable error.

The relative importance of the two sets of values is also shown in Table 26 in the relative assigned weights of the unknowns. The weight of E_1 from Solution V_6 is 2.8 times its weight from Solution BB_4. The weight of E_2 from Solution V_6 is 2.5 times its weight from Solution BB_4.

The poor results obtained on Lake Superior are probably not due to errors in the evaluation of the mean surface of the whole lake by corrections for wind and barometric effects. In fact the mean elevation of the whole surface of Lake Superior, corrected for wind and barometric effects only, can be determined with greater accuracy from the Marquette gage alone than the mean elevation of the whole surface of Lake Michigan-Huron, corrected for wind and barometric effects only, can be determined from observations at the Mackinaw gage alone. The probable error of the mean elevation of the lake surface in the first case is ± 0.014 foot, and in the second case ± 0.016 foot.

The reasons for the absurd negative value of E_1 and the poor determination of E_2 from observations on Lake Superior are probably to be found in the comparatively meager data for obtaining the mean rainfall on the land and lake, wind velocity and saturation deficit. Contrast Plates 1 with 4, and Tables 1 with 2, 3 with 4, and 5 with 6, relating to Lakes Michigan-Huron and Superior, respectively, with reference to the comparative amount of data available on the two watersheds.

THE BEST VALUE OF x IN THE EXPRESSION $\left(\frac{w}{100}-x\right)$

It was mentioned on page 81 that 7 least-square solutions, dealing with evaporation only, were made. The results of the two final solutions have been presented, as representing the final results of this investigation on evaporation.

It is now proposed to give briefly the facts from all seven of the final least-square solutions for determining the evaporation, with the view of presenting principally the evidence leading to the adoption of the value 2.6 as the best value for x. The question of the proper exponent of the wind velocity, and a discussion, besides the one to be here given, of the proper number of terms—two or three—to evaluate properly the evaporation, will be reserved for a later place.[1]

The principal facts from the final 7 least-square solutions for determining the best form of the evaporation equation, as well as for evaluating the constants in it,

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[1] See page 98.

TABLE 30—*Forms of observation equations used in last seven least-square solutions on Lakes Michigan-Huron and Superior for determining evaporation*

Designation of solution	Lake	Form of observation equation
V_2*.................	Michigan-Huron.....	$e\,E_1+\left[e\left(\frac{w}{100}-2.4\right)\right]E_2+I=v$
V_3.................	Michigan-Huron.....	$e\,E_1+\left[e\left(\frac{w}{100}-2.1\right)\right]E_2+I=v$
V_4.................	Michigan-Huron.....	$e\,E_1+\left[e\left(\frac{w}{100}-2.6\right)\right]E_2+I=v$
V_5.................	Michigan-Huron	$e\,E_1+\left[e\left(\frac{w}{100}-2.6\right)\right]E_2+I=v$
BB_2*†	Superior...........	$e\,E_1+\left[e\left(\frac{w}{100}-2.4\right)\right]E_2+\left[e\left(\frac{w}{100}-2.4\right)^2\right]E_3+I=v$
BB_3	Superior...........	$e\,E_1+\left[e\left(\frac{w}{100}-2.1\right)\right]E_2+I=v$
BB_4	Superior...........	$e\,E_1+\left[\left(\frac{w}{100}-2.6\right)\right]E_2+I=v$

† The term $\left[e\left(\frac{w}{100}-2.4\right)^2\right]E_3$ was omitted for all wind velocities above 10.0 miles per hour.

* In these two solutions, the wind travel, *w*, included all winds, both above and below 240 miles per day.

are shown in Tables 30 and 31. Table 30 shows the seven final forms of the observation equations used on Lakes Michigan-Huron and Superior, and Table 31 shows the derived values of the constants E_1, E_2 and E_3 with their probable errors, and other pertinent data. The evaporation, in terms of *e*, computed from results shown in Table 31, is plotted on the upper part of Plate 6.

Referring again to Table 30, note that the observation equations shown are all of the same form, with certain exceptions as to detail. The following is a brief characterization of the differences in the forms of equations there shown.

The equation for Solution V_5 has already been presented in detail, culminating on page 81.

(*a*) The form of observation equation for Solution V_4 was identically the same as that for Solution V_5. Solution V_4 differed from Solution V_5 in that it contained

TABLE 31—*Principal facts from last seven least-square solutions for determining evaporation*

Designation of solution	Lake	E_1	E_2	E_3	No days of observation	No. of observation equations used	Probable error of a single observation of change in elevation of mean lake surface from one day to next	Mean v^2	$\Sigma\ (v^2)$
V_2.........	M-H	+0.448±0.034	+0.367±0.063	....	842	778	±0.016 ft.	562	428,852
V_3.........	M-H	+ .224± .038	+ .661± .085	....	842	774	± .016	553	418,527
V_4.........	M-H	+ .311± .034	+1.019± .153	....	842	773	± .016	554	422,226
V_5.........	M-H	+ .266± .031	+1.272± .148	....	818	755	± .014	455	343,587
BB_2	S	− .045± .059	+ .925± .153	+0.803±0.264	847	759	± .021	1007	760,553
BB_3	S	− .275± .060	+ .857± .138	....	847	756	± .021	998	752,109
BB_4	S	− .154± .052	+1.163± .234	....	847	754	± .021	1006	754,954

more days and more observation equations, as shown in Table 31. In other words, after the completion of Solution V_4, its derived probable error, ±0.016 foot, indicated that certain additional equations should be combined to form one equation, or that certain equations should be rejected, according to the criteria used for combinations and rejections stated on pages 51 and 73. Solution V_5, the successor to Solution V_4, contained these changes, but in other respects they were identical.

(*b*) Solution V_3 differed from Solution V_4 in respect to the same type of difference as mentioned in (*a*) between Solutions V_4 and V_5, and also in the use of the value 2.1 for x in place of 2.6. That is, Solution V_3 served principally to test out the assumption that the minimum wind velocity which affects the evaporation is 210 miles per day, instead of 260.

(*c*) Solution V_2 differed from V_3 in the respects stated in (*a*) and (*b*) and in the additional respect that all wind velocities were used. That is, Solution V_2 served principally to test out the assumption that the evaporation was affected by all wind velocities, both above and below an assumed mean wind velocity of 240 miles per day, or 10 miles per hour.

(*d*) The results from Solution BB_4 have already been presented, culminating on page 90. The form of equation is identical with that of Solution V_5.

(*e*) Solution BB_3 differed from Solution BB_4 in the same manner as Solution V_3 differed from Solution V_4, as mentioned in (*b*), above.

(*f*) Solution BB_2 differed from Solution BB_3 in the same manner that Solution V_2 differed from Solution V_3, as stated in (*c*), above, and also in the fact that it contained an additional term $\left[e\left(\frac{w}{100}-2.4\right)^2\right]E_3$, which was inserted for all days on which the wind velocity was less than 10 miles per hour, or 240 miles per day. Solution BB_2 served the same purpose as Solution V_2, and in addition, served to test out the assumption that the evaporation curve, for winds below 10 m.p.h., is in the form of a parabola.

The results from the seven least-square solutions are concisely shown in Table 31 in the number of observation equations used in each solution, in the derived values of the constants, in their probable errors, in the probable error of a single observation, in the mean v^2, and in the sum of the squares of the residuals. On Lake Michigan-Huron, Solution V_5 was adopted as being nearest the truth as shown by the sum of the squares of the residuals being a minimum, the probable errors of the constants being a minimum, the probable error of a single observation being a minimum, and the mean v^2 being a minimum. On Lake Superior, in so far as the sum of the squares of the residuals is concerned—the principal single test—the choice is in favor of Solution BB_3. But in Solution BB_4, the value of E_1 is a much smaller negative value than in Solution BB_3, and the wind coefficient, E_2, is nearer the value derived in Solution V_5. These two improvements were judged to outweigh the slight deterioration of Solution BB_4 over Solution BB_3 as shown by the sum of the squares of the residuals, and Solution BB_4 was adopted as best representing the facts on Lake Superior.

The two adopted solutions, V_5 and BB_4, both contain the assumption that the evaporation is little affected by winds less than 260 miles per day. They serve to establish the value 2.6 as the most probable value of x in the expression $\left(\frac{w}{100}-x\right)$.

FIRST APPROXIMATION TO MINIMUM WINDS WHICH AFFECT EVAPORATION

In the seven final least-square solutions for determining evaporation, the range in wind velocities tested, below which ventilation is practically ineffectual in increasing the rate of evaporation, was from 8.7 miles per hour to 10.8 miles per hour, corresponding to values of x of from 2.1 to 2.6, respectively. The questions might well be asked: Why stop at these limits? Why not test other values without or within these limits?

Three considerations led to confining the analyses within the limits indicated and to omit testing out other values within those limits. The first was the direct evidence from Solution V_2, and its predecessors, the second the indirect evidence from both Solutions V_2 and BB_2, and the third was the economics of the problem. The discussion of these three considerations follow.

In Solution V_2 all wind velocities were used, and the wind term was written in the form of a difference, i.e. $\left(\frac{w-240}{100}\right)$, because that form lessened the numerical work and also tended to give a better determination of the constants. As mentioned in (*c*) page 92, w here applies to all wind velocities, and is not limited to higher winds only, as defined on page 9. The value, 240 miles, for the mean daily wind travel, was determined statistically to be about right for the two areas concerned. In the Solution V_2 equation, therefore, the second term is subtractive for winds less than 240 miles per day and additive for winds greater than that. Using the values of the two derived constants shown in Table 31, for a zero wind—a day of perfect calm—the computed evaporation is $\left[+0.448+0.367(-2.4)=\right]$ -0.433 in terms of e. This result is obviously absurd.

A study of the probable errors of E_1 and E_2 in connection with this result indicates that in order that the evaporation shall be greater than zero for a zero wind, the value of E_1 would have to be increased, and that of E_2 decreased, by somewhat more than twice their own respective probable errors. *It is highly improbable that the derived values of E_1 and E_2 from Solution V_2 could be in error by the amounts stated as a result of inaccuracies in the absolute term, I. And, on the other hand, it does not appear possible that they are that much in error because of errors in Assumption No. 5. In other words, in order that the expression shall give positive evaporation at all wind velocities, the straight-line formula as in Solution V_2, in which all values of the wind velocity enter the second term, can not be adopted to represent the facts.*

This evidence (from Solution V_2, of negative evaporation with zero wind) could be interpreted in two ways. First, it could be interpreted as meaning that the evaporation curve, plotted with winds as abscissas (Plate 6), is concave upward instead of a straight line. Or it could be interpreted as meaning that the evaporation is inappreciably affected by winds as strong as 10 miles per hour or less. The actual evidence against the first interpretation, obtained from previous least-square solutions on Lake Michigan-Huron indicated that the exponent of the wind term was unity for all wind velocities on that lake. This evidence will be presented later in discussing the wind exponent.[1] The evidence in support of the second possible interpretation will be presented in connection with the discussion of the second consideration mentioned in the third preceding paragraph which led to the range of winds used.[2]

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[1] See page 99. [2] See page 96.

The evidence from Solution V_2 noted in the second preceding paragraph is very strong. Note that the solution contained 778 observation equations, very much the same ones as shown in Table 24. Other values of E_1 and E_2 from solutions containing fewer observations on both Lakes Michigan-Huron and Superior, and with the straight-line form of equation, as in Solution V_2, gave negative evaporation at zero winds, thus corroborating the evidence from Solution V_2.

Before presenting the direct evidence from Solutions V_2 and BB_2 as to the point at which the wind velocity begins appreciably to affect the rate of evaporation, the results of Solution BB_2 will be discussed further. From earlier solutions than BB_2 on Lake Superior, faint evidence was found of a possible curvature in the evaporation curve for low wind velocities on that lake. That is, on that lake and contrary to the findings on Lake Michigan-Huron, faint evidence was obtained that the exponent of the wind term is greater than unity for low winds. This evidence will be given in detail later in discussing the wind exponent.[1] With that evidence as a basis, Solution BB_2 was set up to make a thorough test, based upon a large number of observations. The results are shown in Table 31 and in the evaporation curve shown on Plate 6 computed by substituting those constants in the general equation shown in Table 30 for various winds. (This computation occurs in columns 1 to 6, Table 32.) This parabolic curve is typical of several others obtained from shorter solutions. It shows a minimum—and negative—evaporation for a wind velocity of about 7.5 miles per hour, and an increasing evaporation for decreasing winds, both of which conditions are contrary to our general knowledge of the subject.

Having obtained the strong evidence from Solution V_2 and its predecessors that the evaporation curve is not a straight line for all wind velocities, and from Solution BB_2 that it is not represented by an equation of the second degree for winds less than 10 miles per hour, the residuals of those two solutions were studied for the purpose of estimating the probable shape of the evaporation curve. These studies furnished the second consideration mentioned on page 93 which led to limiting the range of wind velocities tested to from 8.7 to 10.8 miles per hour. The results of these studies are presented in tabular form in Tables 32 and 33, and are plotted on the upper part of Plate 6 in the first three curves mentioned in the legend.

Referring first to Table 32, the first six columns merely represent the computation of the total evaporation in terms of *e* from the Solution BB_2 equation for various assumed wind velocities. In the seventh column is shown a correction, which is stated to be the negative of the mean *v* divided by the mean *e*. The mean *v* and the mean *e* for each wind velocity is the mean of all such values for that wind velocity which occurred in the 759 observation equations used in Solution BB_2. The mean *v* for any wind velocity with the sign changed, divided by the mean *e* for that wind velocity, gives the correction to the computed evaporation in terms of *e* to get the true evaporation at that wind velocity in terms of *e*. The corrected evaporation is shown in column 8. This corrected evaporation in terms of *e* can now be plotted with evaporation as ordinates and wind velocity as abscissæ, and the resulting curve will be an approximation to the true evaporation curve as determinable from observations on that lake. Inasmuch as this curve was very irregular, the observations were combined into larger groups as shown in the last two columns. The values in the last column but one were not obtained from the

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[1] See page 99.

TABLE 32—*Computation of evaporation curve from results of Solution BB_3, Lake Superior*

Total $E_w = \left[E_1 + \left(\frac{w-x}{100}\right)E_2 + \left(\frac{w-x}{100}\right)^2 E_3\right]e$. Unit=0.001 foot.

Wind velocity (m. p. h.)	$\frac{w-x}{100}$	$\left(\frac{w-x}{100}\right)^2$	$\left(\frac{w-x}{100}\right)E_2$	$\left(\frac{w-x}{100}\right)^2 E_3$	Total E_w in terms of e	Correction, $-\frac{\text{mean } v}{\text{mean } e}$ in terms of e	Corrected total E_w in terms of e	By larger groups	
								Correction, $-\frac{\text{mean } v}{\text{mean } e}$ in terms of e	Corresponding total E_w in terms of e
4.2	−1.4	1.96	−1.295	1.574	0.234	−0.75	−0.52		
4.6	−1.3	1.69	−1.202	1.357	0.110	− .67	− .56		
5.0	−1.2	1.44	−1.110	1.156	.001	− .56	− .56	−0.40	−0.40
5.4	−1.1	1.21	−1.018	.972	− .091	.04	− .05		
5.8	−1.0	1.00	− .925	.803	− .167	.31	.14		
6.2	− .9	.81	− .832	.650	− .227	.30	.07	.25	.02
6.7	− .8	.64	− .740	.514	− .271	.25	− .02		
7.1	− .7	.49	− .648	.393	− .300	.07	− .23	.08	− .23
7.5	− .6	.36	− .555	.289	− .311	.09	− .22		
7.9	− .5	.25	− .462	.201	− .306	− .29	− .59	− .29	− .59
8.3	− .4	.16	− .370	.128	− .287	− .15	− .44	− .15	− .44
8.8	− .3	.09	− .278	.0723	− .251	.15	− .10	.15	− .10
9.2	− .2	.04	− .185	.0321	− .198	.08	− .12	.08	− .12
9.6	− .1	.01	− .0925	.0080	− .129	− .08	− .21	.00	− .04
10.0	.0	.00	.000	.0	− .045	.07	.03		
10.4	.1	.01	.0925	....	.047	.00	.05		
10.8	.2	.04	.185	....	.140	1.00	1.14	.64	.74
11.2	.3	.09	.278	....	.233	.50	.73		
11.7	.4	.16	.370	....	.325	− .08	.24		
12.1	.5	.25	.462	....	.417	−1.43	−1.01		
12.5	.6	.36	.555	....	.510	.18	.69		
12.9	.7	.49	.648	....	.603	.85	1.45	.08	.59
13.3	.8	.64	.740	....	.695	.20	.90		
13.8	.9	.81	.832	....	.787	1.20	1.99		
14.2	1.0	1.00	.925	....	.880	.79	1.67		
14.6	1.1	1.21	1.018	....	.973	7.14	8.11		
15.0	1.2	1.44	1.110	....	1.065	.40	1.46		
15.4	1.3	1.69	1.202	....	1.157	−1.60	− .44		
15.8	1.4	1.96	1.295	....	1.250	− .70	.55		
16.2	1.5	2.25	1.388	....	1.343	....	....		
16.7	1.6	2.56	1.480	....	1.435	.85	2.29		
17.1	1.7	2.89	1.572	....	1.527	1.12	2.65		
17.5	1.8	3.24	1.665	....	1.620	−8.80	−7.18		
17.9	1.9	3.61	1.758	....	1.713	− .80	.91	− .11	1.14
18.3	2.0	4.00	1.850	....	1.805	− .62	1.18		
18.8	2.1	4.41	1.942	....	1.897	....	....		
19.2	2.2	4.84	2.035	....	1.990	−5.00	−3.01		
19.6	2.3	5.29	2.128	....	2.083	....	....		
20.0	2.4	5.76	2.220	....	2.175	−4.00	−1.82		
20.4	2.5	6.25	2.213	....	2.267	....	....		
20.8	2.6	6.76	2.405	....	2.360	4.00	6.36		
21.2	2.7	7.29	2.498	....	2.453	....	....		

seventh column, but were computed from the original tabulation of e's and v's for the wind velocities concerned. This extensive tabulation is not shown for want of space. The mean value for each group shown in the second to last column was placed approximately opposite the center of gravity of the observations in that group, and the corrected E_w shown in the last column was taken as the sum of the correction in the second to last column and that value of the computed E_w in the sixth column which happened to be opposite it. The values in the last column are plotted in the upper part of Plate 6, and are represented by the curve marked "Lake Superior" in the legend. This curve starts at the left with an evaporation of -0.40 in terms of e for a 5-mile wind, rises to an evaporation of $+0.02e$ at a 6.2-mile wind, falls to a minimum evaporation of $-0.59e$ for a 7.9 m.p.h. wind, and rises thereafter for increasing winds.

The computation of the evaporation curve for Lake Michigan-Huron, computed from the results of Solution V_2 in the same manner as that just described for Lake Superior, is shown in Table 33 in the first to seventh columns, inclusive. The corrected total evaporation by large groups, shown in the seventh column, is plotted in the upper part of Plate 6 in the curve marked "Lake M-H" in the legend. It will be observed that this curve has the same general shape as the Superior curve. It has a peak between a 6 m.p.h. and a 7 m.p.h. wind, falls on either side, then rises continuously for winds above about 9 m.p.h. It differs from the Lake Superior curve, however, in that it shows an evaporation greater than zero throughout its extent. The two curves were combined into a weighted mean curve shown on Plate 6 by the curve marked "Weighted Mean" in the legend. The computation of this curve is shown in the last two columns of Table 33. The Lake M-H curve was given twice the weight of the Lake Superior curve, which is approximately the relative weights indicated by the reciprocal of the squares of the probable errors as shown in Table 31, $\left[1 \div \left(\frac{0.016}{0.021}\right)^2 = 1.7\right]$.

It is this weighted mean curve which is of prime significance in fixing upon the final form of the evaporation equation as used in all solutions subsequent to V_2 and BB_2 (Table 30, page 91). The peak in the curve at a 6.7 m.p.h. wind probably has no real significance. Nor has the depression at a 7.9 m.p.h. wind. These irregularities are probably due to accidental influences. If one consider the peak thrown into the depression, the curve may be considered as being roughly horizontal up to a wind velocity of about 9 miles per hour, after which it rises rather steeply. It could not reasonably be considered horizontal beyond a wind of about 11 miles per hour. These studies served to fix upon the range of wind velocities of from 210 miles per day to 260 miles per day tested out in Solutions V_3 to V_5 and BB_3 and BB_4. They not only indicated that the lower limit of wind velocity to use was less than 10 miles per hour, as did the direct evidence from Solution V_2 and other solutions with straight-line formulas, but they indicated the probable upper limit also, and besides showed the curve to be essentially horizontal at low wind velocities.

From a consideration of the economics of the problem, the testing out of other wind velocities between 210 and 260 miles per day did not seem justifiable. Probably very little gain in accuracy, if any, would result from the use of some other value than 2.6 for x. The true value may be 2.5. It is probably not greater than 2.6. The value 2.6 was adopted at the culmination of Solutions V_5 and BB_4 as probably representing the best compromise between extreme accuracy, on the one hand, and large expenditure of time on the other hand.

Table 33—*Computation of evaporation curve from results of Solution V_2, Lake Michigan-Huron*

Total $E_w = \left[E_1 + \left(\frac{w-x}{100}\right)E_2\right]e.$ Unit =0.001 foot.

Wind velocity (m.p.h.)	$\frac{w-x}{100}$	Total E_w in term of e	Correction $-\frac{\text{mean } v}{\text{mean } e}$ in terms of e	Corrected total E_w in terms of e	By larger groups		Corresponding point from solution BB_2 curve	Weighted mean E_w in terms of e Solution V_2, *Wt.*2 Solution BB_2, *Wt.*1
					Correction $-\frac{\text{mean } v}{\text{mean } e}$ in terms of e	Corresponding total E_w in terms of e		
4.2	−1.4	−0.07	−1.33	−1.40				
4.6	−1.3	− .03	1.50	1.47				
5.0	−1.2	.01	− .50	− .49				
5.4	−1.1	.04	.35	.39	+0.07	0.15	−0.13	0.06
5.8	−1.0	.08	.06	.14				
6.2	− .9	.12	.07	.19				
6.7	− .8	.16	.00	.16		(.21)	.02	.15
7.1	− .7	.19	.87	1.06	.19	.35	.06	.21
7.5	− .6	.23	.12	.35	.00	.23	− .23	.08
7.9	− .5	.26	− .24	.02		(.23)	− .59	− .04
8.3	− .4	.30	− .07	.23	− .07	.23	− .44	.01
8.8	− .3	.34	− .06	.28	− .06	.28	− .10	.15
9.2	− .2	.38	.27	.65	.27	.65	− .12	.39
9.6	− .1	.41	.00	.41				
10.0	.0	.45	.25	.70	.19	.64	− .04	.41
10.4	.1	.49	.12	.61				
10.8	.2	.52	.14	.66	.14	.66	.74	.69
11.2	.3	.56	.55	1.11				
11.7	.4	.60	.00	.60				
12.1	.5	.64	− .50	.14				
12.5	.6	.67	.23	.90	.08	.72	.63	.69
12.9	.7	.71	.91	1.62		(.83)	.59	.75
13.3	.8	.75	.64	1.39				
13.8	.9	.78	−1.33	− .55				
14.2	1.0	.82	.90	1.72				
14.6	1.1	.85	2.29	3.14				
15.0	1.2	.89	.08	.97				
15.4	1.3	.93	2.43	3.36				
15.8	1.4	.97	− .33	.64				
16.2	1.5	1.01	−1.18	− .17				
16.7	1.6	1.04	1.40	2.44				
17.1	1.7	1.08	− .19	− .11	.60	1.49	1.00	1.33
17.5	1.8	1.11	− .07	1.04				
17.9	1.9	1.15	......	......				
18.3	2.0	1.19	− .17	1.02				
18.8	2.1	1.22	4.25	5.47				
19.2	2.2	1.26	3.00	4.26				
19.6	2.3	1.30	......	......				
20.0	2.4	1.34	......	......				
20.4	2.5	1.37	......	......				
20.8	2.6	1.41	......	......				
21.2	2.7	1.44	3.17	4.61				

Values in parentheses were interpolated between adjacent values.

OTHER EVIDENCE IN FAVOR OF THE ADOPTED FORM OF THE EVAPORATION FORMULA

The character of the evaporation curve derived from Solution BB_3 is a strong indication that the wind term, $e\left[\left(\frac{w}{100}-2.4\right)^2\right]E_3$, does not belong in the evaporation formula. The results of Solution BB_3 were also analyzed in a more orthodox way than the one already presented. An illustration of this simple test is given on page 19 of Running's *Empirical Formulas.* The general problem is there stated as follows:

> "If two variables, x and y, are so related that when values of x are taken in an arithmetical series the nth differences of the corresponding values of y are constant, the law connecting the variables is expressed by the equation
>
> $y=a+bx+cx^2$ . . . qx^n" ....(24)

TABLE 34—*Study of form of observation equation of Solution BB_3*

$\left(\frac{w}{100}-2.4\right)$	Sum of e's	Sum of I's	Number	Mean e	Mean I	$\frac{\text{Mean } I}{\text{Mean } e}$	First differences	Second differences
−1.4	4	+ 3	1	4	+ 3	+ 0.75		
−1.3	23	+ 14	2	12	+ 7	+ .58	− 0.17	
−1.2	90	+ 49	10	9	+ 5	+ .56	− .02	+ 0.15
−1.1	33	− 25	3	11	− 8	− .73	− 1.29	− 1.27
−1.0	415	+ 30	28	15	+ 1	+ .07	+ .80	+ 2.09
− .9	131	−124	12	11	−10	− .91	− .98	− 1.78
− .8	629	+ 9	50	13	0	.00	+ .91	+ 1.89
− .7	326	+ 92	23	14	+ 4	+ .29	+ .29	− .62
− .6	919	+354	71	13	+ 5	+ .38	+ .09	− .20
− .5	443	+132	31	14	+ 4	+ .29	− .09	− .18
− .4	1271	+724	87	15	+ 8	+ .53	+ .24	+ .33
− .3	430	− 71	31	14	− 2	− .14	− .67	− .91
− .2	1363	+ 87	100	14	+ 1	+ .07	+ .21	+ .88
− .1	378	+ 97	27	14	+ 4	+ .29	+ .22	+ .01
.0	1085	−195	68	16	− 3	− .19	− .48	− .70
+ .1	254	+ 30	20	13	+ 2	+ .15	+ .34	+ .82
+ .2	592	−762	52	11	−15	− 1.36	− 1.51	− 1.85
+ .3	231	− 16	16	14	− 1	− .07	+ 1.29	+ 2.80
+ .4	452	−127	31	15	− 4	− .27	− .20	− 1.49
+ .5	203	+ 65	10	20	+ 6	+ .30	+ .57	+ .77
+ .6	296	−234	27	11	− 9	− .82	− 1.12	− 1.69
+ .7	36	−114	3	12	−38	− 3.17	− 2.35	− 1.23
+ .8	134	− 77	13	10	− 6	− .60	+ 2.57	+ 4.92
+ .9	90	−221	8	11	−28	− 2.55	− 1.95	− 4.52
+1.0	71	−117	5	14	−23	− 1.64	+ .91	+ 2.86
+1.1	6	− 75	1	6	−75	−12.50	−10.86	−11.77
+1.2	127	−114	13	10	− 9	− .90	+11.60	+22.46
+1.3	54	− 75	5	11	−15	− 1.36	− .46	−12.06
+1.4	65	+ 8	6	11	+ 1	+ .09	+ 1.45	+ 1.91
+1.5	....	....	....	....	....	....	....	....
+1.6	22	− 56	2	11	−28	− 2.55	....	....
+1.7	....	....	....	....	....	....	....	....
+1.8	5	+ 36	1	5	+36	+ 7.20	....	....
+1.9	8	− 31	1	8	−31	− 3.88	−11.08	....
+2.0	....	....	....	....	....	....	....	....
+2.1	....	....	....	....	....	....	....	....
+2.2	....	....	....	....	....	....	....	....
+2.3	9	+ 21	1	9	+21	+ 2.33	....	....
+2.4	....	....	....	....	....	....	....	....
+2.5	....	....	....	....	....	....	....	....
+2.6	....	....	....	....	....	....	....	....
+2.7	....	....	....	....	....	....	....	....
			759					

In Solution BB_2 the form of the evaporation equation used was

$$e\,E_1+\left[e\left(\frac{w}{100}-2.4\right)\right]E_2+\left[e\left(\frac{w}{100}-2.4\right)^2\right]E_3+I=0 \ldots\ldots\ldots\ldots (25)$$

Dividing this through by e and placing the absolute term on the left-hand side of the equation there is obtained

$$-\frac{I}{e}=E_1+\left(\frac{w}{100}-2.4\right)E_2+\left(\frac{w}{100}-2.4\right)^2 E_3 \ldots\ldots\ldots\ldots (26)$$

which is similar in form to (24); that is, $y=-\frac{I}{e}$, $x=\left(\frac{w}{100}-2.4\right)$, $a=E_1$, $b=E_2$ and $c=E_3$.

In Solution BB_2, for each value of $\left(\frac{w}{100}-2.4\right)$, the corresponding values of e and I were tabulated and their means computed. The ratio of the mean I to the mean e gave the left-hand member of (26). The first and second differences of this ratio were then calculated. They are shown plotted in the lower half of Plate 6, and the computations are shown in Table 34. The curves of first and second differences, considering the curves as a whole, indicate that the first differences are more constant than the second differences, or that $n=1$, in equation (24). That is, for the curve as a whole, the facts can be represented better by a straight-line than by a parabola. However, for the values of $\left(\frac{w}{100}-2.4\right)$ between -0.7 and 0.0 there appears to be about equal constancy in the first and second differences, or either a straight-line or parabola would represent the facts about equally well in that region.

This evidence favors the adopted form of the evaporation formula only in respect of indicating evaporation to be best evaluated by two terms. It says nothing about the ventilating effects of low winds.

EVAPORATION IS A LINEAR FUNCTION OF THE WIND VELOCITY ABOVE 10.8 MILES PER HOUR

Evidence similar to the above from Solution BB_2 on Lake Superior, indicating that the exponent of the wind term is practically unity, was verified earlier in the investigation by other analyses on both lakes—analyses of a different kind than the one just presented. It has already been stated that on Lake Michigan-Huron no departure whatever from a linear relationship could be detected, but that on Lake Superior faint evidence was obtained that there might be curvature in the evaporation curve for wind velocities below 10 miles per hour, and that this evidence was used as a basis for setting up Solution BB_2. It is now proposed to set forth this evidence from both lakes.

The evidence is from Solution U_3, Lake Superior, and Solution T_6, Lake Michigan-Huron. Each of these solutions contained six months of observations, May to October 1910. These solutions represent early attempts to determine simultaneously the laws of evaporation from the lake surface and of run-off into the lake. The form of observation equation used in each is as follows:

$$e\,E_1+\left[e\left(\frac{w}{100}-2.4\right)\right]E_2-r_1R_1-r_2R_2-r_3R_3-r_4R_4-r_5R_5-r_6R_6+I=v \ldots\ldots\ldots\ldots (27)$$

The terms eE_1, $\left[e\left(\frac{w}{100}-2.4\right)\right]E_2$ and I of equation (27) have already been defined. The first two terms are the same as the corresponding terms in Solutions V_2 and BB_2. The absolute term, I, is the same as in all the V—and BB—solutions shown in Table 30, and is defined on page 9.

The other terms in equation (27), viz, $-r_1R_1, -r_2R_2, -r_3R_3, \ldots r_6R_6$, represent the variable part of the run-off into the lake in one day. As it is the intention at present to limit the attention to the wind term, the definitions of the run-off terms will be reserved for a later place.[1]

The analysis of the residuals of Solution U_3 is shown in Table 35, and of Solution T_6 in Table 36. The third column in these tables shows the number of residuals which occurred in the solution for the wind velocity shown in the second column, corresponding to values of $\left(\frac{w}{100}-2.4\right)$ in the first column. The algebraic sum of all of the residuals corresponding to each wind velocity is shown in the fourth column. The mean residual is shown in the fifth column, and the probable error of the mean residual appears in the sixth column. This is computed from the relation $r_0=\frac{r}{\sqrt{n}}$, in which r_0 is the probable error of the mean residual, r the probable error of a single observation for the solution (± 20.5 for Sol. U_3 and ± 16.4 for Sol. T_6, in units of 0.001 foot), and n equals the number of residuals in the group.

After the first trial grouping, the residuals were grouped in larger groups as shown in the second (and subsequent) trial groupings, in order to make the analysis more powerful. In order to avoid bias in deciding which residuals to include in the larger groups in the trial groupings higher than the first, the rules adopted were as follows:

> In the second trial grouping, the maximum number of residuals which shall constitute any one group shall not exceed the maximum number of residuals for any single wind velocity in the first trial grouping.
>
> In the third and fourth trial groupings, the maximum number of residuals which shall constitute any group shall not exceed the maximum number of residuals obtainable by combining any two adjacent groups in the next earlier trial grouping.

As an illustration of the first rule, it was found that the maximum number of residuals for any single wind velocity in the first trial grouping in Table 35 was 23 for a wind velocity of 9.2 m.p.h. Hence in the second trial grouping the number of residuals combined into a single group was not allowed to exceed 23.

In illustration of the second rule, it was found that the maximum number of residuals obtainable by combining any two adjacent groups of residuals in the second trial grouping was 45. Hence in the third trial grouping, the number of residuals which were combined into a single group were not allowed to exceed 45.

The residuals from Solution U_3, considered as a whole, tabulated with respect to wind velocity as an argument, showed but little departure from the laws of accidental error. On the other hand the following considerations tended to indicate that the evaporation curve, plotted with respect to wind velocities as abscissas, is concave upward—at least for wind velocities below 10 m.p.h.

(*a*) All four groupings as shown in Table 35 showed a tendency for more large residuals than would occur if the errors were all in the accidental class.

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[1] See page 104.

Table 35—*Analysis of residuals of Solution V_3, Lake Superior, for evidence of wind exponent*

$\left(\frac{w}{100}-2.4\right)$	Wind velocity w (m. p. h.)	First trial grouping				Second trial grouping				Third trial grouping				Fourth trial grouping			
		No. of v's	Sum of v's	Mean v	P. E. of mean v	No. of v's	Sum of v's	Mean v	P. E. of mean v	No. of v's	Sum of v's	Mean v	P. E. of mean v	No. of v's	Sum of v's	Mean v	P. E. of mean v
−1.2	5.0	1	− 15	−15	±20												
−1.1	5.4	0	....	....	......	11	− 67	− 6	± 6	11	− 67	− 6	±6				
−1.0	5.8	5	− 59	−12	± 9												
− .9	6.2	5	7	1	± 9												
− .8	6.7	13	−189	−14	± 6	20	−263	−13	± 5					86	75	1	±2
− .7	7.1	7	− 74	−11	± 8					38	− 86	− 2	±3				
− .6	7.5	18	177	10	± 5	18	177	10	± 5								
− .5	7.9	12	142	12	± 6	12	142	12	± 6								
− .4	8.3	21	84	4	± 4	21	84	4	± 4	37	227	6	±3				
− .3	8.8	4	1	0	±10	4	1	0	±10								
− .2	9.2	23	49	2	± 4	23	49	2	±4								
− .1	9.6	7	6	1	± 8	20	69	3	±5	43	118	3	±3				
.0	10.0	13	63	5	± 6												
.1	10.4	2	29	14	±14												
.2	10.8	9	− 44	− 5	± 7	22	−184	−14	± 4								
.3	11.2	4	14	3	±10												
.4	11.7	7	−183	−26	± 8												
.5	12.1	2	90	45	±14									88	−166	− 2	±2
.6	12.5	8	− 49	− 6	± 7					45	−284	− 6	±3				
.7	12.9	1	− 14	−14	±20												
.8	13.3	4	90	22	±10												
.9	13.8	4	− 93	−23	±10	23	−100	− 4	± 4								
1.0	14.2	1	− 24	−24	±20												
1.1	14.6	1	− 72	−72	±20												
1.2	15.0	2	− 28	−14	±14												
1.3	15.4	2	16	8	±14												
1.4	15.8	2	48	24	±14	5	63	13	± 9	5	63	13	±9	5	63	13	±9
1.5	16.2	......	....	....	......												
1.6	16.7	1	− 1	− 1	±20												
		179				179				179				179			

In the First Trial Grouping there are 27 mean v's. 13 are less than their own p.e.'s; 2 are greater than 3 times their own p.e.'s; 8 are $\gtrless$ 2 times their own p.e.'s; 1 is greater than 3.5 times its own p.e., viz, −72±20.

In the Second Trial Grouping, there are 11 mean v's. 3 are < their own p.e.'s; 1 is 3.5 times its own p.e., viz, −14±4.

In the Third Trial Grouping, there are 6 mean v's. Of these, 1 is less than its own p.e.; 2 are as great as 2 times their own p.e.'s.

In the fourth trial grouping, there are 3 mean v's. 1 is less than, and 1 is equal to its own p.e. 1 is $1\frac{4}{9}$ as large as its own p.e.

(*b*) The first, second and third trial groupings showed a tendency for + residuals between wind velocities of 7.5 to 10.4 m.p.h. This was shown also in the original detailed tabulation, which is not presented here for want of space.

(*c*) From the original tabulation and the four trial groupings there appeared but little—and certainly only inconclusive—evidence of any systematic groupings of signs in the residuals, and therefore of curvature in the evaporation curve.

(*d*) For wind velocities of 7.1 m.p.h. or lower $\left(\left(\frac{w}{100}-2.4\right)=-0.7\right.$ or less) the first trial grouping showed negative mean v's in four out of five groups, and three out of four of these negative means are larger than their own p.e. and the one positive value is only $\frac{1}{9}$ of its own p.e. This evidence was also confirmed by the second and third trial groupings.

The above faint evidence of curvature for wind velocities below 10 m.p.h. led to the test of that idea in Solution BB_2 with the results already noted. One other consideration, obtained largely from other Solutions than U_2, led to the belief that the evaporation curve is concave upward. This has already been noted in connection with Solution V_2, and was the fact that with the best determinable values of E_1 and E_2 from the straight-line form of evaporation equation, the computed evaporation for zero winds came out negative.

The distribution of the residuals with respect to wind velocity in Solution T_6, shown in Table 36, indicates that, within the allowable limits of accuracy of that solution, it is probably impossible to determine any departure of the wind exponent from unity.

The proper conclusions from these studies, and as embodied in equation (22), appear to be, (*a*) that the evaporation in terms of e is a constant for wind velocities up to 10.8 m.p.h., and (*b*) that it is a linear function of the wind velocity for winds higher than that.

RUN-OFF INTO THE LAKE

The method of obtaining a first approximation to the constant part of the run-off into the lake was stated on pages 16 to 19, and it was stated in connection with the discussion of the results of Solution V_6 on pages 84 to 86 and on page 80 that the only method devised for evaluating the variable part of the run-off into the lake failed.

It is now proposed to show how it was attempted in this investigation to evaluate the variable part of the run-off into each of Lakes Michigan-Huron and Superior, and to show how the first approximate values of I. were revised by the least-square computations by successive trials until more correct values were obtained.

VARIABLE PART OF RUN-OFF INTO LAKE MICHIGAN-HURON

One form of observation equation used in the attempt to evaluate simultaneously the evaporation from the lake surface and the run-off into the lake from the adjacent drainage area is equation (27), which is here repeated for convenience as equation (28), namely

$$eE_1+\left[e\left(\frac{w-240}{100}\right)\right]E_2-r_1R_1-r_2R_2-r_3R_3-r_4R_4-r_5R_5-r_6R_6+I=v \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

This is the same as equation (1), page 8, except in two particulars, namely, (*a*) that the run-off terms $-r_1R_1, -r_2R_2, \ldots r_6R_6$, expressing the variable part of

Table 36—*Analysis of residuals of Solution T_6. Lake Michigan-Huron. for evidence of wind exponent*

$\left(\frac{w}{100}-2.4\right)$	Wind velocity, w. (m. p. h.)	First trial grouping				Second trial grouping				Third trial grouping				Fourth trial grouping			
		No. of v's	Sum of v's	Mean v	P. E. of Mean v	No. of v's	Sum of v's	Mean v	P. E. of Mean v	No. of v's	Sum of v's	Mean v	P. E. of mean v	No. of v's	Sum of v's	Mean v	P. E. of mean v
−1.0	5.8	4	− 3	− 1	± 8	4	− 3	−1	± 8	4	− 3	−1	±8				
− .9	6.2	1	42	42	±16	20	36	2	± 4								
− .8	6.7	19	− 6	0	± 4					39	−28	−1	±3	43	−31	−1	±2
− .7	7.1	4	−60	−15	± 8	19	−64	−3	± 4								
− .6	7.5	15	− 4	0	± 4												
− .5	7.9	12	−19	− 1	± 5	12	−19	−1	± 5								
− .4	8.3	14	−30	− 2	± 4	14	−30	−2	± 4	41	35	1	±3				
− .3	8.8	15	84	6	± 4	15	84	6	± 4								
− .2	9.2	21	1	0	± 4	21	1	0	± 4					82	−17	0	±2
− .1	9.6	9	−12	− 1	± 5	20	−53	−3	± 4	41	−52	−1	±3				
.0	10.0	11	−41	− 4	± 5												
.1	10.4																
.2	10.8	13	39	3	± 5	14	33	2	± 4								
.3	11.2	1	− 6	− 6	±16					32	130	4	±3				
.4	11.7	14	44	3	± 4	18	97	5	± 4								
.5	12.1	4	53	13	± 8												
.6	12.5	7	56	8	± 6												
.7	12.9	3	−32	−11	±10												
.8	13.3	4	−82	−20	± 8	21	−68	−3	± 4					56	56	1	±2
.9	13.8	1	21	21	±16												
1.0	14.2	6	−31	− 5	± 7					24	−74	−3	±3				
1.1	14.6	2	−14	− 7	±12												
1.2	15.0					3	− 6	−2	±10								
1.3	15.4																
1.4	15.8	1	8	8	±16												
		181				181				181				181			

In First Trial Grouping, there are 22 mean v's. 14 are $<$ their own p.e.'s; 1 is as much as 2.5 times its own p.e.

In Second Trial Grouping, there are 12 mean v's. 10 are $<$ their own p.e.'s and only 1 is as much as 1½ times its own p.e.

In Third Trial Grouping, there are 6 mean v's. 4 are $<$ their own p.e.'s, and only 1 is as much as 1.3 times its own p.e.

In Fourth Trial Grouping, there are 3 mean v's, all less than their own p.e.'s.

the run-off, have been introduced, and (*b*) that the wind travel, w, applies to all wind velocities instead of being limited to wind velocities above x. In equation (28),

r_1 = (Observed rainfall on land drainage area tributary to lake) − (estimated evaporation from land) − (estimated run-off) on the current day, expressed in units of 0.001 foot of depth on the lake area.

r_2 = (Observed rainfall on land drainage area tributary to lake) − (estimated evaporation from land) − (estimated run-off) on the preceding day, expressed in units of 0.001 foot of depth on the lake area.

r_3 = (Observed rainfall on land drainage area tributary to lake) − (estimated evaporation from land) − (estimated run-off) on the day before the preceding day, expressed in units of 0.001 foot of depth on the lake area.

r_4 = (Observed rainfall on land drainage area tributary to lake) − (estimated evaporation from land) − (estimated run-off) on the two days preceding the day named in defining r_3, expressed in units of 0.01 foot of depth on the lake area.

r_5 = (Observed rainfall on land drainage area tributary to lake) − (estimated evaporation from land) − (estimated run-off) on the four days preceding the two days named in defining r_4, expressed in units of 0.01 foot of depth on the lake area.

r_6 = (Observed rainfall on land drainage area tributary to the lake) − (estimated evaporation from land) − (estimated run-off) on the eight days preceding the four days named in defining r_5, expressed in units of 0.01 foot of depth on the lake area.

R_1 is a physical constant which expresses the effect of the change in storage in the land part of the drainage area on the current day, r_1, on the rise of the lake surface that current day.

Similarly, R_2, R_3, . . . R_6, are physical constants which express the effects of the change in storage in the ground on the preceding day, day before the preceding day, . . . next preceding eight days, respectively, upon the rise of the lake surface the current day.

The similarity between the run-off part of equation (28) and the stream-flow equations in Part II of this publication should be evident.

It was proposed to determine the unknowns, R_1, R_2, . . . R_6 indirectly from the least-square solutions by successive trials. The unknowns E_1 and E_2 were to be derived directly. The mean value of the wind travel determined statistically to be 240 miles per day was to be adjusted by the least-square computations to that value which, in combination with the best values of the various unknowns, was to reduce $\Sigma(v^2)$ to a minimum.

The evaporation terms have previously been defined, and $+I$ is the same as defined on page 9, and shown in column 10 of Table 23.

Equation (28) expresses the fact that the lake surface is caused to fall by the natural evaporation from it, which is assumed at first to follow the straight-line law expressed in the first two terms. It further expresses the fact that the lake surface is caused to fall by the negative of the run-off into it, as expressed by the R-terms. If the total fall, on any day, as computed from all terms to the left of $+I$ is just equal to $+I$, that is to the observed rise of the lake surface corrected for wind and barometric effects, inflow, outflow, rainfall on the lake, and constant part of run-off into the lake, then the residual, v, on that day will be zero.

EXAMPLE OF COMPUTATION OF r_1, r_2, r_3, . . . r_6

From previous examples the method of computing all of the quantities in the observation equation, (28), except the run-off terms, r_1, r_2, r_3, . . . r_6, should be clear. In order to clarify the method of computing the r's, a numerical example, shown in Table 37, is taken from the least-square solution designated as V_1 pertaining to Lake Michigan-Huron.

Attention is invited to Table 37, in connection with the definitions of r_1, r_2, r_3, . . . r_6 on page 104. In order to estimate the evaporation from land, E_l, it is necessary to assume values for E_1 and E_2, and first estimate the evaporation from the water (lake) surface, E_w, on the basis of these assumed values, the assumed straight-line evaporation law and the observed weather elements e and w. The best values of E_1 and E_2 obtainable from previous least-square solutions were $E_1=+0.581$ and $E_2=+0.624$. These were accordingly used in Solution V_1 in estimating the evaporation. The sum of the products eE_1 and $e\left(\frac{w-240}{100}\right)E_2$ are shown in column 4. This is the estimated evaporation, E_w, from the lake surface in units of 0.001 foot of depth per day. Using the ratio $\frac{E_l}{E_w}=0.62$ obtained on pages 16 to 18, and the ratio of land to lake area for Lake Michigan-Huron shown in Table 7, page 17, $\frac{92{,}493}{45{,}314}=2.04$, the estimated evaporation from the land surface, E_l, in units of 0.001 foot of depth on the lake area is $E_w\times 0.62\times 2.4=1.26\ E_w$. This is shown in column 13, Table 37.

Similarly, in order to estimate the run-off from the land into the lake on any day, it is necessary to assume the run-off constants, R_1, R_2, R_3, . . . R_6. This is more difficult than assuming values of E_1 and E_2. The estimated constant part of the run-off into the lake each day in 0.001 foot of depth is shown in column 5 as $I_c=6$. The value originally estimated was 8 (page 19). The method of revising the original value, 8, to get the final value, 6, has yet to be stated.[1] The estimated variable part of the run-off into the lake each day is shown in columns 6 to 11 inclusive, based upon the following estimated values, (29), or R_1, R_2, R_3, . . . R_6.

$$\left.\begin{array}{ll} R_1=+0.01 & R_4=+0.80 \\ R_2=+0.07 & R_5=+0.20 \\ R_3=+0.17 & R_6=+0.27 \end{array}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

The quantity r_1, by definition, is the total change in storage in the ground within the drainage area of the lake on the current day, and it is expressed in the same units as the absolute term of the equation, I. Now, from equation (28) it should be evident that R_1 is that fraction of r_1 which is delivered to the lake on the current day. From this, and the definitions of r_1, r_2, r_3, . . . r_6, it can be shown that, if one assume that a change in storage on the current day, r_1, is followed by no change in storage for a long time thereafter,

100 R_1 per cent of r_1 is delivered to the lake on the current day, or 0th day;
100 R_2 per cent of r_1 is delivered to the lake on the first following day;
100 R_3 per cent of r_1 is delivered to the lake on the second following day;
5.0 R_4 per cent of r_1 is delivered to the lake on *each* of the third and fourth following days;
2.5 R_5 per cent of r_1 is delivered to the lake on *each* of the fifth to eighth following days, inclusive;
1.25 R_6 per cent of r_1 is delivered to the lake on *each* of the ninth to sixteenth following days, inclusive.

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[1] See page 117.

And the total amount of r_1 which is delivered to the lake during the seventeen-day interval beginning with and following the current day is [100 R_1+100 R_2+ 100 R_3+(5.0)(2) R_4+(2.5)(4) R_5+(1.25)(8) R_6] per cent, on the assumption that, following the change in storage on the current day, (r_1), the sum of the evaporation from land and run-off from land are just equal to the rainfall on land on each day for a long time thereafter.

Substituting the assumed values of R_1, R_2, R_3, . . . R_6, (29) in this expression, it appears that 37 per cent of the change in storage in the ground on any day, r_1, is assumed to reach the lake by the end of the sixteenth day thereafter, if, during the 16 days following the r_1 there is no change in storage in the ground.

In estimating the values of R_1, R_2, R_3, . . . R_6 to use in Solution V_1 and previous solutions, they were first estimated as percentages, represented by the products of the various R's by the factors 100, 5.0, 2.5 and 1.25. Thus, the assumed R's, (29), expressed as percentages are

$$\left.\begin{array}{ll} R_1=+\ 1 & R_4=+4.0 \\ R_2=+\ 7 & R_5=+0.5 \\ R_3=+\ 17 & R_6=+0.3 \end{array}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(30)$$

An example of the meaning of this is that (5.0) (0.80) = 4.0 per cent of r_1 is assumed to be delivered to the lake on each of the third and fourth following days, if no change in storage occurred after the current day or day on which the change in storage, r_1, took place.

The reason for estimating the values of the R's as percentages (30) is that it is easier to do it that way. The estimates so made are converted to the absolute values as in (29), by dividing by the appropriate factor 100, 5.0, 2.5, or 1.25.

The estimates of the R's and of the E's (used in estimating the evaporation) depended upon (*a*) judgment, (*b*) internal evidence from previous least-square solutions, and (*c*) external sources of information in the engineering literature. In the very first least-square solution of a series, in which the observation equation of the form of (28) was used, the estimates of E_1, E_2, I_c, R_1, R_2, . . . R_6 depends entirely upon judgment and information, such as could be obtained in the engineering and scientific literature. As an example, for the first least-square solution on Lake Michigan-Huron in which it was attempted to evaluate simultaneously the evaporation and run-off by equation (28), a value for E_1 was estimated as follows: The mean *e* for June 1910 for the Lake, computed as in the illustration on pages 25 to 29, was 20, and for July 1910, 24. From the Monthly Weather Review, September 1888, the depth of evaporation, in inches, at signal service stations in thermometer shelters, computed from the means of the tri-daily determination of dew-point and wet-bulb observations, varied from 2.5 at Duluth to 4.9 at Rochester, with a mean of 4.1 inches for June 1888 for the 17 stations in the Upper and Lower Lake Regions. This averages 0.14 inch per day.

Similarly from the same publication for July 1887, for the same 17 stations the computed evaporation varied from 3.4 at Marquette to 6.0 at Toledo, with a mean of 4.8 inches for the month, or 0.15 inch per day. Assuming all the evaporation to be accounted for upon an average by the term $e\,E_1$ and that June 1910 = June 1888 and July 1910 = July 1887, for June, $E_1=\frac{0.14}{20}=0.0070$ and for July $E_1=\frac{0.15}{24}=$ 0.0062. Or, as a mean, $E_1=0.0066$ when the unit is the inch. But if the unit is

0.001 foot as in this evaporation investigation, $E_1=(0.0066)\times\left(\frac{1000}{12}\right)=0.55$. If it is now assumed that part of the evaporation in these two months is represented by the $\left[e\left(\frac{w-240}{100}\right)\right]$ E_2–term, the value of E_1 may be arbitrarily reduced to 0.50, as this value lends itself easily to calculation. The value of E_2 was arbitrarily assumed to be $\frac{2}{5}$ as large as E_1 or $+0.20$ in the first solution. These values were revised many times from the actual observations in the many least-square solutions which preceded Solution V_1 before the values assumed in that solution were secured.

The first estimate of values of $R_1, R_2, R_3, \ldots R_6$ to use in the first least-square solution was nothing more than a guess. The internal evidence from the succeeding least-square solutions gave positive proof that this original estimate was too large. Accordingly the values were reduced in each succeeding solution so long as the evidence persisted in proving the assumed values still too large. Solution V_1 represents the eighth least-square solution on Lake Michigan-Huron in which observation equations of the form of (28) was used. In these eight solutions, the original estimate of the R's was revised several times before the final values, (29), were secured. As will be shown later, these values are still probably much too large.

The principal type of evidence from the least-square solutions which indicated an assumed set of R's to be too large was the fact that the estimated run-off as shown in column 12, Table 37, would sometimes come out negative. Theoretically, negative run-off would be an absurdity. It would correspond to a slope of the water "table," downward from the lake, instead of downward toward the lake. This condition might exist locally, but probably not generally for the whole lake region. Also a ground-water level sloping downward away from a lake or reservoir surface could exist in the case of a lake located on top of a hill or mountain—a lake whose content was maintained by precipitation directly upon its surface, and whose only outlets would be, say, seepage and evaporation.

The observed rainfall on the land drainage area tributary to the lake, used in computing $r_1, r_2, r_3, \ldots r_6$ was obtained from observations made at the 142 stations shown in Table 5 and on Plate 1. The rainfall observed at each station in inches was multiplied by the station-factor shown in Table 5, and the sum of these products gave the rainfall on the land drainage area in units of 100 inch-miles. This was then converted to units of 0.001 foot of depth on the lake surface by multiplying it by the factor 0.184. Except for the conversion stated in the preceding sentence, the computation was exactly analogous to the illustration given of the computation of the rainfall on the lake, I_2, shown on pages 32 to 38.

The rainfall, computed as stated in the preceding paragraph, is shown in column 14, Table 37, preceded by a negative sign.

To compute $-r_1$ for any day, one adds together algebraically the three quantities $-$(observed rainfall), $+$(estimated evaporation from land), and $+$(estimated run-off) for that day. The method of computing the first two quantities has been explained.

Estimated run-off for any day, shown in the twelfth column of Table 37, is the sum of the constant part of the run-off, $I_c=6$, shown in column 5, and the variable part of the run-off, $R_1r_1+R_2r_2+R_3r_3+\ldots+R_6r_6$, shown in columns 6 to 11 inclusive, computed by multiplying the assumed values of $R_1, R_2, R_3, \ldots R_6$, (29), into the computed values of $r_1, r_2, r_3, \ldots r_6$ respectively. To be specific, the

TABLE 37—*Example of computation of $r_1, r_2, r_3, \ldots r_6$, Solution V_1, Lake Michigan-Huron*

Unit=0.001 foot of depth on lake area, except in last three columns

Date	eE_1	$e\left(\frac{w-240}{100}\times E_2\right)$	Estimated E_w	$I_c=6$	Estimated values of run-off based upon $R_1=+0.01$, $R_2=+0.07$, $R_3=+0.17$, $R_4=+0.80$, $R_5=+0.20$ and $R_6=+0.27$						+Estimated run-off	+Estimated E_l	−Observed rainfall on land	$-r_1$	$-r_2$	$-r_3$	Unit=0.01 foot		
					$+R_1r_1$	$+R_2r_2$	$+R_3r_3$	$+R_4r_4$	$+R_5r_5$	$+R_6r_6$							$-r_4$	$-r_5$	$-r_6$
Column	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
1911 Apr. 2	+3	−2	+1	+6	0	......	......	......	......	......	+6	+1	−2	+5	......	......	......	......	......
3	+2	+1	+3	+6	0	0	......	......	......	......	+6	+4	−2	+8	+5	......	......	......	......
4	+1	+1	+2	+6	0	−1	−1	......	......	......	+4	+3	−17	−10	+8	+5	......	......	......
5	+1	+1	+2	+6	0	+1	−1	......	......	......	+6	+3	−40	−31	−10	+8	......	......	......
6	+2	+1	+3	+6	0	+2	+2	−1	......	......	+9	+4	−43	−30	−31	−10	+1.3	......	......
7	+2	−1	+1	+6	0	+2	+5	0	......	......	+13	+1	−20	−6	−30	−31	−0.2	......	......
8	+3	−4	−1	+6	0	0	+5	+3	......	......	+14	−1	−1	+12	−6	−30	−4.1	......	......
9	+2	−3	−1	+6	0	−1	+1	+5	......	......	+11	−1	0	+10	+12	−6	−6.1	......	......
10	+5	−4	+1	+6	0	−1	−2	+3	+1	......	+7	+1	0	+8	+10	+12	−3.6	−2.8	......
11	+7	−3	+4	+6	0	−1	−2	0	+1	......	+4	+5	0	+9	+8	+10	+0.6	−6.3	......
12	+5	−2	+3	+6	0	−1	−1	−2	+2	......	+4	+4	−11	−3	+9	+8	+2.2	−7.7	......
13	+4	0	+4	+6	0	0	−2	−1	+1	......	+4	+5	−27	−18	−3	−9	+1.8	−5.5	......
14	+8	+4	+12	+6	0	+1	+1	−1	0	......	+7	+15	−40	−18	−18	−3	+1.7	−1.4	......
15	+8	+4	+12	+6	0	+1	+3	0	0	......	+10	+15	−26	−1	−18	−18	+0.6	+2.4	......
16	+10	0	+10	+6	0	0	+3	+2	−1	......	+10	+13	−2	+21	−1	−18	−2.1	+3.9	......
17	+9	−9	0	+6	0	−1	0	+3	0	......	+8	0	0	+8	+21	−1	−3.6	+2.4	......
18	+13	−7	+6	+6	0	−1	−4	+2	0	+1	+4	+8	−1	+11	+8	+21	−1.9	−0.4	−4.2
19	+1	−1	0	+6	0	−1	−1	−2	+1	+1	+4	0	−18	−14	+11	+8	+2.0	−3.0	−3.9
20	+4	−4	0	+6	0	+1	−2	−2	+1	+1	+5	0	−21	−16	−14	+11	+2.9	−4.0	−3.8
21	+9	−7	+2	+6	0	+1	+2	−2	0	+1	+8	+3	−6	+5	−16	−14	+1.9	−1.6	−3.1
22	+8	−3	+5	+6	0	0	+3	0	0	0	+9	+6	−3	+12	+5	−16	−0.3	+1.0	−1.8
23	+5	−2	+3	+6	0	−1	−1	+2	−1	0	+5	+4	−2	+7	+12	+5	−3.0	+3.9	−0.6
24	+5	−6	−1	+6	0	0	−2	+1	−1	0	+4	−1	0	+3	+7	+12	−1.1	+2.6	−0.1
25	+9	−11	−2	+6	0	0	−1	−1	0	0	+4	−3	0	+1	+3	+7	+1.7	−1.1	+0.8
26	+11	−12	−1	+6	0	0	−1	−2	0	0	+3	−1	0	+2	+1	+3	+1.9	−1.4	+0.6
27	+14	−9	+5	+6	0	0	0	−1	0	0	+5	+6	0	+11	+2	+1	+1.0	−1.3	+0.9
28	+10	−7	+3	+6	0	−1	0	0	0	0	+5	+4	−1	+8	+11	+2	+0.4	+0.8	−1.4
29	+3	−3	0	+6	0	−1	−2	0	−1	+1	+3	0	−6	−3	+8	+11	+0.3	+2.7	−2.7
30	−2	+2	0	+6	0	0	−1	−1	0	0	+4	0	−19	−15	−3	+8	+1.3	+2.3	−0.4
May 1	+1	0	+1	+6	0	+1	+1	−2	0	−1	+5	+1	−45	−39	−15	−3	+1.9	+1.3	+2.6
2	+3	+4	+7	+6	0	+3	+3	0	0	−1	+11	+9	−36	−16	−39	−15	+0.5	+1.7	+3.4
3	+3	+1	+4	+6	0	+1	+7	+1	0	0	+15	+5	−6	+14	−16	−39	−1.8	+2.2	+1.6
4	+6	−1	+5	+6	0	−1	+3	+4	0	0	+12	+6	−1	+17	+14	−16	−5.4	+1.8	+0.9
5	+7	−3	+4	+6	0	−1	−2	+4	0	0	+7	+5	0	+12	+17	+14	−5.5	+0.1	0.0
6	+8	−4	+4	+6	0	−1	−3	0	+1	−1	+2	+5	0	+7	+12	+17	−0.2	−4.9	+2.5
7	+10	−2	+8	+6	0	0	−2	−2	+1	−1	+2	+10	−1	+11	+7	+12	+3.1	−7.3	+4.9
8	+12	−2	+10	+6	0	−1	−1	−2	+1	−1	+2	+13	−3	+12	+11	+7	+2.9	−5.6	+4.1
9	+8	−6	+2	+6	0	−1	−2	−2	0	0	+1	+3	−6	−2	+12	+11	+1.9	−2.4	+1.4
10	+8	−2	+6	+6	0	0	−2	−1	−1	+1	+3	+8	−22	−11	−2	+12	+1.8	+2.7	−3.2

method of computation of the estimated run-off for the first few days in Table 37 is as follows: On April 2, 1911, the only known part of the run-off is the constant part, 6. Hence $-r_1$ for that day $=-2+1+6=+5$, shown in column 15. This is an approximation to the true $-r_1$, for April 2 in the sense that the variable part of the run-off has been omitted because it is unknown. This approximate $-r_1$, for April 2, by definition, becomes $-r_2$ for April 3, and $-r_3$ for April 4, shown in the sixteenth and seventeenth columns, respectively, Table 37, opposite April 3 and 4.

On April 3, the product of $-(-r_2)$ and R_2, viz, $+R_2r_2=0.07\times-5=-0.35=-0.00035$ foot, is that part of the change in storage on April 2 which is assumed to be delivered to the lake on April 3. As this is less than 0.5 of 1/1000 foot, the unit used in the equations, it is entered as zero in the $+R_2r_2$ column on April 3. Thus far on April 3 our total knowledge of the run-off is given by the sum of the constant part, 6, and that part of the variable part, $+R_2r_2=0$. The sum of these two is 6, which, added to the evaporation and rainfall for that day, $+4-2$, gives 8 as the estimated value of $-r_1$ for April 3, which is an approximation in the sense that that part of the variable part of the run-off for April 3 represented by the $R_3r_3+R_4r_4+R_5r_5+R_6r_6$ terms can not be known, and the R_1r_1 part can not be known at the beginning of April 3 because r_1 is not known at that time. With the estimated value of $-r_1=+8$, we now obtain the product $+R_1r_1=(0.01)(-8)$ for April 3. This is zero in the units used and is so recorded in the R_1r_1 column for April 3. Since the addition of this quantity has not changed the first estimate of the run-off for April 3, $+6$, the value of $-r_1$ for that date, $+8$, stands as the best obtainable. This becomes, by definition, $-r_2$ for April 4, $-r_3$ for April 5 and combines with r_3 for April 4 to give $-r_4$ for April 6 which is in units 10-times as large, or $+1.3$.

On April 4 the first estimate obtainable for the run-off is $I_c+R_2r_2+R_3r_3$ or $6+(0.07)(-8)+(0.17)(-5)$ or $+4$, and of $-r_1$, $+4+3-17=-10$. With this first estimate, we compute the quantity $R_1r_1=(0.01)(10)=0$ in the units used. This now makes the second estimate of the run-off $+6+0-1-1=4$, still. Hence the first estimate of $-r_1$ remains unchanged at -10. This becomes $-r_2$ for April 5, $-r_3$ for April 6 and added to $+8$ and divided by 10 gives the $-r_4=-0.2$ of April 7.

On April 5 the first estimate obtainable of the run-off is $I_c+R_2r_2+R_3r_3$, or $6+(0.07)(10)+(0.17)(-8)$, or $+6$, and of $-r_1$, $+6+3-40=-31$. With this first estimate, an approximate $+R_1r_1$ can be computed, viz, $(0.01)(+31)=0$ in the units used. This now makes the second estimate of the run-off $+6+0+1-1=+6$, still. Hence the first estimate of $-r_1$ remains unchanged at -31. This becomes, by definition, $-r_2$ for April 6, $-r_3$ for April 7; added to -10 and divided by 10 gives $-r_4$ of April 8, viz, -4.1; and added to $-10+8+5$ and divided by 10 gives $-r_5$ of April 10, viz, -2.8.

On April 6 the first estimate obtainable of the run-off is $I_c+R_2r_2+R_3r_3+R_4r_4$, or $6+(0.07)(31)+(0.17)(10)+(0.80)(-1)$, or $6+2+2-1=+9$, and of $-r_1$, $+9+4-43=-30$. With this first estimate, an approximate $+R_1r_1$ can be computed, viz, $(0.01)(+30)=0$, in the units used. This now makes the second estimate of the run-off $+6+0+2+2-1=+9$, still. Hence the first estimate of $-r_1$ remains unchanged at -30. This is written into the table, according to definition, as $-r$'s for succeeding dates.

The process indicated is carried forward day by day. On April 10 it becomes possible to compute all $-r$'s to $-r_5$ inclusive, and on April 18 all of the $-r$'s can be computed. Beginning with April 18, the sixteenth day after starting the computation, the first estimate obtainable of the run-off is $I_c+R_2r_2+R_3r_3+R_4r_4+R_5r_5+R_6r_6$, or $6+(0.07)(-8)+(0.17)(-21)+(0.80)(+2)+(0.20)(0)+(0.27)(+4) = 6-1-4+2+0+1=+4$. The first estimate of $-r_1$ is $+4+8-1=+11$, with which a first estimate of $+R_1r_1$ for that day can be computed, viz, $(0.01)(-11) = 0$, in the units used. Since this, added to the first estimate of the run-off, $+4$, does not change it, the first estimate of $-r_1$ for April 18, $+11$, remains unchanged at that value.

After April 18, to the end of October 31, 1911, the last day for which an observation equation is written in that year, $-r_1$ for any day is computed as for April 18; that is, the first estimate of the run-off is always $I_c+R_2r_2+R_3r_3+R_4r_4+R_5r_5+R_6r_6$. The first estimate of $-r_1$ is $[(I_c+R_2r_2+R_3r_3+R_4r_4+R_5r_5+R_6r_6)+$ (Estimated E_l)$-$(Observed rainfall)]. With this value, the first estimate of $+R_1r_1$ is obtainable, which added to the first estimate of the run-off, gives the second estimate. If this second estimate of the run-off now fails to change the first estimate of $-r_1$, the computation is carried forward to the next day.

If the first estimate of the $+R_1r_1$ for any day differs from zero, this will change the first estimate of the run-off, $I_c+R_2r_2+R_3r_3 \ldots R_6r_6$, for that day by the amount $+R_1r_1$. This, in turn will change the first estimate of $-r_1$ by the same amount. Using this second estimate of $-r_1$, a second estimate of $+R_1r_1$ may be obtained. If this agree with the first estimate of $+R_1r_1$ the computation may proceed to the next day. If it differ from the first estimate of $+R_1r_1$, the cycle must be repeated, and this repetition must continue until the nth estimate of $-r_1$ is the same as the $(n-1)$th estimate of it. Note that, in this particular case, since $R_1=0.01$, the first estimate of $-r_1$ will have to be numerically greater than 50 in order to make a second estimate of it necessary. Thus, a second estimate is rarely necessary (note the range in value of $-r_1$, in Table 37), and the computation proceeds more rapidly than might be supposed from the above explanation.

It should be apparent that the estimated run-off, beginning with an approximation on April 2, rapidly converges toward the true run-off; that is, the true run-off in so far as the assumed law of run-off and the assumed constants make possible the evaluation of the true run-off. The first date in 1911 for which an observation equation is written is May 2. By starting the computation of r_1, r_2, r_3, $\ldots$ r_6 one month earlier, the convergence is approximately complete by May 2. Careful study will show that the convergence is not complete on April 18, as at first might be supposed.

If the proper conception of the run-off terms, $I_c+R_1r_1+R_2r_2+\ldots+R_6r_6$, has been obtained, *it should be clear that they represent the flow of a composite river emptying into the lake.* These terms represent not only the flow into the lake through all of the rivers except the St. Mary's River, but also that part of the flow which reaches the lake directly by underground travel.

EXAMPLE OF OBSERVATION EQUATIONS FOR DETERMINATION OF EVAPORATION AND RUN-OFF

The method of computing all of the quantities in equation (28) should be clear from preceding examples. As a sample of the observation equations, those for May 2 to 10, 1911, are shown in Table 38. In that table the unknowns E_1, E_2, R_1, R_2, $\ldots$ R_6 shown at the tops of the columns in parentheses should be considered as repeated down the columns.

The coefficients of the unknowns may be verified as follows: The coefficient of E_1, e, from Table 24. The coefficients of R_1, R_2, R_3, $\ldots$ R_6, viz, $-r_1$, $-r_2$, $-r_3$, $\ldots$ r_6, from Table 37. The absolute term, I, from Table 23.

TABLE 38—*Sample of observation equations from Solution V_1, Lake Michigan-Huron*
Unit is 0.001 foot of depth on the lake.

Date	Terms of the equation								
	$+e\ (E_1)$	$+\left[e\left(\frac{w-240}{100}\right)\right](E_2)$	$-r_1\ (R_1)$	$-r_2\ (R_2)$	$-r_3\ (R_3)$	$-r_4\ (R_4)$	$-r_5\ (R_5)$	$-r_6\ (R_6)$	$+I=v$
1911									
May 2	$+6\ (E_1)$	$+7\ (E_2)$	$-16\ (R_1)$	$-39\ (R_2)$	$-15\ (R_3)$	$+0\ (R_4)$	$+2\ (R_5)$	$+3\ (R_6)$	$-13=v_1$
3	$+6$	$+1$	$+14$	-16	-39	-2	$+2$	$+2$	$+161=v_2$
4	$+10$	-2	$+17$	$+14$	-16	-5	$+2$	$+1$	$-8=v_3$
5	$+12$	-5	$+12$	$+17$	$+14$	-6	0	0	$+2=v_4$
6	$+13$	-6	$+7$	$+12$	$+17$	0	-5	$+2$	$+23=v_5$
7	$+18$	-4	$+11$	$+7$	$+12$	$+3$	-7	$+5$	$-17=v_6$
8	$+20$	-4	$+12$	$+11$	$+7$	$+3$	-6	$+4$	$+30=v_7$
9	$+13$	-10	-2	$+12$	$+11$	$+2$	-2	$+1$	$-11=v_8$
10	$+14$	-4	-11	-2	$+12$	$+2$	$+3$	-3	$-6=v_9$

The coefficient of E_2, $e\left(\frac{w-240}{100}\right)$, can not be verified from previous sample computations. Its method of computation should be clear, however, from observing the method of computation of w shown in the last column but one in Table 11. If from such a column of values, one subtracts 2.4 and multiplies the difference by e, the result will be the coefficient of E_2, which now is either + or − because the w used in (28) refers to any wind velocity.

FINAL NORMAL EQUATIONS FOR SOLUTION V_1, LAKE MICHIGAN-HURON

An observation equation like those shown in Table 38 was written for each day of the 28 months July to October 1909 and May to October of 1910, 1911, 1912, and 1913. In some cases two or more days were combined into one equation, and in others certain equations were rejected, in accordance with the rules adopted for combinations and rejections already stated on pages 51 and 73. In all, there were 845 days of observation representing 787 observation equations used in Solution V_1.

The final normal equations formed from the 787 observation equations like and including those in Table 38 are shown in Table 39.

TABLE 39—*Final normal equations for Solution V_1, Lake Michigan-Huron*

$$\left.\begin{array}{l}
+244008E_1 - 44512E_2 + 28088R_1 + 10385R_2 + 4661R_3 + 830R_4 - 2020R_5 - 1031R_6 - 94169=0\\
-44512E_1 + 71090E_2 - 7721R_1 - 21652R_2 - 10603R_3 + 1822R_4 - 845R_5 + 4077R_6 - 8213=0\\
+28088E_1 - 7721E_2 + 291773R_1 + 151238R_2 - 9144R_3 - 7389R_4 + 4333R_5 - 836R_6 + 37093=0\\
+10385E_1 - 21652E_2 + 151238R_1 + 296816R_2 + 165826R_3 - 5231R_4 - 702R_5 - 131R_6 - 19665=0\\
+4661E_1 - 10603E_2 - 9144R_1 + 165826R_2 + 291696R_3 + 12663R_4 - 8663R_5 + 3402R_6 - 33669=0\\
+830E_1 + 1822E_2 - 7389R_1 - 5231R_2 + 12663R_3 + 8622R_4 - 837R_5 + 788R_6 - 2686=0\\
-2020E_1 - 845E_2 + 4333R_1 - 702R_2 - 8663R_3 - 837R_4 + 19458R_5 + 97R_6 + 4294=0\\
-1031E_1 + 4077E_2 - 836R_1 - 131R_2 + 3402R_3 + 788R_4 + 97R_5 + 33162R_6 + 1106=0
\end{array}\right\}\ldots(31)$$

The solution of the normal equations, (31), gives the following values for the unknowns:

$$\left.\begin{array}{ll}
E_1=+0.493\pm0.035 & R_3=-0.035\pm0.044\\
E_2=+0.466\pm0.065 & R_4=+0.121\pm0.188\\
R_1=-0.291\pm0.039 & R_5=-0.086\pm0.117\\
R_2=+0.253\pm0.049 & R_6=-0.081\pm0.089
\end{array}\right\}\ldots\ldots\ldots\ldots\ldots(32)$$

The probable errors were computed rigorously from the normal equations, (31), and the residuals, v, of the 787 equations.

If the theory is correct, if the constants E_1, E_2, R_1, R_2, R_3, . . . R_6 were assumed correctly and if there were no errors in the absolute term I, the derived values of the unknowns, (32), would be the same as the assumed values. The following is a comparison between the assumed and derived values.

(1) Assumed	(2) Derived	(3) Difference (1) − (2)	(4) Difference ÷ p.e. = (3)/p.e.	Mean
$E_1=+0.581$	$+0.493\pm0.035$	$+0.088$	2.5	2.5
$E_2=+\ .624$	$+\ .466\pm\ .065$	$+\ .158$	2.4	
$R_1=+\ .01$	$-\ .291\pm\ .039$	$+\ .301$	7.7	4.3
$R_2=+\ .07$	$+\ .253\pm\ .049$	$-\ .183$	3.7	
$R_3=+\ .17$	$-\ .035\pm\ .044$	$+\ .205$	4.7	
$R_4=+\ .80$	$+\ .121\pm\ .188$	$+\ .679$	3.6	
$R_5=+\ .20$	$-\ .086\pm\ .117$	$+\ .286$	2.4	
$R_6=+\ .27$	$-\ .081\pm\ .089$	$+\ .351$	3.9	

It is apparent from the above tabulation that the assumed values of the run-off into the lake were farther from the truth than the assumed values of the evaporation constants. The derived values of the latter differ by an average of only 2.5 times their own p.e.'s from the assumed values, whereas the derived values of the run-off constants differ by amounts varying from 2.4 times its own p.e. for R_5 to 7.7 times its own p.e. for R_1, with an average difference for all six derived values of 4.3 times their own p.e.'s.

From the derived values of the R's, the total excess (or defect) of storage on the current day, r_1, which is delivered to the lake by the end of the sixteenth day thereafter, if during those 16 days the storage is held constant, is $(-0.291)(100)+(0.253)(100)+(-0.035)(100)+(0.121)(5)(2)+(-0.086)(2.5)(4)+(-0.081)(1.25)(8)=-7.8$ per cent, as compared with the assumed value of +37 per cent (page 106), a strong indication that the assumed values of the R's were too large.

VARIABLE PART OF RUN-OFF INTO LAKE SUPERIOR

On Lake Superior a least-square solution designated as Solution BB_1 was made in which the form of observation equation was identical with equation (28) except that an additional wind term, $\left[e\left(\frac{w-240}{100}\right)^2\right]E_3$, was introduced for winds below 10 miles per hour, as in Solution BB_2. In this solution a much better basis was obtainable for estimating the values of R_1, R_2, R_3, . . . R_6. Previous to starting this solution, approximate values for the percentage of change in storage on any day in the drainage area, which reaches the stream on the current or later days, assuming the storage to be held constant after the current day, had been obtained on Streams A and B, Wagon Wheel Gap, Colorado, the detailed exposition of which appears in Part II of this publication. These values furnished a much more sound basis for estimating the R's than had been available up to that time on the Great Lakes. For example, on Lake Michigan-Huron, the values first estimated by the methods at first available were much too high, as previously mentioned, and were reduced successively in various least-square solutions until the final values used, for Solution V_1, were obtained. The evidence from the first studies on the two Colorado streams indicated that even the values finally used in Solution V_1 are probably too large, and vindicates the evidence from that solution itself. The first approximate values of the *R*'s obtained on Streams *A* and *B*, expressed as percentages, as in (30) are

Value of—	For Stream *A*	For Stream *B*	Mean for Streams *A* and *B*
R_1........	+0.71	+0.08	+0.40
R_2........	+ .41	+ .19	+ .30
R_3........	+ .42	+ .16	+ .29
R_4........	+ .28	+ .11	+ .20
R_5........	+ .17	+ .002	+ .09
R_6........	+ .15	− .039	+ .06

It should be stated at this point that in equation (28), the symbol *R* with a subscript corresponds with the symbol R', with the corresponding subscript, used in the stream-flow studies.

Compare the mean *R*'s shown in the above tabulation with (30), page 106. These first approximate values from the stream-flow studies indicate that 0.40 per cent of the change in storage in the ground on the current day is delivered to the stream on that day. In Solution V_1, it was assumed to be 1 per cent. Similarly, from the above tabulation, 0.30 per cent of the change in storage in the ground on the current day reaches the stream the following day, if the addition to the storage in the ground on the following day is assumed to be zero. In solution V_1 it was assumed that 7 per cent (or 23 times as much) of the change in storage in the ground on the current day is delivered to the lake on the following day, under the same assumption. Similarly, a comparison of the other values shows that in Solution V_1 the assumed percentages varied from 59 times larger in the case of R_3 to 5 times larger in the case of R_6, than the first estimates of the actually derived percentages on the two Colorado streams. Clearly it would be unreasonable to assume that actually such a large difference does exist. The underground drainage system of the Great Lakes can not be as different from the underground drainage system of the Rocky Mountain region in the vicinity of Streams *A* and *B* as the difference in these figures indicate. In fact, the character of the underground drainage system of the Great Lakes is probably much nearer like that of the drainage areas of Streams *A* and *B*, as represented by the percentages shown in the preceding tabulation, than it is like the drainage system which would correspond to the percentages of delivery shown in (30).

On the basis of this reasoning, the assumed *R*'s for Lake Superior were made like the means shown in the preceding tabulation, except that they were slightly modified according to the general considerations stated in the following lettered paragraphs:

(*a*) The maximum run-off was assumed to come on the second day after the change in storage takes place, on Lake Superior, instead of on the first day, as on the Colorado streams, according to the mean values shown above, because the streams emptying into Lake Superior are much longer and the drainage area is much larger than those studied in Colorado.

(*b*) The upward slope to the maximum of the run-off curve, plotted with percentages as ordinates and days as abscissas, is probably steeper than the downward slope.

(*c*) The curve probably becomes concave upward soon after the maximum. On the basis of these considerations, the means for Streams *A* and *B* were modified as shown in the last two columns of the following tabulation:

Value of—	(1) Means from Streams *A* and *B* as %	(2) Factor to reduce to absolute value	(3) Absolute values (1)÷(2)	(4) Adopted absolute values for Solution *B* B_1	(5) Adopted values expressed as percentages (4)×(2)
R_1	0.40	100	0.0040	0.0020	0.20
R_2	.30	100	.0030	.0040	.40
R_3	.29	100	.0029	.0030	.30
R_4	.20	5.0	.040	.025	.125
R_5	.09	2.5	.036	.020	.050
R_6	.06	1.25	.048	.009	.01125

From column (5) in the above tabulation, it may be verified that only 1.44 per cent of the change in storage in the ground on any day, r_1, was assumed in Solution *B* B_1, to reach the lake (Superior) by the end of the sixteenth day after the change took place, if, during the last 16 days of that 17-day period, the storage is assumed to be constant. This solution contained 847 days and 759 observation equations. The following is a comparison between the assumed values of the constants used in computing $-r_1$, $-r_2$, $-r_3$, . . . $-r_6$ and the values derived from the least-square solution:

Value of—	(1) Assumed	(2) Derived	(3) Difference (1)−(2)	(4) Difference÷ p.e. = (3)/p.e.	
E_1	+0.46 +0.56	−0.051±0.059	+0.511 + .611	8.7	Mean =3.6
E_2	+0.69	+0.721± .155	− .031	0.2	
E_3	+1.17	+0.653± .261	+ .517	2.0	
R_1	+0.0020	−0.023± .062	+ .025	0.4	
R_2	+0.0040	−0.383± .080	+ .387	4.8	
R_3	+0.0030	+0.244± .068	− .241	3.5	Mean =2.0
R_4	+0.025	+0.662± .263	− .637	2.4	
R_5	+0.020	−0.057± .148	+ .077	0.5	
R_6	+0.009	−0.049± .090	+ .058	0.6	

The assumed values of E_1, E_2, and E_3 need further explanation. It has already been shown in Table 35 that slight evidence was found in Solution U_3 that the evaporation curve might be concave upward for winds below 10 m.p.h., and that this evidence was used in setting up the equation to use in Solution BB_2 for testing out the evidence with a larger number of observations—28 months instead of 6 months of observations. As a matter of fact Solution U_3 was succeeded by a 6-months solution designated as Solution U_4, before Solution BB_1, immediate predecessor of BB_2, was made. In Solution U_4 the form of the observation equation was the same as equation (27), the one used in Solutions U_3 and T_6, except that it contained the additional wind-term, $\left[e\left(\frac{w}{100}-2.4\right)^2\right]E_3$, for winds below 10 m.p.h.

The weighted-mean values of E_1, E_2, and E_3 derived from the two six-months solutions, U_4 and T_6, were as follows, the method of weighing being the same as that shown in Table 26, page 82:

$$E_1=+0.561\pm0.065 \qquad E_2=+0.690\pm0.125 \qquad E_3=+1.167\pm0.485$$

These values were studied in the light of the following criteria:

(1) That a decrease in the wind velocity can not be accompanied by an increase in the evaporation.

(2) That the evaporation can not be negative for any positive value of e.

The above values are grossly inconsistent with the first criterion, and no assumed value of E_3 near that quoted would be consistent with both criteria, E_1 and E_2 remaining unchanged. From the studies which had been made of the evaporation computations up to this time it appeared that the values of E_1 and E_2 were much more reliable than the values of E_3. It appeared also that the reliability of the evaporation curve was high for wind velocities of 10 m.p.h. and above, and that this reliability decreased very rapidly as the wind velocity drops below 10 m.p.h. At this time it appeared, moreover, that the evaporation might become independent of the wind velocity for such low wind velocities that there is but very slight ventilation at the evaporation surface—water, ground or vegetation.

In view of the best values of E_1, E_2, and E_3 quoted above as having been obtained up to this time, of the two criteria mentioned, and of the three considerations mentioned in the three preceding sentences, it was concluded at this time that the relation between evaporation and wind velocity (*a*) is a straight line relation from 10 m.p.h. upward with increasing evaporation corresponding to increasing wind velocity, (*b*) is a curved line between 8.8 m.p.h. and 10 m.p.h. with increasing evaporation corresponding to increasing wind velocity, (*c*) is independent of the wind velocity below 8.8 m.p.h., (*d*) that the evaporation from a free-water surface is best expressed in units of 0.001 foot of depth per day by the following three formulas:

If the wind velocity is 10 m.p.h. or above $+0.56\,e+0.69\,e\left(\frac{w}{100}-2.4\right)$

If the wind velocity is 8.8 to 10 m.p.h. $+0.56\,e+0.69\,e\left(\frac{w}{100}-2.4\right)+1.17\,e\left(\frac{w}{100}-2.4\right)^2$

If the wind velocity is less than 8.8 m.p.h., $+0.46\,e$.

The estimated evaporation from land is taken as 0.62 times the evaporation from water, as in Solution V_1.

The derived values of E_1, E_2, and E_3 from Solution BB_1 are grossly inconsistent with both criteria above, as they were in Solution BB_2—strong indication that the $\left[e\left(\frac{w}{100}-2.4\right)^2\right]$ E_3-term does not belong in the equation. This evidence is strengthened by the internal evidence of the solution.

This evidence shows that the very large difference between the assumed and the derived value of E_1—the largest of all the differences—is due mainly to the influence of the term $\left[e\left(\frac{w}{100}-2.4\right)^2\right]E_3$.

The derived values of the run-off constants from Solution BB_1 differ much less from the assumed values than was the case in Solution V_1. Only one derived value, R_2, differs by more than 3.5 times its own p.e. from the assumed value. The mean difference of all the derived R's from their assumed values is only 2 times their own probable errors, as against 4.5 times their own p.e.'s in Solution V_1. This evidence is a strong indication that the assumed R's in Solution BB_1 are much nearer the truth than they were in Solution V_1, or that the amount of the change in storage on the current day, which is delivered to the lake by the end of the following sixteenth day, if, during the last 16 days of that 17-day period, the storage in the ground does not change, is much nearer 1.44 per cent of such change than 37 per cent of it. In fact, the derived run-off constants indicate that −10.6 per cent of the change in storage in the ground on any day is delivered to the lake by the end of the sixteenth day thereafter, which is an absurdity and an indication that even the assumed 1.44 per cent is possibly too large for the Lake Superior drainage basin. On the other hand a comparison of the run-off values as finally derived on Streams A and B, shown in Table 51, page 195, with the first approximate values derived on those streams and used in Solution $B\,B_1$, shows that the correct mean value for Streams A and B is 1.68 per cent rather than 1.44 per cent, as shown in the accompanying tabulation. The two lines of evidence are contradictory, if the assumption is true that the natural underground drainage systems of the two regions is substantially the same.

Value of—	Assumed in Solution $B\,B_1$	Mean of final values derived on Streams A and B
	p. ct.	*p. ct.*
R_1...........	0.20	0.186
R_2...........	0.40	0.152
R_3...........	0.30	0.124
R_4...........	0.125	0.106
R_5...........	0.050	0.092
R_6...........	0.01125	0.080
Total.........	1.44	1.68

From another viewpoint, however, it appears that whether the R's be assumed in such sizes and relations to each other that the total amount of excess storage on a day which is delivered to the lake by the end of the sixteenth day thereafter be 1.44 per cent or 1.68 per cent of such excess storage, if the rainfall on each day after the first of the 17-day period is just equal to the evaporation from land plus the run-off, is relatively immaterial. *The probability is that in neither case could the R's be derived from the observations.* This is true because the variable part of the estimated run-off with such small R's as were assumed in Solution BB_1 never totaled to as much as 0.001 foot of depth on the lake area. This means that the assumed constant part of the run-off into that lake, 0.004 foot of depth on the lake area per day, was never increased or decreased on any day by the assumed variable part of the run-off. On Lake Michigan-Huron, with much larger assumed R's, the variable part of the run-off was considerable. Note that in Table 37 the estimated variable part of the run-off shown in the columns headed $+R_1r_1$, $+R_2r_2$, $+R_3r_3$, . . . amounted to +0.009 foot on May 3, which increased the assumed constant part of the run-off, 0.006 foot to a total of 0.015 foot for that day. On May 9, on the other extreme, the estimated variable part of the run-off was -0.005 foot, which decreased the constant part of the run-off into Lake Michigan-Huron to 0.001 foot.

The conclusion, that the variable part of the run-off is so small that it can not be evaluated by the form of expression adopted, would be affected but little, it is believed, by the errors in the estimated evaporation from land, used in computing r_1, r_2, r_3, . . . r_6. The difference between that estimated evaporation and the true evaporation would probably introduce a negligible error into the evaluation of R_1, R_2, R_3, . . . R_6, in comparison with other unavoidable errors.

CONSTANT PART OF RUN-OFF INTO LAKE MICHIGAN-HURON

The original estimate of the constant part of the run-off into the lake in one day, $I_c = 0.008$ foot, page 19, was revised in the following manner. Consider the change in storage in the drainage area, represented by the r_1 – term. It is evident that over any long period of time for which the constant part of the run-off has been assumed correct, $\Sigma(r_1) = 0$. Any variation from zero must be attributable largely to errors in the estimated run-off rather than to errors in the estimated evaporation, since the former was the more uncertain quantity involved in the computation of r_1. Between two least-square solutions in which the variable part of the run-off is held the same, i.e., computed by the same assumed R's, but the constant part of the run-off, I_c—and that only—is caused to vary, one can get a variation in $\Sigma(r_1)$ and in its mean value $\Sigma(r_1)/n$ where n is the number of observations used in the solution. By comparing the change in $\Sigma(r_1)/n$ between the two solutions with the assumed change in I_c in connection with the knowledge that $\Sigma(r_1)/n$ should equal zero, that value of I_c may be estimated which will render $\Sigma(r_1)$ zero. This reasoning is believed not to be vitiated even though the assumed R's used in the two solutions be so much as 40 per cent from the true values.

In Solution T_5, containing 169 observation equations of the form of equation (28), and in which $I_c = 8$ was used, the value of $\Sigma(r_1)/(169)$ was $+0.75$. In Solution T_6, exactly like Solution T_5 except that $I_c = 6.5$ was used, the value of $\Sigma(r_i)/(169)$ was $+0.18$. That is, the change of assumption of I_c from 8.0 to 6.5 changed the mean value of r_1 from $+0.75$ to $+0.18$, from which it is seen that the value of 6.0 for I_c would make the mean $r_1 = 0$. This final adopted value of 0.006 foot of depth per day on the lake area for the constant part of the run-off into Lake Michigan-Huron or, otherwise stated, for the run-off when the ground water is at its average level, and the quantity [(rainfall) *minus* (evaporation from land) *minus* (run-off)] has been zero for a long time, it is believed to be in error by not more than ± 0.0005 foot. The equivalent of 0.006 foot of depth per day on Lake Michigan-Huron is 87,900 cubic feet per second.

The evidence in favor of the reduction from 8 to 6.5 was of four kinds, each independent of the others, and only one of the four being internal evidence from the least-square computations, in which case it was of different character from the type already presented. Each of these four lines of evidence indicated 8 to be too large, and after rejecting one of the values, which appeared to have been determined from uncertain premises, the mean of the other three values was 6.7. The value was arbitrarily reduced to 6.5 to facilitate computations. These four lines of evidence need not be given here, for the reasons that the final reduction from 6.5 to 6.0 involved a more potent method of analysis than any one of the three involved in the reduction from 8 to 6.5, and moreover that analysis showed even 6.5 to be too large.

CONSTANT PART OF RUN-OFF INTO LAKE SUPERIOR

The original value of the constant part of the run-off into Lake Superior as estimated on page 19 at 0.004 foot of depth on the lake area per day was never revised in the manner indicated for Lake Michigan-Huron. The equivalent of 0.004 foot of depth per day on Lake Superior is 41,400 cubic feet per second.

ENTANGLEMENT BETWEEN CONSTANT PART OF RUN-OFF AND EVAPORATION

The ideal method of evaluating the constant part of the run-off into the lake would be to determine it directly from the least-square computations, instead of by the roundabout way adopted. This was tried. The observation equation was written in the form of equation (28) except that the constant part of the run-off, I_c, was not included in the I-term, but was written in as $(I_c=)$ 10 R and inserted in the equation just before the r_1R_1-term. In this case $R=\frac{1}{10}I_c=$ the constant part of the run-off into the lake in $\frac{1}{10}$ day, was to be determined from the least-square computations by trial in the same manner as, and simultaneously with, R_1, R_2, R_3, . . . R_6. The multiplication of R by 10 merely served to make the coefficient of R in the observation equations about the same numerical size as the other coefficients as a device to facilitate the computations.

Many attempts to determine R, and from it I_c, in the manner indicated proved futile for the reason, which gradually emerged from the computations, that the entanglement between R and the evaporation constants E_1 and E_2 is very great, especially with the former. Because of the entanglement, the derived values of all three constants, E_1, E_2, and R suffered. It was of primary importance to evaluate E_1 and E_2, with as much accuracy as possible, and it appeared that by dropping 10 R from the observation equations and putting it into the absolute term in the manner already described, the accuracy of determination of these two constants would be greatly increased. This action was later justified, and the conclusion corroborated that to determine R from the least-square computations was impossible.

EFFECT OF VARIABLE PART OF RUN-OFF ON DERIVED VALUES OF E_1 AND E_2

As stated above it was of primary importance to fix the values of E_1 and E_2 with the greatest accuracy possible. The inclusion or exclusion of any element in the problem was justifiable on that basis. The rejection of an element was particularly justifiable, if its inclusion actually decreased that accuracy. On this latter basis the constant part of the run-off was determined in the direct manner adopted.

The variable part of the run-off, which has already been shown to have been indeterminable, apparently affected the evaporation constants but little. This

Value of—	Solution V_1	Solution V_2	Difference V_1-V_2	Solution BB_1	Solution BB_2	Difference BB_1-BB_2
E_1.......	$+0.493\pm0.035$	$+0.448\pm0.034$	$+0.045$	-0.051 ± 0.059	-0.045 ± 0.059	-0.006
E_2.......	$+0.466\pm.065$	$+.367\pm.063$	$+.099$	$+.721\pm.155$	$+.925\pm.153$	$-.204$
E_3.......	....	....	....	$+.653\pm.261$	$+.803\pm.264$	$-.150$

may be seen by a comparison of the values derived in Solution V_1 with V_2 and BB_1 with BB_2. The only difference between these solutions was that Solutions V_1 and BB_1 contained the run-off terms, $-R_1r_1-R_2r_2-R_3r_3 \ldots -R_6r_6$, and Solutions V_2 and BB_2 did not. The comparison is given in the accompanying tabulation, from which it may be seen that the change in these constants caused by ignoring entirely the variable part of the run-off into the lake is of the order of their own probable errors. Although these changes appear to be systematic with reference to each lake, they are essentially accidentally distributed with reference to both lakes.

SUMMARY OF GENERAL CONCLUSIONS TO PART I

The principal conclusions are here briefly summarized for the convenience of anyone who wishes to get a general view of the results of this part of the investigation. Page references enable those interested to look up the basis of particular conclusions.

The best expression of the laws of evaporation from large water surfaces under natural conditions which could be determined in this investigation is equation (22) (page 82).

The probable errors of the constants in that equation were computed directly from the residuals on the assumption that all the errors involved are accidental in character. As there is some evidence of systematic errors, these probable errors should be slightly increased to represent the actual uncertainties in the values (pages 85 to 90).

As indicated by the probable errors, the evaporation on the lake on any day may be computed from the formula developed in this investigation with an error of about ±10 per cent (pages 121 and 122).

The evaporation from the surface of Lake Michigan-Huron apparently varies between the limits 0.000 foot and 0.029 foot of depth on the lake area per day, with a mean value for the months May to October, inclusive, of 0.0054 foot of depth per day. The equivalent of 0.001 foot of depth per day on Lake Michigan-Huron is 14,650 cubic feet per second (pages 84-85).

Wind velocities below 10.8 miles per hour or 260 miles per day are too feeble to have an appreciable effect upon evaporation from the surface of water, ground or vegetation. The wind velocity here referred to is that as measured about 100 feet above the ground or water surface (pages 90 to 99).

For winds above 260 miles per day, evaporation is a linear function of wind velocity, with increasing evaporation corresponding to increasing wind velocity (pages 98 to 102).

The rate of increase of evaporation from the surfaces of the Great Lakes with increasing wind velocity for winds above 10.8 miles per hour, as measured at the Weather Bureau Stations, is many times greater than the corresponding rate of increase as determined from observations on small-scale apparatus (pages 122 to 130).

The run-off into the lake when the ground-water is at its average level, and the change in storage in the ground, i.e. (rainfall on land) *minus* (evaporation from land) *minus* (run-off), has been zero for a long time, is equivalent to 0.006 foot of depth per day on the area of Lake Michigan-Huron, or 87,900 cubic feet per second, and 0.004 foot of depth per day on Lake Superior, or 41,400 cubic feet per second (pages 16 to 19, 117 to 118).

The total part of the change in storage in the ground on any day,viz (rain fall on land) *minus* (evaporation from land) *minus* (run-off), which reaches the lake during a period of 17 days thereafter (and including the day the change takes place), if the rainfall is just sufficient to balance the evaporation from land plus the run-off on each day of the 16-day period after the current day, is in the order of 1.5 per cent of such change. This figure refers to either Lake Michigan-Huron or Superior, and would probably apply, locally, to any of the region around the Great Lakes. On very rare days of exceptionally heavy general rainfall, particularly if the ground had been soaked by previous rains, this figure may be augmented, but the figure given probably holds for most of the days (pages 102 to 117).

The accuracy with which the elevation of the mean surface of the whole of Lake Michigan-Huron may be determined by correcting the observed elevations at the three gages, Milwaukee, Harbor Beach and Mackinaw for wind and barometric effects, inflow, outflow, rainfall on the lake, run-off into the lake and evaporation from the lake surface, has been increased. It appears that the mean elevation of the whole lake surface on any day may now be determined with a probable error of only ±0.010 foot. This is less than ⅛-inch; an accuracy hitherto unattainable (pages 81 to 82).

APPENDIX TO PART I

ACCURACY OF COMPUTED EVAPORATION, AND COMPARISON OF FORMULA DEVELOPED IN THIS INVESTIGATION WITH OTHERS

ACCURACY OF COMPUTED EVAPORATION

What is the accuracy with which the evaporation may be computed on any day by use of the formula developed in this investigation? The final formula adopted as being the best obtainable is (22) or (23), page 82. Consider the latter, in which the evaporation is expressed in units of 0.01 inch of depth per day. The value of the constant E_1 is $+0.319$ and its probable error is ± 0.037. The probable error is a measure of the accuracy which is the best that can be obtained, provided the errors in the derived constants are all accidental in character. Assuming, for the moment, that the errors are all accidental in nature, it is an even chance that the actual error in the constant E_1 is greater or less than its computed probable error as shown. That is, the chances are even for and against the proposition that the true value of E_1 lies within $0.319-0.037$ and $0.319+0.037$. In other words there is one chance in two that the value $+0.319$ is correct within about one-ninth of itself $\left(\frac{0.037}{.319}=\frac{1}{8.6}\right)$. On this basis the conclusion is that the computed evaporation constant E_1 is subject to an error which stands 1 chance in 2 of being less than 11.6 per cent $\left(\frac{1}{8.6}=0.116\right)$. Inasmuch as there is, however, evidence of systematic error (see page 85, etc.) allowance should be made for this fact. Just how much the probable error should be increased to make allowance for the systematic errors is a matter difficult, if not impossible, to determine exactly, hence the error in E_1 will henceforth be discussed as if it were the same as that represented by its probable error, with the understanding that the accuracy so indicated is probably too high. On any day, therefore, on which the wind velocity is less than 10.8 miles per hour, assuming no systematic or constant errors present, and no errors in the observed saturation deficit, the computed evaporation stands 1 chance in 2 of being in error by less than 11.6 per cent.

Proceeding in the same manner with reference to the constant E_2, its value is $+1.49$ and its probable error is ± 0.150. Hence, assuming no constant or systematic errors present, the chances are 1 in 2 that the value $+1.49$ is correct within one-tenth of itself $\left(\frac{0.150}{1.49}=\frac{1}{9.9}\right)$; or, stated in a different way, on the above basis the computed evaporation constant E_2 is subject to an error which stands 1 chance in 2 of being less than 10 per cent as large as E_2. As the effect of the systematic errors on this probable error is unknown, ± 10 per cent is probably an understatement of the true error of E_2.

It is interesting to note that the accuracy with which the evaporation curve for winds above 10.8 miles per hour has been determined is greater than is that for winds less than 10.8 miles per hour. Suppose the following inquiry were made: What is the error in the computed evaporation when a wind is blowing at the rate of 30 miles per hour, as measured by the Weather Bureau anemometers, and the saturation deficit is 0.01 inch of mercury? Under these conditions the total computed evaporation will be at the rate of 0.0717 inch per day, 0.0685 inch, or 96 per cent, of which will be contributed by the wind term. If the entanglement between the wind term and the other term is neglected, the computed evaporation on such a day will stand 1 chance in 2 of being in error less than 10 per cent, assuming no systematic or constant errors present, and no errors in the measured wind velocity and saturation deficit.

COMPARISON OF FORMULA DEVELOPED IN THIS INVESTIGATION WITH OTHERS

The presentation of the methods used and the results obtained in that part of this investigation dealing primarily with evaporation would be incomplete without comparing the results with those obtained by other investigators. Accordingly, the remaining paragraphs of this appendix are devoted to making such comparisons. No attempt is made to present an exhaustive comparison. The number of evaporation formulas extant in the scientific and engineering literature is very large. These formulas differ from the one derived in this investigation in three ways. (*a*) They are, so far as the writer knows, all based upon observations on small-scale apparatus, on evaporation pans a few square inches or a few square feet in area, placed either in a laboratory or out in the open in different positions with reference to the surface of the ground, water or vegetation. In these artificial conditions lie the fundamental weakness of all such formulas, namely, the assumption that the laws of evaporation so obtained will be correct when applied to natural conditions. (*b*) In most of the evaporation formulas based on small-scale apparatus, the observations of the meteorological elements have been taken at or near the apparatus. The formulas developed, therefore, express primarily the relationship between the rate of evaporation in the pan and the meteorological elements so observed. Such formulas, then, are of limited application, even if the defect mentioned in (*a*) were not present. This follows because one can not use them with existing, standard observations of the weather elements as made by the Weather Bureau. In this sense the evaporation in any locality in the past can not be investigated, and special observations are required for estimating current evaporation. (*c*) As compared with the evaporation formula developed in this investigation, most other formulas are based on relatively few observations. Moreover, the precision with which the formulas express the stated relationships has not usually been determined in the sense that the probable errors of the proportionality constants have not usually been derived.

In comparing and contrasting the results of this investigation with others it is important to bear in mind these three points of difference. The comparisons presented are probably typical. They serve to bring out the principal differences obtained by the two radically different lines of attack.

COMPARISON WITH FREEMAN'S (AND MEYER'S) FORMULA

The formula developed by John R. Freeman[1] is as follows:

$E_w=0.5\ (V-v)\ +0.05\ (V-v)\ w$, in which .... (A)

V = Saturation vapor-pressure corresponding to the temperature of the water, in inches of mercury;
v = Saturation vapor-pressure corresponding to the dew-point temperature, in inches of mercury;
w = Average velocity of wind, in miles per hour;
E_w = Evaporation in inches per day.

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[1] *Regulation of the Great Lakes*, by John R. Freeman (1926), Appendix 2.

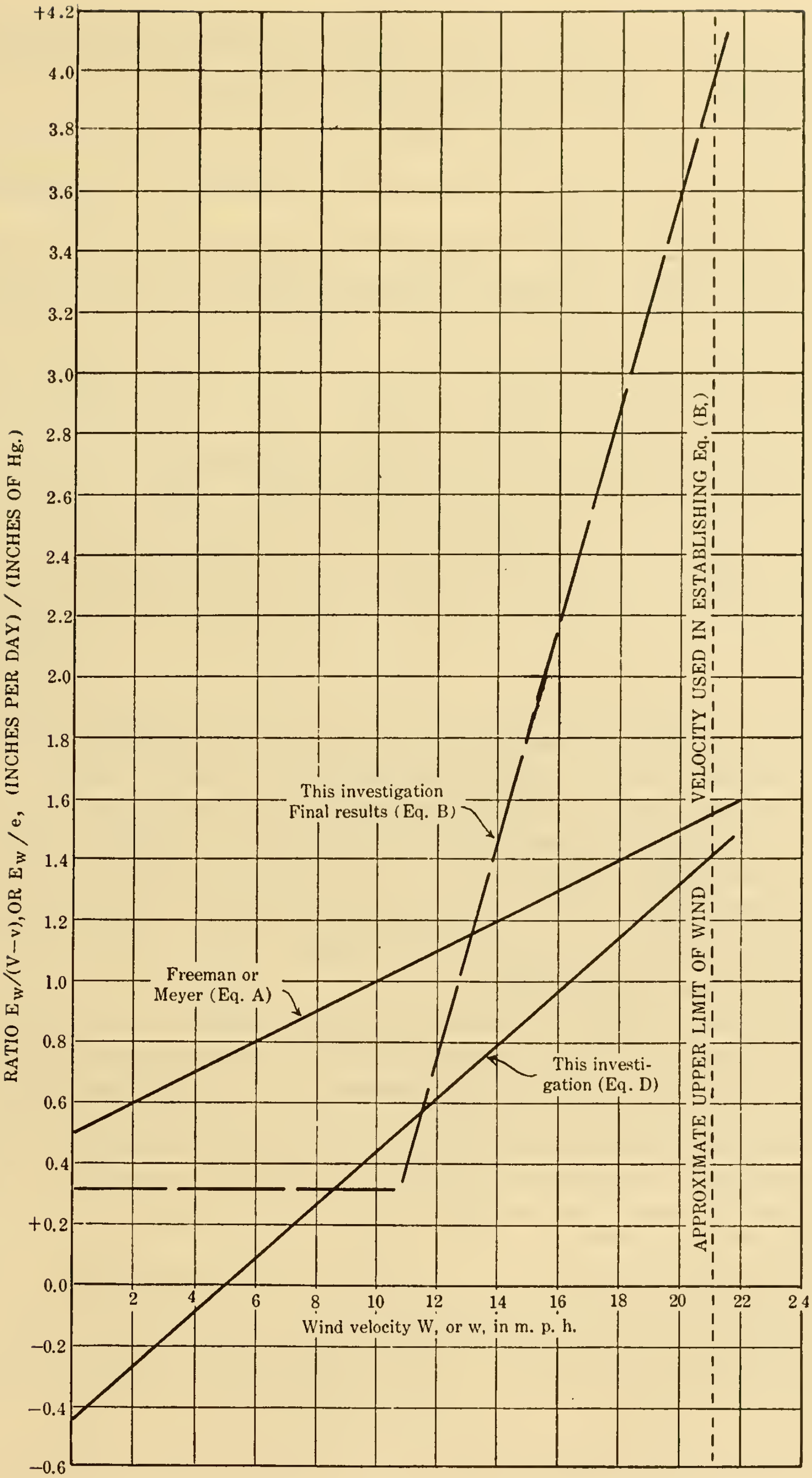

FIG. A—Comparison of Freeman's (or Meyer's) Evaporation Curve with those Developed in this Investigation.

This formula is identical with that of Professor Adolph Meyer,[1] and was developed independently, at a later date, than the latter. The formula developed in this investigation, converted to the same units as Freeman's, is

$E_w = 0.319\ e + 0.358e\ (w - 10.8)$, in which ....(B)
$(\pm 0.037)(\pm 0.036)$
$e = e_a - e_d$, and e_a = Saturation vapor-pressure corresponding to the temperature of the air, in inches of mercury;
e_d = Saturation vapor-pressure corresponding to the dew-point temperature, in inches of mercury;
w = average velocity of wind in miles per hour for winds above 10.8 miles per hour; that is, the expression $0.358\ (w - 10.8)$ enters only for wind above 10.8 m.p.h.

The above two formulas differ from each other in three respects, viz, in the manner of expressing the wind-term, in the saturation vapor-pressure difference, and in the derived proportionality constants. These three differences to some extent offset one another, as will be shown later in comparing the aggregate evaporation computed from them on Lake Michigan-Huron. The formulas are shown graphically in figure *A*. A comparison of them is given in the following paragraphs.

(*a*) If there is no error in Assumption No. 5 (page 10), $(V - v) = (e_a - e_d)$, and the two equations are directly comparable. For a zero wind, the evaporation computed by equation (*A*) is $(0.5 - 0.319 =)\ 0.181e$ greater than that computed by equation (*B*). This difference is 4.9 times the probable error of 0.319 $\left(\frac{0.319}{.037} = 49.\right)$. Assuming that no systematic or constant errors affect the results, i.e., that the probable error ± 0.037 represents the true inaccuracy in the constant 0.319, the constant 0.5 in equation (*A*) is certainly too *large* to represent the truth.

(*b*) For a wind velocity of 10.8 miles per hour, assuming no error in Assumption No. 5, the evaporation computed by equation (*A*) is $(1.04 - 0.319 =) 0.721e$ larger than that computed by equation (*B*). This difference is 19.5 times the probable error of 0.319 $\left(\frac{0.721}{.037} = 19.5\right)$. Assuming that no systematic or constant errors affect the results, the evaporation computed by equation (*A*) for a wind velocity of 10.8 m.p.h. is certainly too *large.*

(*c*) For a wind velocity of 21 miles per hour, assuming no error in Assumption No. 5, the evaporation computed by equation (*A*) is $(1.55 - 3.97 =) 2.42e$ smaller than that computed by equation (*B*). Neglecting the entanglement between the two terms in the right-hand member of equation (*B*) when the wind velocity is greater than 10.8 m.p.h., the probable error of 3.97 is about ± 0.37. The difference 2.42 is, therefore, 6.5 times this probable error and, if no systematic or constant errors affect the results, the evaporation computed by equation (*A*) for a 21-mile wind is certainly too *small.*

(*d*) The above paragraphs, (*a*), (*b*) and (*c*) contain strong indications—proofs—that the Freeman or Meyer formula, developed from small-scale apparatus, does not represent the true laws of evaporation from a natural body of water. This may be indicated in another way. Consider the evaporation formula developed in Solution V_2 from observations on Lake Michigan-Huron. This formula is (Tables 30 and 31).

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[1] Trans. A. S. C. E., 1915, p. 1074.

$$E_w = \underset{(\pm 0.034)}{+0.448e}\ \underset{(\pm 0.063)}{+0.367}\left[e\left(\frac{w}{100}-24\right)\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(C)$$

It is based upon 778 observation equations covering 842 days. Converted to the same units as in equations (A) and (B), it is

$$E_w = \underset{(\pm 0.034)}{+0.448e}\ \underset{(\pm 0.015)}{+0.0881}e(w-10)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(D)$$

in which w has the same meaning as in (A) and e the same meaning as in (B). This equation is also shown plotted on figure A. Equation (D) is of the same form as equation (A). In its derivation all wind velocities were used. It is directly comparable with the Freeman and Meyer formula, if there are no errors in Assumption No. 5. The impossibility of representing the variation in evaporation by this form of expression has already been pointed out (page 93, etc.). It proves that the assumption that evaporation is affected by all wind velocities on the Great Lakes—wind velocities as measured at the Weather Bureau Stations—is not true. Moreover, it shows that if this assumption be made, as in the Freeman and Meyer formulas, the rate of increase of evaporation with increasing wind velocity is greater than that represented by the Freeman and Meyer formula. In support of this last statement, note that the constant representing the slope of the curve (D) is $(0.0881-0.05=)$ 0.0381 greater than that of the curve (A). This difference is 2.5 times the probable error of 0.0881 $\left(\frac{0.0381}{.0151}=2.5\right)$, strong indication in itself that the slope of the Freeman curve is too small. Additional proof that that slope is too small is seen in the large increase in the slope between Solutions V_2 and V_3, where the increase was from $+0.367\pm0.063$ to $+0.661\pm0.085$ (Table 31), an increase due solely to the assumption that, for small winds, evaporation is practically independent of wind velocity.

(*e*) The two equations, (A) and (B), give the same evaporation at a wind velocity of about 13.1 m.p.h., where the curves intersect. For winds below this velocity, equation (A) gives results which are certainly too high, and for winds above that, certainly too low as judged by the probable errors of the constants in equation (B), the best available criteria. The area between these curves to the left of this point of intersection is about 35 per cent greater than the area to the right of that point up to winds of 21 m.p.h. This means that if, over a long interval of time—say a year—there are as many winds between 13.1 and 21 m.p.h. as there are below 13.1 m.p.h. the overall evaporation computed from (A) will be 35 per cent greater than that computed from (B). The upper limit of 21 m.p.h. is approximately the maximum average wind velocity which entered into the observations on which the constants in equation (B) are based. The average referred to is that computed from the ten stations near Lake Michigan-Huron, or from the five stations near Lake Superior (Tables 1 and 2). With an average based on ten stations—an average velocity assumed to be representative of the whole lake area—the maximum wind at a single station might be much above, or the minimum wind much below, 21 miles per hour. When the average daily wind velocity exceeded this amount, the observations were rejected because, as noted elsewhere, the observed change of elevation of the water surface on the day in question was abnormal and due to the first oscillation of a new seiche affecting the gage record at the station, or that it was abnormal because of extremely rapid and irregular changes in barometric gradients over the lake, which departed widely from the conditions postulated in the approximate theory used in the investigation of the effects of winds and of barometric pressures.

(*f*) The average total evaporation on Lake Michigan-Huron for the months May to October inclusive, as computed by Freeman, using his formula, is 15.33 inches of depth on the lake area, or 0.00694 foot of depth per day (page 145, *Regulation of the Great Lakes*). This is 0.00144 foot of depth per day, or 28 per cent, greater than that computed by the formula developed in this investigation $\left(\frac{0.00694}{.0054} = 1.28\right)$, and is a difference of the same sign, and approximately of the same amount, as that noted in paragraph (*e*), above. The difference between the overall evaporation computed by the two formulas is probably too great to be accounted for by errors in Assumption No. 5.

(*g*) The average daily rate of evaporation for the summer months May to October inclusive, as finally adopted by Freeman (column 7 of table on page 145, *Regulation of the Great Lakes*) is 0.0056 foot of depth per day, an amount in almost exact agreement with the value 0.0054 derived in this investigation. But the comparison month by month is not so close. The average monthly values, and their accumulated sums, expressed in units of one foot of depth on the lake area are as follows:

Month	Average evaporation (ft.)		Accumulated sums of evaporation (ft.)	
	This investigation	Freeman	This investigation	Freeman
May........	0.169	0.0250	0.169	0.025
June........	.165	.0333	.334	.058
July........	.220	.133	.554	.191
Aug........	.158	.250	.712	.441
Sept........	.114	.308	.826	.749
Oct........	.165	.300	.991	1.049

From this tabulation it is apparent that the rate of evaporation as computed in this investigation is larger in May, June and July, and smaller in August, September and October than the rate as finally adjusted by Freeman.

COMPARISON WITH HORTON'S RESULTS

A comparison of Horton's formula[1] directly with the one developed in this investigation is not feasible on account of its difference in form.

The average monthly evaporation on Lake Michigan-Huron for the six months May to October of 1909 to 1913 inclusive—the years of observation used in this investigation—as computed by Horton is 22.74 inches or 1.895 foot (Table 71, page 184). This is at the rate of 0.0103 foot of depth per day, which is 91 per cent greater than the value 0.0054 foot of depth per day derived in this investigation.

COMPARISON WITH FORMULAS OF ABASSIA, FITZGERALD, CARPENTER AND STELLING

In the monthly Weather Review for July 1907 (page 313), Professor Bigelow presents a comparison between four evaporation formulas, all based on the Dalton

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[1] *Report of the Engineering Board of Review of the Sanitary District of Chicago on the Lake Lowering Controversy and a Program of Remedial Measures. Part III—Appendix II. Hydrology of the Great Lakes,* by Robert E. Horton in collaboration with C. E. Grunsky, page 174 (1927).

Law. These formulas are shown below, together with the one developed in this investigation, the evaporation being expressed in millimeters per hour.

Abassia:

$$E,\ \text{mm/hr} = 0.0122\,(e_w-e_d)+0.00029\,(e_w-e_d)\,v \ldots\ldots (E)$$

Fitzgerald:

$$E,\ \text{mm/hr} = 0.0166\,(e_w-e_d)+0.000783\,(e_w-e_d)\,v \ldots\ldots (F)$$

Carpenter:

$$E,\ \text{mm/hr} = 0.0161\,(e_w-e_d)+0.0000895\,(e_w-e_d)\,v \ldots\ldots (G)$$

Stelling:

$$E,\ \text{mm/hr} = 0.0351\,(e_w-e_d)+0.00044\,(e_w-e_d)\,v \ldots\ldots (H)$$

Mean,

$$E,\ \text{mm/hr} = 0.0200\,(e_w-e_d)+0.000401\,(e_w-e_d)\,v$$

This investigation:

$$E,\ \text{mm/hr} = \underset{(\pm 0.00154)}{0.0133}\,(e_a-e_d) + \underbrace{\underset{(\pm 0.00336)}{0.0334}\,(e_a-e_d)\,(v-4.84)}_{\text{(Enters only for } v \underset{>}{=} 4.84)} \ldots\ldots (I)$$

The saturation vapor-pressures corresponding to the surface-water temperature, to the dew-point temperature, and to the air temperature are e_w, e_d and e_a, respectively, and are expressed in millimeters of mercury. The wind velocity, v, is in meters per second. The five equations are shown graphically in Figure *B*. The chief point of difference between (*I*) and (*E*), (*F*), (*G*) and (*H*) is in the wind term.

Comparing the formulas in detail, the mean coefficient of (e_w-e_d), +0.0200, is larger than the coefficient of (e_a-e_d) in (*I*) by 0.0067, an amount 4.4 times the probable error of the coefficient of the latter $\left(\frac{0.0067}{.00154}=4.4\right)$. Similarly, the coefficient of (e_a-e_d) is about twice its own probable error smaller than the corresponding coefficients derived by Fitzgerald and Carpenter. It differs from the Abassia value, however, by an amount less than its own probable error.

The coefficient of $(e_a-e_d)(v-4.84)$ *in* (*I*), $+0.0334\pm0.00336$, *is 83.3 times larger than the mean coefficient of* $(e_w-e_d)v$ *in equations* (*E*), (*F*), (*G*), *and* (*H*), *+0.000401.* The difference between these values is $\left(\frac{0.0334-0.000401=}{0.00336}\right)9.8$ times the probable error, ±0.00336. Neglecting errors in Assumption No. 5, and the effects of systematic and constant errors, this is a strong indication—proof—that the wind terms in equations (*E*), (*F*), (*G*) and (*H*) are too small to represent the true rate of increase of evaporation on a natural body of water with increasing wind velocity for winds above 4.84 meters per second. For small winds—those less than 4.84 meters per second—the differences are not nearly so serious, except in the case of the Stelling formula.

Using a value of (e_w-e_d), or (e_a-e_d), of 8.85 mm. and a value of v of 10 meters/sec., the rate of evaporation as computed from equations (*E*) to (*H*) by Professor Bigelow, are compared below with that computed from equation (*I*).

Abassia, E, mm/hour = 0.1080 + 0.0257 = 0.1337

Fitzgerald, E, mm/hour = 0.1469 + 0.0693 = 0.2162

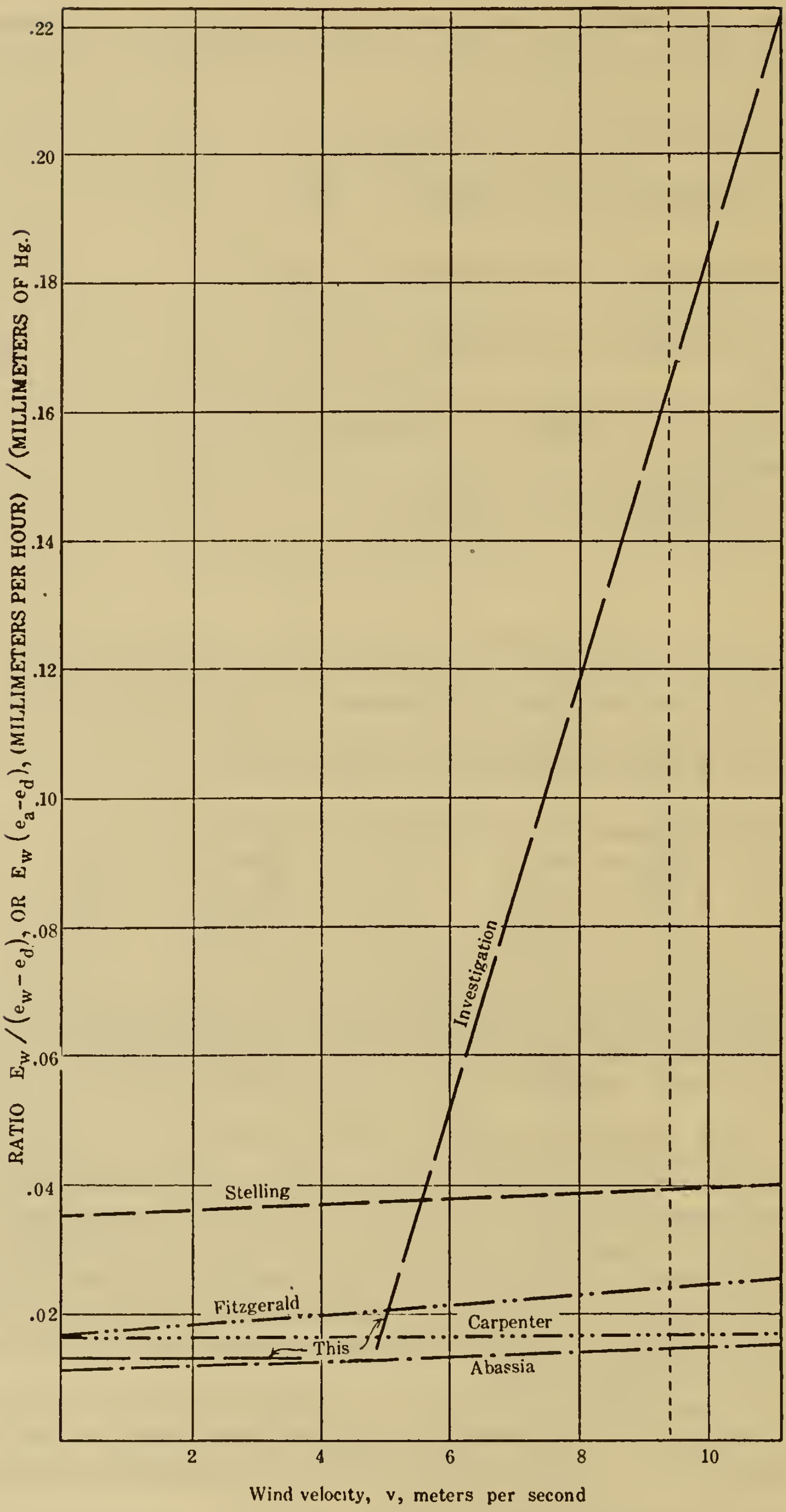

FIG. B—Comparison of Evaporation Curve Developed in this Investigation with those of Abassia, Fitzgerald, Carpenter and Stelling.

Carpenter, E, mm/hour $= 0.1425 + 0.0079 = 0.1504$

Stelling, E, mm/hour $= 0.3106 + 0.0389 = 0.3495$

Mean of above four, E, mm/hour $= 0.1770 + 0.0354 = 0.2124$

This investigation, E, mm/hour $= 0.1177 + 1.5264 = 1.6441$
$(\pm 0.0136) \qquad (\pm 0.153)$

Here, as with the comparison of the general formulas, the difference between the total evaporation, 1.6441 m.m./hour and the average total from the other four formulas, 0.2124 m.m./hour, is much too great to be accounted for by the probable errors.

COMPARISON WITH FORMULA PROPOSED BY MARVIN

The evaporation formula proposed by Marvin[1] is

$$\frac{de}{dt} = \frac{c}{B}(e_s + e_a - 2e_d)\, f\,(e)\, f\,(v) \dots\dots (J)$$

The form of this formula is in substantial agreement with the one developed in this investigation (B). In the latter, the effect of barometric pressure on evaporation, denoted by B in equation (J), has been thrown into the constants E_1 and E_2 because the effect of the variation in barometric pressure on evaporation is small in comparison with other, unavoidable errors. Also, by Assumption No. 5, $e_s = e_a$ in equation (J). On the basis of the two preceding sentences, we might rewrite (J) as follows:

$$\frac{de}{dt} = E\,(e_a - e_d)\, f\,(e)\, f\,(v) \dots\dots (K)$$

Equations (K) and (B) are the same in form, neglecting errors in Assumption No. 5, and the variation in the effect of barometric pressure on evaporation. In this investigation the functions $f(e)$ $f(v)$ have been determined in the manner set forth in detail in the text, from the actual observations of real evaporation on a natural lake under natural conditions.

CONCLUSIONS ON COMPARISON OF EVAPORATION FORMULAS

The chief difference between the evaporation formula developed in this investigation and the formulas based upon small scale apparatus lies in the wind term. The rate of increase of evaporation from the natural lake surface with increasing wind velocity, as established in this investigation, is *many times greater* than the corresponding rate from evaporation pans, as established in other studies. Two causes of the difference are apparent. In the first place, the wind velocity used in the establishment of the formula in this investigation is observed at an elevation of about 100 feet above the water surface. This, in itself, would tend to make the derived wind coefficient larger than it would be if the observations of wind velocity had been taken at a lower elevation, as in most evaporation pan experiments. In the second place, the water-surface exposed to evaporation at high winds is very much greater on a large, natural body of water, such as on any one of the Great Lakes, than it is in evaporation pans. Consider a natural lake under the influence of a wind of increasing velocity, beginning at zero. The water surface, from a plane surface (if one neglects seiches, tides and barometric effects) becomes disturbed, first by ripples, then later by waves. As the waves increase in size, the wind is caused to strike the water with a component at right-angles to the water surface which is greater than that in the case of an evaporation pan in which at best only small ripples form. Such action, in which the wind is brought forcibly into contact with the sides of the waves, must act greatly to increase the rate of evaporation.

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[1] *A Proposed New Formula for Evaporation*, by C. F. Marvin, Monthly Weather Review, vol. XXXVII, No. 2, page 57 (Feb. 1909).

Besides this effect, one might distinguish another. Suppose the wind velocity to increase beyond the point where white-caps begin to form (say beyond 11 miles per hour). In the action of white-caps, when the top of the wave is caught by the wind and transported forward at a greater rate than the lower part of the wave, air is trapped under it, later on bubbling through. Moreover, bits of spray are caught and thrown high into the air and far over the leeward shore of the lake. Under such conditions the exposed area of water in contact with the air, and the increased effectiveness and intimacy of that contact, together with the water carried inland, must act greatly to augment the rate of increase of evaporation with increasing wind velocity. Viewed in this manner this rate of increase must necessarily be much greater than in the case in small evaporation pans.

Part II. A New Method of Estimating Stream-Flow

A NEW METHOD OF ESTIMATING STREAM-FLOW

INTRODUCTION

In the preceding pages, the method of deriving the evaporation formula has been explained. It is the purpose in the pages which follow to indicate the progress made to date in this investigation of the laws of stream-flow, using as a basis the evaporation equation (23), previously presented (page 82).

Stream-flow, as used hereinafter, is understood to be the discharge of a stream at point of measurement, expressed in cubic feet per second or some similar unit.

Stream-flow is characteristically irregular. After very heavy rains or spring thaws, it goes up to values many times the average flow. It becomes very small at the end of long drouths. There are large differences in flow between streams at which the conditions are apparently similar.

The standard books upon the subject and the writings of experts are essentially qualitative, not quantitative. They lead one to an explanation of the variations of stream-flow through a maze of qualitative statements as to the manner in which the stream-flow is dependent upon rainfall, temperature, evaporation, vegetation, seepage, surface conditions, topography, geology, and many other factors. They give one but few *quantitative* relations between stream-flow and the various matters just mentioned. The writings by the experts do not give one such quantitative relations as will enable him to compute the effect upon stream-flow of these various matters. These statements are made to indicate the state of the present knowledge in this difficult field, and not as criticisms of, nor reflections upon, the authors of the books or the experts.

DATA USED AND ACKNOWLEDGMENTS

The principal data used in this study were those pertaining to two small streams at Wagon Wheel Gap, Colorado, at latitude 37° 46′ north, longitude 106° 35′ west and elevation about 10,000 feet, designated as Streams *A* and *B*. Each of these streams drains about one-third of a square mile. Plate 7, reproduced through the kind permission of Professor Marvin, shows the topography, etc., of the drainage areas.[1] On these two streams the Weather Bureau and the Forest Service made continuous observations of the various meteorological elements, and of the run-off, with an unusually high degree of accuracy from 1911 to 1926.[2] These streams were chosen for the first intensive study, mainly because of the extreme care and accuracy with which the observations have been made. The complete record of the observations for the period 1911–1915 were made available for use in this investigation through the kindness of Professor Charles F. Marvin, Chief of the U. S. Weather Bureau. Professor Henry J. Cox, meteorologist in charge of the Weather Bureau at Chicago, and his assistants, particularly Mr. C. A. Donnel, have facilitated the work on this part of this investigation in many ways, but especially by granting free access to the data at the Chicago Weather Bureau Office. Mr. L. F. Harza, Consulting Hydro-Electric Engineer in Chicago, has taken much interest in this study. His criticisms and encouragement are hereby gratefully acknowledged.

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[1] For dimensions of these watersheds, see Table 40, page 151.

[2] For complete description of this project the reader is referred to *Forest and Stream-flow Experiment at Wagon Wheel Gap, Colorado.* Final Report on Completion of the Second Phase of the Experiment, by C. G. Bates and A. J. Henry. Monthly Weather Review Supplement No. 30 (1928).

OUTCOME OF THIS INVESTIGATION

The outcome of this part of this investigation may be briefly stated as follows:

(1) Approximate quantitative expressions of the fundamental laws of flow of two streams, *A* and *B*, Wagon Wheel Gap, Colorado, have been derived. These numerical expressions enable one to compute the daily flow of these two streams from the observed weather elements of wind velocity, air temperature,vapor pressure, rainfall and snow gagings with a reasonably good degree of approximation.

(2) The general method has been developed by which such expressions can be derived for any stream anywhere in the eastern two-thirds of the United States where the annual rainfall is 20 inches or more from parallel observations of stream flow, as measured by the United States Geological Survey, or otherwise, and from the meteorological elements as observed by the Weather Bureau.

(3) The relation of the new knowledge stated in (1) and (2) to three important problems has become evident. The three problems are:

(*a*) The problem of increasing the length of record of flow of a stream, the hydrograph, as a basis for greater accuracy in the design of works for power, irrigation, sanitation and navigation.

(*b*) The problem of forecasting the flow of a stream as a basis for increasing the economy of operation of hydro-electric power plants.

(*c*) The problem of determining the effect of forest cover on the run-off from watersheds.

ORDER OF PRESENTATION

The order of presentation, briefly stated, is as follows:

(1) The manner of setting up the least-square solution to determine the normal flow of a stream is first given, including the theoretical basis for the observation equations. The principal facts in regard to the two final normal flow solutions are given, including the values computed from them. The manner of using these values to compute daily normal flows is set forth.

(2) The manner of setting up the least-square solutions to determine the flood-flow of a stream is given, including the theoretical basis for the observation equations. The principal facts in regard to the two final flood-flow solutions is given, including the values computed from them. The manner of using these values to compute daily flood-flows is set forth.

(3) The manner of setting up the least-square solutions to determine the laws of freezing and melting is given. The principal facts in regard to the two final solutions are given, including the values computed from them.

(4) A statement of the accuracy of the computed normal flow is presented.

(5) A statement of the accuracy of the computed flood-flow is presented.

(6) A statement of the accuracy of the laws of freezing and melting is presented.

(7) The over-all accuracy of the computed flow of the stream is discussed, and the evidence presented.

(8) Some general conceptions with reference to the travel of water from the time it reaches a watershed as rain or in the frozen form to the time it is measured as stream-flow are presented. These conceptions are intended to aid one in making a first estimate of the equations of flow of any stream. The method of correcting this first estimate from the actual observations is presented.

(9) The relation of this research to three important problems is briefly given.
(10) A brief summary of conclusions is given.

FORM OF EQUATION FOR NORMAL STREAM-FLOW

In the present discussion, consideration will be limited to a perennial stream in a moist climate, which, for the present purpose, is defined as one in which the annual precipitation is 20 inches or more. At present, consideration will further be limited to what will be called the *normal flow* of the stream, or the flow which is maintained by water which travels the *whole* of the distance *underground* from the point where it falls as rain, or occurs as melted snow and ice, to the stream.

On the basis of this investigation to date, it is believed that the normal flow of any stream in the eastern two-thirds of the United States, in any region where the annual rainfall is 20 inches or more, may be expressed by the following formula:

$$S_c+r_1R'_1+r_2R'_2+r_3R'_3+r_4R'_4+r_5R'_5+r_6R'_6+r_7R'_7+r_8R'_8+r_9R'_9+r_{10}R'_{10}=D' \quad\ldots\ldots\ldots(33)$$

S_c is the constant part of the flow.

r_1 is the *change in storage* in the drainage area on the current day, or day to which the equation pertains. Specifically,

$$r_1=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on current day;}$$

$$r_2=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on preceding day;}$$

$$r_3=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on day before preceding day;}$$

$$r_4=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on \textit{next} preceding \textit{two} days (the two days preceding } r_3\text{);}$$

$$r_5=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on \textit{next} preceding 4 days;}$$

$$r_6=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on \textit{next} preceding 8 days;}$$

$$r_7=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on \textit{next} preceding 16 days;}$$

$$r_8=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on \textit{next} preceding 32 days;}$$

$$r_9=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on \textit{next} preceding 64 days;}$$

$$r_{10}=\left\{\begin{matrix}\text{Rainfall}\\+\\\text{Net melting}\end{matrix}\right\}-\text{(evaporation from land)}-\text{(run-off), on \textit{next} preceding 128 days.}$$

From the above definitions of the r's, it is evident that it is only necessary to compute r_1 for each day; that r_2, r_3, . . . r_{10} on any day may be written in directly from the values of r_1 computed for preceding days; thus,

r_2 for current day $= r_1$ for preceding day;

r_3 for current day $= r_1$ for day before preceding day $= r_2$ for preceding day;

r_4 for current day $= \Sigma(r_1)$ for *two* days preceding day named in defining r_3 in terms of r_1; $= \Sigma(r_3)$ on preceding and day before preceding days;

r_5 for current day $= \Sigma(r_1)$ for *four* days preceding days named in defining r_4 in terms of r_1; $= \Sigma(r_4)$ on *second* and *fourth* preceding days;

r_6 for current day $= \Sigma(r_1)$ for the *eight* days preceding days named in defining r_5 in terms of r_1; $= \Sigma(r_5)$ on *fourth* and *eighth* preceding days;

r_7 for current day $= \Sigma(r_1)$ for the 16 days preceding days named in defining r_6 in terms of r_1; $= \Sigma(r_6)$ on eighth and sixteenth preceding days;

r_8 for current day $= \Sigma(r_1)$ for the 32 days preceding days named in defining r_7 in terms of r_1; $= \Sigma(r_7)$ on sixteenth and thirty-second preceding days;

r_9 for current day $= \Sigma(r_1)$ for the 64 days preceding days named in defining r_8 in terms of r_1; $= \Sigma(r_8)$ on thirty-second and sixty-fourth preceding days;

r_{10} for current day $= \Sigma(r_1)$ for the 128 days preceding days named in defining r_9 in terms of r_1; $= \Sigma(r_9)$ on sixty-fourth and one hundred and twenty-eighth preceding days.

It is thus apparent that by defining the r's in the manner stated, every r with subscript 4 or greater is the sum of two r's with subscript one less than the r wanted; and every r with subscript 3 or less (except r_1) can be copied from earlier r's. This device effects rapidity in computation.

The above definitions will become clearer later in connection with the numerical examples of the computations to be given.

D' is the flow of the stream, usually, expressed in cubic feet per second, or some similar unit.

R'_1, R'_2, R'_3, . . . R'_{10} are physical constants which express the effects of the total *change in storage* in the drainage area both above and below ground surface on the current day, preceding day, day before the preceding day . . . next preceding 128 days, respectively, on the flow of the stream the current day, or day to which the equation pertains.

The evaporation from land is computed from the following formula:

$$\text{Evaporation from land} = E_l = \left[0.319e + 1.49e\left(\frac{w}{100} - 2.6\right)\right]\left(\frac{E_l}{E_w}\right) \ldots\ldots\ldots\ldots\ldots (34)$$

In equation (34), $E_w = +0.319e + 1.49e\left(\frac{w}{100} - 2.6\right)$ is the evaporation from any free, open surface of water in units of 0.01 inch of depth per day anywhere in the United States where the observations of e and w are derived from standard Weather Bureau observations. This will be recognized as the evaporation formula derived from observations on the Great Lakes, or equation (23), page 82. In equation (34) $e = (e_a - e_d)$ in units of 0.01 inch of mercury, in which e_a = saturation vapor-pressure corresponding to the mean air temperature over the watershed in 0.01 inch of mercury; e_d = saturation vapor-pressure corresponding to the temperature of the dew-point, in units of 0.01 inch of mercury; w is the total travel of the wind, in miles per day, over the watershed, as measured by the nearest Weather Bureau anemometer or anemometers located about 100 feet above the ground surface. The expression $1.49e\left(\frac{w}{100}-2.6\right)$ enters the equation only when it is positive, or only when w exceeds 260 miles per day or 10.8 miles per hour.

$\frac{E_l}{E_w}$ is a constant for each watershed. It is derived from observations on the watershed in terms of the observed precipitation as rain, computed net melting of the snow and ice, observed run-off, and computed evaporation from water. Apparently the best basis for the evaluation of the ratio $\frac{E_l}{E_w}$ is the idea that in general for any long period, especially for a long period which begins and ends at the same season of the year, the net rate of addition to storage must be nearly zero. That is

$$\frac{\text{(Rainfall)} + \text{(Net melting)} - \text{(Evaporation from Land)} - \text{(Run-off)}}{\text{Length of period}} \quad \ldots\ldots\ldots\ldots (35)$$

must be numerically small, and must tend to get smaller as the length of the period is increased.

In equation (35) the rainfall is observed, the net melting is computed in a manner presently to be described, the run-off is observed, the evaporation from a water surface is computed, hence setting the equation equal to zero and solving for the only unknown,

$$\frac{E_l}{E_w} = \frac{\text{Rainfall}+\text{net melting}-\text{Run-off}}{E_w} \quad \ldots\ldots\ldots\ldots (35a)$$

In equation (35a) since E_w and the net melting are in units of 0.01 inch per day the observed run-off (stream-flow) must be converted into the same unit before substitution therein.

The use of equation (34) to compute the evaporation from land in the r's in equation (33) involves three assumptions, which may be stated at this point, namely:

Assumption No. (6)—It is assumed that the evaporation from a land surface (including the surfaces of trees, shrubs, etc.) in the summer months, when the precipitation is all in the form of rain, follows the same laws, to the degree of accuracy which can be detected from the observations themselves, as the evaporation from a free, open, water surface as stated in equation (23), modified only by the constant $\frac{E_l}{E_w}$.

Assumption No. (7)—It is assumed that the loss by evaporation from a land surface in the warmer months, to which assumption No. 6 applies, follows the same laws, to the degree of accuracy which can be detected from the observations themselves, when the trees and shrubs are covered with leaves in the spring, or when the crops are growing, as in the fall when the branches are bare and the ground covered with leaves, or after the crops have been harvested.

Assumption No. (8)—It is assumed that the evaporation from the surface of ice and snow in the winter months, when precipitation is all in the frozen form, follows the same laws, to the degree of accuracy which can be detected from the observations themselves, as the evaporation from a free, open, water surface as stated in equation (23), modified only by the constant $\frac{E_l}{E_w}$.

It is realized that Assumptions Nos. (6), (7) and (8) are approximations. Existing knowledge on these matters did not make possible a more exact expression at the outset. It was hoped that the deviation of the true laws of evaporation from land surfaces at all seasons of the year from those expressed in equation (34) could be detected from the observations themselves in this investigation, and that these deviations could be evaluated or expressed as modifications of equation (34) in definite terms. This hope was not realized, or rather, no proof was obtained that these approximations are far from the truth on Watersheds *A* and *B*. This evidence will be presented at an appropriate place later.

$$\text{Net melting on any day} = +C + \left\{ \begin{matrix} -(T''-t)F \\ \text{or} \\ +(t-T'')M \end{matrix} \right\} \dots\dots\dots\dots (36)$$

Equation (36) is dependent on there being plenty of snow and ice on and in the ground of the watershed. In equation (36), t is the mean observed air temperature on the watershed for the day in degrees Fahrenheit; C, F and M are physical constants to be derived from the observations. T'' is the mean air temperature for the watershed for the day in degrees F. at which there is assumed to be the same amount of melting as of freezing and therefore no change in storage in the drainage area due to melting and freezing.

Equation (33) thus expresses the mean flow of a stream on any day as a function of the changes in storage of water in the drainage area for a total period of 256 days or 8.4 months preceding the current day.

The constants S_c, R'_1, R'_2, R'_3, . . . R'_{10}, $\frac{E_l}{E_w}$, C, F, M and T'' must ordinarily be derived from parallel observations of stream-flow and the weather elements. The last four constants, C, F, M and T'' are believed to be the same for all watersheds. They have been evaluated in this investigation from the observations on Streams *A* and *B*. The method of deriving the constants in equations (33) to (36) is discussed later.

To compute the normal stream-flow for any day from the equation after its constants are evaluated, the observed weather elements of rainfall, temperature, vapor-pressure and wind velocity for the past period are substituted into the formula in accordance with the definitions of the quantities already given. The only quantity in Equation (33) which is not known on the current day, after the constants have all been derived, is r_1, since the run-off for the current day is not known. It is the quantity wanted. The procedure is to estimate the run-off for the current day and compute an approximate r_1. This enables one to compute D' for the current day. If this, when converted to the same units in which the r's are expressed, agrees with the estimated run-off for the current day, the computed D' for the current day is correct, and the computation may proceed to the computation of D' for the next day. If the D' computed for the current day differs from the run-off estimated for computing r_1, this estimated run-off must be changed until the computed D' for the current day agrees with the estimated run-off used in computing r_1. For flows to which Equation (33) applies, that is, stream-flows which result entirely from percolation and not from waters traveling over the surface of the ground to the stream, the estimate of the run-off for the current day is usually correct on the first trial, and is best estimated in terms of the computed D' for the preceding day. During a rising stream it may be necessary to add to the D' of the preceding day (converted to the same units as r_1) to get the estimated run-off of the current day. During a falling stream, it may be necessary to subtract. In either case the change one way or the other is gradual for the class of flows to which equation (33) applies, hence the estimate, with a little practice, is easily made correct the first time.

FORM OF EQUATION FOR FLOOD-FLOW

During periods of heavy rain or rapid melting of snow and ice, the ground becomes saturated, the "water-table" rises above the surface of the ground, and there is a rapid travel of some of the water toward the stream by surface flow. This surface travel of the water is much more rapid than travel through the ground, and gives rise to quick and large flows which follow a different law of stream flow at such times than if the normal underground lines of flow were followed. Two lines of evidence lead to this conclusion; one being the internal evidence of the least-square solutions themselves and the other certain external evidence. This evidence will be presented at an appropriate place in connection with discussing the "Form of Observation Equation for Determining Flood-Flow."

Whenever the rainfall (or its equivalent in melted snow and ice) exceeds, in a given drainage area in any day, a quantity which will be called G (in depth over watershed), on the basis of this investigation to date, the flood-flow of the stream, or that part of the stream-flow which travels over the surface of the ground to the stream, may be effectively expressed by the following formula:

$$r_{f1}R'_{f1}+r_{f2}R'_{f2}+r_{f3}R'_{f3}+r_{f4}R'_{f4}+r_{f5}R'_{f5}+r_{f6}R'_{f6}+r_{f7}R'_{f7}=\text{Flood-flow}\ldots\ldots(37)$$

In equation (37), $r_{f1}=(r_1-G)$, $r_{f2}=(r_1-G)$ for the preceding day, $r_{f3}=(r_1-G)$ for the day before the preceding day, and so on, the quantities r_{f1}, r_{f2}, r_{f3}, . . . r_{f7} bearing the same relation to each other as has been indicated for r_1, r_2, r_3, . . . r_{10}. All negative values of r_{f1} are ignored.

G for any day bears a fixed relation, determined from the computations, to the discharge, D', on the day before.

R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7} are physical constants which express the effects of only that part of the change in storage in the drainage area which occurs above the ground surface on the current day, preceding day, day before the preceding day . . . next preceding sixteen days, respectively, on the surface run-off or flood-flow the current day. That is, the R'_f's bear the same relations to the change in storage above the ground surface as the R''s bear to the total change in storage, both above and below ground.

During periods of flood-flow, the total flow of the stream is given by equations (33) and (37). It should be evident therefore that during periods of flood-flow, to compute the total stream-flow for the current day, one must, besides estimating r_1 for the current day as described on pages 138 and 139, also estimate r_{f1}. This makes the procedure of computing the total stream-flow from equations (33) and (37) a slower process in times of flood than when the stream is fed by percolation only.

THE FREEZING-MELTING THEORY

If the flow of a stream is not affected by current or earlier freezing and melting on its watershed, equation (33) becomes simplified by the fact that the net melting as expressed by equation (36) is zero. This would be true of any stream in which little or none of the precipitation on its watershed is in the frozen form, as for instance, roughly speaking, streams below latitude 35° to 37° N in the eastern two-thirds of the United States. For streams affected by freezing and melting of the snow and ice on their watersheds, equation (33), involving equation (36), becomes more complicated by virtue of the lag between precipitation in the frozen form and its subsequent run-off as water, and because of the reverse process—a subtraction from the storage in the drainage area by freezing.

Equation (33), involving equation (36), when applied to normal stream-flow in cold weather, is based upon the following assumptions:

Assumption No. (9)—There is a mean air temperature on the watershed for a day which will be called T' for which the melting during the 24-hour period is just balanced by the freezing during that period, and there is therefore no *net* addition to or subtraction from the stored *water* in the drainage area by melting and freezing.

Assumption No. (10)—The stored water in the drainage area moves according to the *same laws*, to the degree of accuracy which can be detected by observations of stream-flow, in cold weather as in warm weather. The stored ice and snow does not move.

Assumption No. (11)—Whenever the mean air temperature for a day in the drainage area is below T' the amount of water which is changed into ice on that day, and thereby withheld from storage as water, is $(T'-t)F$, in which t is the observed mean air temperature for the day and F is a constant to be determined from the observations.

Assumption No. (12)—Whenever the mean air temperature for a day in the drainage area is above T', and there is an abundant amount of ice or snow on and in the ground available for melting, the amount of water which is added to the storage in the drainage area on that day is $(t-T')M$; in which M is a constant to be derived from the observations.

On the above four assumptions r_1, the addition to the water stored in the drainage area on the current day, is

$$r_1 = (\text{Observed } \textit{rainfall}) - (\text{Estimated evaporation from land} - (\text{Observed discharge})$$

$$+\left\{\begin{array}{c} -(T'-t)F \\ \text{or} \\ +(t-T')M \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Note that equation (38) takes no account of precipitation in any other form than *rain*. It is written as if T' were known. It can not be known from the start and must be determined from the observations. Let T'' be an assumed value of T', and let it be assumed that when the mean air temperature for the day is T'', there will be an amount of melting C, that the melting at higher temperatures will be $C+(t-T'')M$, and that the amount of water held back from storage by freezing at lower temperatures will be $+C-(T''-t)F$. Then from a given set of observation equations, as indicated later, a value of C may be derived. If this derived value of C is positive, the necessary correction to the assumed temperature T'' to give the true temperature T' at which melting and thawing are just balanced is $T = -\frac{C}{M}$. If the derived value of C is negative, the necessary correction to T'' is $T = -\frac{C}{F}$.

On the basis just indicated, in which C is taken into account, the expression for r_1 as given in equation (38) must be modified to read as follows:

$$r = (\text{Observed rainfall}) - (\text{Estimated evaporation from land}) - (\text{Observed discharge}) + \left\{ \begin{matrix} +C-(T''-t)\ F \\ \text{or} \\ +C+(t-T'')\ M \end{matrix} \right\} \qquad (39)$$

In equation (39) if we designate by n_1, the following:

$$n_1 = (\text{Observed rainfall}) - (\text{Estimated evaporation from land}) - (\text{Observed discharge}) \qquad (40)$$

on the current day, then

$$r_1 = \left[n_1 + C + \left\{ \begin{matrix} -(T''-t)\ F \\ \text{or} \\ +(t-T'')\ M \end{matrix} \right\} \right] \qquad (41)$$

The values for $r_2, r_3, r_4, \ldots r_{10}$ of equation (33) may now be written in the forms corresponding to that shown for r_1 above, using the definitions given after equation (33).

Each $r_1, r_2, r_3, \ldots r_{10}$, for any date which reaches back in the computation to a day on which there was melting or freezing, will consist of a *known numerical part* made up of (observed rainfall), (estimated evaporation from land), and (observed discharge)—the n_1 of equation (40)—and an *unknown literal part* of the form $+C+ \left\{ \begin{matrix} -(T''-t)\ F \\ \text{or} \\ +(t-T'')\ M \end{matrix} \right\}$, the net melting of equation (36), in which the values of $(T''-t)$ or $(t-T'')$ are known from the observed temperatures and the assumed T''.

FORM OF OBSERVATION EQUATION TO DETERMINE C, F AND M

On the basis of what has preceded, using equations (33), (37) and (41), we may now write the form of observation equation for determining C, F and M as follows:

$$S_0 + R'_1 \left[n_1 + C + \left\{ \begin{matrix} -(T''-t)\ F \\ \text{or} \\ +(t-T'')M \end{matrix} \right\} \right] + R'_2 \left[n_2 + \Sigma_2(C) + \left\{ \begin{matrix} -\Sigma_2(T''-t)\ F \\ \text{or} \\ +\Sigma_2(t-T'')M \end{matrix} \right\} \right] + R'_3 \left[n_3 + \Sigma_3(C) + \left\{ \begin{matrix} -\Sigma_3(T''-t)\ F \\ \text{or} \\ +\Sigma_3(t-T'')M \end{matrix} \right\} \right] +$$

$$+R'_4 \left[n_4 + \Sigma_4(C) - \Sigma_4(T''-t)F + \Sigma_4(t-T'')M \right] + R'_5 \left[n_5 + \Sigma_5(C) - \Sigma_5(T''-t)F + \Sigma_5(t-T'')M \right] +$$

$$+R'_6 \left[n_6 + \Sigma_6(C) - \Sigma_6(T''-t)F + \Sigma_6(t-T'')M \right] + R'_7 \left[n_7 + \Sigma_7(C) - \Sigma_7(T''-t)F + \Sigma_7(t-T'')M \right] +$$

$$+R'_8 \left[n_8 + \Sigma_8(C) - \Sigma_8(T''-t)F + \Sigma_8(t-T'')M \right] + R'_9 \left[n_9 + \Sigma_9(C) - \Sigma_9(T''-t)F + \Sigma_9(t-T'')M \right] +$$

$$+R'_{10} \left[n_{10} + \Sigma_{10}(C) - \Sigma_{10}(T''-t)F + \Sigma_{10}(t-T'')M \right] + R'_{11} \left[(n_1 - G_1) + C + (t-T'')M \right] +$$

$$
\begin{aligned}
&+R'_{f2}\Big[(n_2-G_2)+\Sigma_2(C)+\Sigma_2(t-T'')M\Big]+R'_{f3}\Big[(n_3-G_3)+\Sigma_3(C)+\Sigma_3(t-T'')M\Big]+R'_{f4}\Big[(n_4-\\
&\qquad G_4)+\Sigma_4(C)+\Sigma_4(t-T'')M\Big]+\\
&+R'_{f5}\Big[(n_5-G_5)+\Sigma_5(C)+\Sigma_5(t-T'')M\Big]+R'_{f6}\Big[(n_6-G_6)+\Sigma_6(C)+\Sigma_6(t-T'')M\Big]+\\
&+R'_{f7}\Big[(n_7-G_7)+\Sigma_7(C)+\Sigma_7(t-T'')M\Big]-D'=-D \ldots\ldots\ldots (42)
\end{aligned}
$$

In equation (42), the sum of the terms S_c to $R'_{10}[n_{10}+\Sigma_{10}(C)-\Sigma_{10}(T''-t)F+\Sigma_{10}(t-T'')M]$, inclusive, express the normal part of the stream-flow. This is equation (33) rewritten with r_1, r_2, r_3, . . . r_{10} defined as in (41). The terms $R'_{f1}[(n_1-G_1)+C+(t-T'')M]$ to $R'_{f7}[(n_7-G_7)+\Sigma_7(C)+\Sigma_7(t-T'')M]$, inclusive, express the "Flood-flow" part of the stream-flow, or equation (37) rewritten with r_1, r_2, . . . r_7 defined as in equation (41), except that the terms like $-(T''-t)F$, $-\Sigma_2(T''-t)F$, . . . $-\Sigma_7(T''-t)F$ have been omitted because a flood-flow can not be initiated on a day when $t<T''$.

D in that part of the stream flow which is not accounted for in terms of $S_c+R'_1n_1+R'_2n_2+\ldots+R'_{10}n_{10}$ and $R'_{f1}(n_1-G_1)+R'_{f2}(n_2-G_2)+\ldots+R'_{f7}(n_7-G_7)$; in other words D is that part of the stream-flow which is to be accounted for as fully as possible by C, F, M and T''.

For convenience in equation (42) let

$$S_c+R'_1n_1+R'_2n_2+R'_3n_3+\ldots+R'_{10}n_{10}=D_n \ldots\ldots\ldots (43)$$

and

$$R'_{f1}(n_1-G_1)+R'_{f2}(n_2-G_2)+R'_{f3}(n_3-G_3)+\ldots+R'_{f7}(n_7-G_7)=D_f \ldots\ldots\ldots (44)$$

the latter equation being confined to days affected by flood-flows, when there is plenty of snow and ice available for melting, then

$$D_n+D_f-D'=-D \ldots\ldots\ldots (45)$$

in which D is that part of the stream-flow to be accounted for in terms of C, F, M and T''.

Now, in equations (42), (43), (44) and (45), the following quantities are assumed to be known before C, F and M can be determined from (42). These quantities (I) to (IX) are assumed to be known as estimated, derived from previous least-square computations, or directly observed:

(I) S_c, R'_1, R'_2, R'_3, . . . R'_n, which are assumed to be known, as assumed or derived from previous least-square computations confined to normal stream-flows in the summer months;

(II) R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{fn}, which are assumed to be known, as assumed or derived from previous least-square computations confined to flood-flows in summer months;

(III) The n's, which can be computed from observed or estimated quantities of rainfall, evaporation and run-off as defined in equations (40) and (34);

(IV) D', which is the directly observed stream-flow;

(V) Values of G (the r_1 required to glut the ground before a surface flow is initiated) for various D''s, which are determined from the computations which yield (I) and (II);

(VI) T'', which is an assumption at first, later verified by least-square computations;

(VII) Values of t, which are directly observed;

(VIII) Values of $(T''-t)$ and their Σ's, which can be computed from (VI) and (VII); and

(IX) Values of $(t-T'')$, and their Σ's, which can be computed from (VI) and (VII).

If now all these known quantities, (I) to (IX), are substituted in the observation equation, equation (42), and all similar quantities grouped and combined as far as possible, the observation equation for determining C, F and M may be written in the following form, one for each day:

$$xC+yF+zM-D=v \quad \text{(46)}$$

In this equation, x, y and z are *definite, known, numerical quantities*, fixed by previous computations and by observed, assumed, or derived quantities as indicated in (I) to (IX) above. The specific definitions of x, y and z are as follows, as given in equations (47) to (55) inclusive:

$$x=x_n+x_f \quad \text{(47)}$$

$$y=y_n+y_f \quad \text{(48)}$$

$$z=z_n+z_f \quad \text{(49)}$$

In equations (47), (48) and (49), and referring to equation (42),

$$x_n=[R'_1C+R'_2\Sigma_2(C)+R'_3\Sigma_3(C)+R'_4\Sigma_4(C)+R'_5\Sigma_5(C)+R'_6\Sigma_5(C)+R'_7\Sigma_7(C)+R'_8\Sigma_8(C)+R'_9\Sigma_9(C)+R'_{10}\Sigma_{10}(C)]/C \quad \text{(50)}$$

and

$$x_f=[R'_{f1}C+R'_{f2}\Sigma_2(C)+R'_{f3}\Sigma_3(C)+R'_{f4}\Sigma_4(C)+R'_{f5}\Sigma_5(C)+R'_{f6}\Sigma_5(C)+R'_{f7}\Sigma_7(C)]/C \quad \text{(51)}$$

the latter equation being confined to days affected by flood-flows, when there is plenty of snow and ice available for melting.

$y_n=R'_1(t-T'')+R'_2\Sigma_2(t-T'')+R'_3\Sigma_3(t-T'')+R'_4\Sigma_4(t-T'')+ \ldots +R'_{10}\Sigma_{10}(t-T'')$ confined to those cases in which $t< T''$ .... (52)

$y_f=0$, since for values of $t< T''$ no flood-flow can be initiated .... (53)

$z_n=R'_1(t-T'')+R'_2\Sigma_2(t-T'')+R'_3\Sigma_3(t-T'')+ \ldots +R'_{10}\Sigma_{10}(t-T'')$, confined to those cases in which $t > T''$ and there is plenty of snow and ice on and in the ground of the watershed available to be melted; .... (54)

$z_f=R'_{f1}(t-T'')+R'_{f2}\Sigma_2(t-T'')+R'_{f3}\Sigma_3(t-T'')+ \ldots +R'_{f7}\Sigma_7(t-T'')$, confined to days affected by flood-flows and on which there is an abundant supply of snow and ice on and in the ground of the watershed available to be melted .... (55)

In equation (46) v is the residual, or the discrepancy between the computed net melting $(xC+yF+zM)$, and D, that part of the discharge not accountable for in terms of equations (43) and (44). From a set of observation equations, values of C, F and M may be computed by the least-square method of computation. The substitution in observation equations furnishes a group of v's which serve to determine the accuracy with which C, F and M have been derived from the set of equations.

The complete meanings of all the terms involved in equations (42) to (55) will be made clear later in connection with numerical illustrations.

FORM OF OBSERVATION EQUATION FOR DETERMINING NORMAL STREAM-FLOW

In what has preceded, culminating in equation (55), there have been presented: (*a*) the form of equation which expresses normal stream-flow, viz, equation (33) involving equation (34), (35*a*) and (36); (*b*), the form of equation which expresses flood flow, viz, equation (37); and (*c*), the form of observation equation for determining C, F and M, viz, equation (46).

It should be obvious that the constants, S_c, R'_1, R'_2, . . . R'_{10} in equation (33) could not be determined at the outset in this research from observations in the winter months without knowing how to compute the net melting, equation (36), or the $\frac{E_l}{E_w}$ of equation (35a). Likewise it was impossible to evaluate all of the R''s in equation (33) from simultaneous observations of stream-flow and the meteorological elements in the summer months only, when net melting was not involved, because on the Wagon Wheel Gap watersheds the air temperature is above freezing for only about seven months out of the year, and to compute r_{10} one must go back 8.4 months from the date for which r_{10} is wanted. (See definition of r_{10} after equation (33).)

The method of procedure was as follows: It was assumed at the outset that $\frac{E_l}{E_w}$ was the same as for the Great Lakes' region, viz, 0.62; that is, it was assumed that the rate of evaporation from the land surface of the Wagon Wheel Gap area was 62 per cent as large as that from a water surface, as determined for the Great Lakes' drainage area. Further, the observations were confined to the summer months only, which eliminated net melting from equation (33). The observation equation for the first least-square solution then had the following form:

$$S_c+r_1R'_1+r_2R'_2+r_3R'_3+r_4R'_4+\ .\ .\ .\ +r_nR'_n-D'=v\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(56)$$

In this equation $r_1=$(observed *rainfall*)$-E_w(0.62)-$(observed run-off) on the current day, r_2 has the same definition but relates to the preceding day, r_3 to the day before the preceding day, and so on, similarly, to the definitions already given in connection with equation (33).

D' is the observed stream-flow on the current day, or day to which the equation pertains, expressed in 0.001 c.f.s.

S_c is the constant part of the stream-flow in the same units as D', and R'_1, R'_2, . . . R'_n express the effects of current and earlier changes in storage in the drainage area, on the normal flow of the stream on the current day.

The right-hand member of equation (56) v is a residual and represents the discrepancy between the stream-flow as computed from theory—the S_c and rR''s—and the observed stream flow, D'.

One observation equation in the form of equation (56) was written for each day. From these the least-square solution served to determine the most probable values of the unknowns, and from the residuals, v, the accuracy of the unknowns (their probable errors) was computed.

Because of the relatively short period of the summer months on the Wagon Wheel Gap watersheds; because, to compute r_{10} for any day, one must use data which goes back a period of 8.4 months; and because only three years of data were available for use in deriving the laws of flow (viz, 1911–1913)—as it was desirable to apply the formulas to years (1914–1915) not used in their derivation—equation

(56) served to determine less than 10 R''s. The last R' determinable is indicated in equation (56) by the subscript n.

On each of Streams A and B, equation (56) was applied and as many R''s derived as possible from simultaneous observations of the weather elements and stream discharges in the summers of 1911, 1912 and 1913. An R' was considered real when it came out greater than 3.5 times its own probable error.

The laws of normal flow so derived were first approximations, because $\frac{E_t}{E_w}$ was assumed and only part of the R''s were evaluated.

These first approximations were made more exact by the step-by-step process outlined on pages 146 and 147.

FORM OF OBSERVATION EQUATIONS FOR DETERMINING FLOOD-FLOW

From any least-square solution in which equation (56) was used, a set of v's was obtained in the substitution in observation equations. From these v's the probable error of a single observation was computed. Normally, any residual greater than 3.5 times the probable error of a single observation was considered "suspicious," and a candidate for careful scrutiny. A residual greater than five times the probable error of a single observation was considered a candidate for rejection in the next least-square solution of the series.

If a residual in equation (56) came out negative, the meaning was that the computed stream-flow, computed according to the assumed law of normal flow, was less than the observed stream-flow, D'. If it was negative and greater than 3.5 times the probable error of a single observation, this was considered an indication that the flow concerned was a flood-flow. According to the law of probability, only 18 equations per thousand should have residuals greater than the suspicion limit if the law of flow as expressed by equation (56) is complete; that is, if there were no approximations involved in the theory, or if the residuals followed the normal law of error. The appearance of many more v's beyond the suspicion limit than are accountable for on the basis of accidental errors formed a clear indication, proof, that the law of flow as expressed by equation (56) is not complete; that in times of large flows, one must account for that part of the flow which equation (56) fails to account for by another expression. The interpretation of the large number of $-v$'s during heavy rain-falls when equation (56) alone is used is that, as already stated, equation (56) accounts for only that part of the stream-flow which is produced by water which travels largely through the ground to the stream. When the ground becomes glutted with water, when the water-table has risen to the level of the ground surface, a surface flow is started. It has been found that this surface flow is expressible in the form of equation (37).

The internal evidence from the least-square solutions that the stream-flow follows a different law during heavy rains or rapid melting is very strong, but if one is inclined to be skeptical about it, the following sample of external evidence, not connected with the least-square solutions, is given:

At the Wagon Wheel Gap streams the maximum R', expressed as a percentage of the total change in storage which reaches the stream by percolation on any day, is less than 1 per cent. The maximum flow is about 6 c.f.s. per square mile of drainage area. At the Miami Conservancy District, according to figure 4, page 30, of *Hydraulics of the Miami Flood Control Project* by Sherman M. Woodward, the maximum R' expressed as a percentage, during the March 1913 flood, was probably between 40 and 50 per cent. Also according to figure 5, page 33, the maximum flow at various places in the Miami Valley during the 1913 flood was from 68 to 571 c.f.s. per square mile, and for the whole drainage area it was 94 c.f.s. per square mile.

The more than 40 or 50 to 1 ratio of the R''s and the 11 to 1, to 90 to 1, ratio of discharges in c.f.s. per square mile, show that the law of flood-flow is probably different, to a decided degree, from the law of normal stream-flow.

The meaning of the quantity "Maximum R' expressed as a percentage, etc.," as used in the second preceding paragraph has not yet been defined. The definition appears subsequently in the proper context.[1]

The form of observation equation for determining flood-flow is as follows:

$$r_{f1}R'_{f1}+r_{f2}R'_{f2}+r_{f3}R'_{f3}+r_{f4}R'_{f4}+r_{f5}R'_{f5}+\ \dots\ +F'=v \qquad (57)$$

The r_f's and the R'_f's have already been defined in connection with equation (37). G for any day is that addition to storage for the day which is just sufficient to saturate the ground on that day.

A least-square solution for determining the constants in equation (56) gives rise to a set of v's. Using normally only those which are negative and greater than the suspicion limit of the solution to which equation (56) pertains, there is obtained a set of absolute terms, the F''s, of equation (57), from which one can derive the flood coefficients R'_{f1}, R'_{f2}, R'_{f3}, . . . provided G is known for each day. Apparently the best criterion by which to determine G for any day, the size of the r_1 for that day required to glut the ground, is the size of the stream-flow the preceding day, when it was not appreciably affected by abnormal surface flow. An examination of all the F''s of the solution to which equation (56) applies, in connection with the D''s of the preceding day, enables one to establish a curve from which any G can be taken for the corresponding D' on the preceding day. This enables one to compute (r_1-G) or r_{f1}, and thereafter, the other r_f's, the r_f's with subscripts larger than 1 being obtained from r_{f1} in the same manner as has been explained for the r's. Knowing the r_f's, a set of observation equations in the form of equation (57) can be written, from which the R'_f's can be determined from the least-square solutions. The v's, or right-hand members in equation (57), serve to indicate the over-all accuracy of the theory, and enable one to compute the individual probable errors of the unknowns.

METHOD OF PROCEDURE ON STREAMS *A* AND *B*

It is now proposed to state, in outline only, the method of procedure on Streams A and B in establishing numerical values for the S_c's, the R''s, the R'_f's, $\frac{E_l}{E_w}$, C, F, M and T'''. A detailed exposition of these processes would be outside the limits of space and time justified for them. It is hoped that a sufficiently clear statement can be made to enable one to grasp the principles involved, so that he may apply them to any streams anywhere to which this method is at present limited. The method of procedure is that indicated in the following numbered steps:

(1) Using, first, an observation equation of the form of equation (56), in which $\frac{E_l}{E_w}$ was assumed to be 0.62, approximate numerical expressions of the normal

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[1] See page 163.

stream-flow were obtained on both Streams A and B. These observation equations were confined to the summer months only, and no r was used which reached back in its computation into the spring or winter freezing-melting period. The least-square solutions of this kind contained from 30 to 60 observation equations each, one equation for each day, normally in August and September of 1912 and 1913. Into each observation equation were written the known, or assumed, quantities, the r's and the D''s. By the method of least-squares, the unknowns S_c to R'_n were evaluated. The result was an approximate, numerical expression for the flow of the stream when the addition to storage in the drainage area was all from *rain* (except some of it—an uncertain part—which came from springs).

(2) The residuals from the least-square solutions mentioned in step (1) which were negative and greater than 3.5 times the probable error of a single observation of the solution, became approximate F''s for use in equation (57) and made possible a determination of approximate flood-flow expressions for the two streams. It is important to note here also that the approximate numerical expression so derived applied to the flood-discharges in times of heavy *rain.*

(3) The approximate expressions of normal flow and flood-flow obtained in steps (1) and (2), that is, the approximate S_c's, R''s and R'_f's for both streams, were then used in equations (42) to (55), and approximate values of C, F and M computed, with a T' at first assumed at 30° F. This procedure of using the approximate expressions of the stream-flow derived from observations in the *summer* months to evaluating that part of the stream flow in the *winter* months which is not accounted for in terms of C, F and M, is justifiable on the basis of assumption No. 10, page 140.

(4) With approximate values of C, F and M derived as stated in step (3), it now became possible to (a) derive an approximate value of $\frac{E_t}{E_w}$ from equation (35a); and (b) to revise the first expression for the normal flow derived in step (1) above, and to derive additional R''s in equation (56). This process gave a more exact expression of the normal stream-flow than was obtained in step (1) above.

(5) Using the results of step (4) it now became possible to revise the results of step (2), and so secure a better expression for the flood-flow than that obtained in step (2).

(6) Using the results of steps (4) and (5), it now became possible to repeat step (3), and so secure better values of C, F, M, and T'', than those derived in step (3).

The method of procedure is thus seen to be a cyclical one, which gradually converges toward the truth, or toward as much of the truth as is obtainable with the adopted assumptions and forms of expression. Steps (1), (2) and (3) constitute one cycle; steps (4), (5) and (6) another. The convergence toward the truth is indicated by a decrease in the probable errors of the constants themselves, by a general decrease in size of the residuals, and by a tendency for the residuals to become accidental in their time distribution.

In the study on Streams A and B, in such steps as indicated in (1) to (6) above, a total of 69 least-square solutions was made, each containing from one to twelve unknowns, and each containing from nineteen to 280 observation equations. The final numerical results on the two streams, as hereinafter presented, were based upon only 8 of the above 69 solutions. The other solutions were necessary from the nature of the problem involed. They served to build up gradually the method to its present status as already partially presented in general terms.

EXAMPLE OF OBSERVATION EQUATION FOR NORMAL STREAM-FLOW

The first form of observation equation for a least-square solution to determine the normal flow of a stream is that shown in equation (56). Since it is the intention here to present an example from the final solution which served to fix the unknowns for Stream *A*, equation (56) must be extended to include all the rR' terms indicated in equation (33) and otherwise modified in the manner to be indicated. The inclusion of all the rR''s is possible in the final solution because the laws of freezing and melting are assumed to be known, as derived in such steps as (1) to (6) above. Also $\frac{E_l}{E_w}$ is assumed to be known. With these quantities known the complete observation equation for Stream *A* can be written in the following form:

$$S_c+r_1R'_1+r_2R'_2+r_3R'_3+r_4R'_4+r_5R'_5+r_6R'_6+r_7R'_7+r_8R'_8+r_{9a}R'_{9a}+r_{9b}R'_{9b}+r_{10}R'_{10}-D'=v \qquad (58)$$

The only differences between this equation and equation (33) are that D' has been placed on the left of the equality sign, the residual, v, indicated on the right of it, and the term $r_9R'_9$ has been divided into two parts, $r_{9a}R'_{9a}$ and $r_{9b}R'_{9b}$. The last-named alteration was made in this particular case merely to determine whether the effect of the change in the storage as defined in the first half of r_9 (r_{9a}) was different from the effect as defined in the last half (r_{9b}), on the stream-flow on the current day. The final numerical values to be presented later show that the difference is small for Stream *A*, and the splitting up of the $+r_9R'_9$ term into two parts is not justifiable.

Besides the alterations mentioned with reference to equation (58), one other alteration of equation (56) is advantageous. Instead of computing directly the unknowns S_c, R'_1, R'_2, . . . R'_{10} from the least-square solution in which the observed D' is used as absolute term, it is advantageous to make a preliminary substitution in observation equations, using approximate values of S_c, R'_1, R'_2, . . . R'_{10} as determined from previous solutions—as indicated in steps (1) to (6) above, or as estimated, or as obtained in any manner whatsoever—and to compute corrections to the assumed values of S_c, R'_1, R'_2, . . . R'_{10} from the least-square solutions, using the residuals in the preliminary substitution in observation equations as the absolute term in the observation equations for determining the corrections.

Thus, let the assumed values of S_c, R'_1, R'_2, R'_3, . . . R'_{10} be designated by R'', R''_1, R''_2, R''_3, . . . R''_{10}, respectively, and let the residuals resulting from a preliminary substitution in observation equations be designated by $-D$. Then, using the r's, an example of the computation of which will be presently presented, the substitution in observation equation takes the form

$$R''+r_1R''_1+r_2R''_2+r_3R''_3+r_4R''_4+r_5R''_5+r_6R''_6+r_7R''_7+r_8R''_8+r_{9a}R''_{9a}+r_{9b}R''_{9b}+r_{10}R''_{10}-D'=-D \qquad (59)$$

$-D$ is the discrepancy between the stream-flow computed from the assumed values of S_c, R'_1, R'_2, . . . R'_{10}; that is from R'', R''_1, R''_2, . . . R''_{10} (with the use of the r's) and the observed stream-flow, D'. (Note that this is not the same as the $-D$ in equations (42), (45) and (46).) Using $-D$ as the absolute term, the observation equation, (58), now takes this form

$$R+r_1R_1+r_2R_2+r_3R_3+r_4R_4+r_5R_5+r_6R_6+r_7R_7+r_8R_8+r_{9a}R_{9a}+r_{9b}R_{9b}+r_{10}R_{10}-D=v \qquad (60)$$

in which $R, R_1, R_2, R_3, \ldots R_{10}$ are corrections, respectively, to $R'', R''_1, R''_2, R''_3, \ldots R''_{10}$ to give the true constants $S_c, R'_1, R'_2, R'_3, \ldots R'_{10}$, respectively; that is,

$$\left.\begin{array}{lll} S_c=R''+R & R'_4=R''_4+R_4 & R'_8=R''_8+R_8 \\ R'_1=R''_1+R_1 & R'_5=R''_5+R_5 & R'_{9a}=R''_{9a}+R_{9a} \\ R'_2=R''_2+R_2 & R'_6=R''_6+R_6 & R'_{9b}=R''_{9b}+R_{9b} \\ R'_3=R''_3+R_3 & R'_7=R''_7+R_7 & R'_{10}=R''_{10}+R_{10} \end{array}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (61)$$

The v of equation (60) is the same as the v of equation (58).

It is this form of observation equation, (60), which was used throughout this investigation for normal stream-flows in every solution in which it was possible to estimate values of $S_c, R'_1, R'_2, \ldots R'_{10}$, with any degree of confidence. This became possible, in fact, after the very first solution on Stream A.

The meanings of the r's in equation (60) are here repeated for convenience and because they differ slightly from those previously given.

The current day is the day to which the equation is assigned in listing the equations.

$r_1=\left\{\begin{matrix}\text{Observed rainfall}\\ + \\ \text{Net melting}\end{matrix}\right\}$ −(estimated evaporation from land) − (observed run-off), on current day. Or otherwise stated,

$r_1=$ (total addition to storage) − (total subtraction from storage), in the drainage area on current day,

= net change in storage on current day.

$r_2=$ change in storage on preceding day.

$r_3=$ change in storage on day before preceding day.

$r_4=$ change in storage on next preceding two days; (the two days preceding r_3).

$r_5=$ change in storage on next preceding 4 days.

$r_6=$ change in storage on next preceding 8 days.

$r_7=$ change in storage on next preceding 16 days.

$r_8=$ change in storage on next preceding 32 days.

$r_{9a}=$ change in storage on next preceding 32 days.

$r_{9b}=$ change in storage on next preceding 32 days.

$r_{10}=$ change in storage on next preceding 128 days.

The unit in which r_1 to r_4 inclusive are expressed is 0.01 inch of depth on the watershed; r_5 to r_{9b}, 0.1 inch of depth; r_{10}, 1.0 inch of depth.

The sample computations to be given were taken from Solution M, Stream A. At the time this solution was made the best available values of C, F, M, T'' and $\dfrac{E_l}{E_w}$ were as follows:

$$\left.\begin{array}{l} C=+6.40\pm0.38 \\ F=+0.453\pm0.037 \\ M=+8.39\pm0.015 \\ T''=28^\circ\ \text{F}. \end{array}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

It should be emphasized here that the above are not the final values of C, F and M derived in this investigation. The final (best) values will be presented later.[1] The values in equation (62) are used in the illustration to be given because time will not permit remaking the computations with the best values.

$$\frac{E_l}{E_w}=2.3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (63)$$

Substituting equation (62) in equation (36) there is obtained

$$\text{Net melting}=+6.40+\left\{\begin{matrix} -(28^\circ-t)0.453 \\ \text{or} \\ +(t-28^\circ)8.39 \end{matrix}\right\} \text{in units of 0.01 inch of depth per day on the watershed} \quad \ldots\ldots \quad (64)$$

Substituting equation (63) in equation (34) there is obtained

$$E_l=\left[0.319\ e+1.49\ e\ \left(\frac{w}{100}-2.6\right)\right]\left[2.3\right] \text{ in units of 0.01 inch of depth per day on the watershed} \quad \ldots\ldots \quad (65)$$

For definitions of e and w, see pages 136 and 137.

The use of equations (64) and (65), together with the observed *rainfall* on the watershed, and the observed stream-flow (converted to 0.01 inch of depth per day) enabled one to compute r_1, r_2, r_3, . . . r_{10}. Having computed these, the next step is to use equations (59), (60) and finally (61).

The mean rainfall as used in the computation of r_1, r_2, r_3, . . . r_{10} was the arithmetic mean of the observed rainfall as made by the Weather Bureau at Stations A_1, A_2, C, and D on and near Watershed A. The exact locations of these meteorological stations is shown on Plate 7.

The mean air temperature, t, used in equations (64) and (65) was the arithmetic mean of that observed at Stations A_1, A_2, C and D on and near Watershed A.

The mean vapor-pressure corresponding to the temperature of the dew point was taken as the mean of that obtained from wet and dry bulb observations at Stations A_1 and C, on and near Watershed A.

The saturation vapor-pressure corresponding to t was obtained from *Psychrometric Tables for Obtaining the Vapor Pressure, Relative Humidity, and Temperature of the Dew Point*, by Professor C. F. Marvin.

The observed run-off, D', was the observed run-off from Watershed A as taken by the Weather Bureau. On account of the small flow of these streams the run-off was measurable by a v-notch weir, and was computed to 0.001 cubic foot per second from the well-known formula $Q=CH^{5/2}$, in which H is the vertical height in feet of the still water in the pond above the vertex of the weir, Q is the flow in cubic feet per second and C had a mean value of about 2.6 and was determined for the particular weir.

The conversion factor for converting the discharge of Stream A in c.f.s. to units of 0.01 inch of depth on Watershed A per day is 10.7. The area of the watershed, from which this may be verified, is shown in Table 40.

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[1] See page 192.

TABLE 40—*Dimensions of watersheds A and B*

	A	B
Total area	222.5 acres	200.4 acres
Extreme length	7,300 feet	4,600 feet
Computed mean width	1,328 feet	1,898 feet
Absolute elevations	9,373–11,355 feet	9,245–10,952 feet

The observation equations (page 152) for Stream *A* as used in the least-square solution designated as Solution *M*, serve as a typical illustration. This is part of the observation equations for that solution which included in all 242 such equations covering the months February, August and September 1912 and June to September inclusive and December 1913—8 months in all.

The equations (page 152) are written in the form of equation (60). In each column, the unknowns (R), (R_1), (R_2), . . . (R_{10}) should be considered as repeated down the column. The method of computing $-D$ for each of these equations will be next illustrated, and thereafter an illustration will be given of the method of computation of r_1, r_2, r_3, . . . r_{10}.

The computation of $-D$, the absolute term in the above equations, was effected by use of equation (59) in which the following values of the unknowns were used:

$$\left.\begin{array}{lll} R'' = +61 & R''_4 = +0.17 & R''_8 = +0.87 \\ R''_1 = +\ 0.26 & R''_5 = +1.13 & R''_{9a} = +\ .314 \\ R''_2 = +\ \ .23 & R''_6 = +1.05 & R''_{9b} = +\ .058 \\ R''_3 = +\ \ .15 & R''_7 = +\ .83 & R''_{10} = +\ .58 \end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots (66)$$

These values were based partly upon previous least-square solutions and were partly estimated. In this sense the derivation of the *R*'s from Solution *M*—and by equation (61), S_e, R'_1, R'_2, . . . R'_{10}—may be considered as some multiple of the process stated in step (4), page 147. The table on page 153 is an illustration of the method of computing $-D$ for each of the above observation equations:

In the computation of $-D$, the second column and the quantities in parentheses at the beginning of subsequent columns are the R''s of (66) as shown in the headings of the columns. The other quantities, excepting the last two columns, are the *r*'s. The last column in the left-hand member of the equations is the observed discharge, D', expressed in units of 0.001 c.f.s. The right-hand member of the equations, or last column to the right, is $-D$, which values are copied into the observation equations already presented on page 152.

Referring to Plate 10, D' or the observed flow is shown as a solid line.

Table 41 is an illustration of the computation of r_1, from which r_2, r_3, . . . r_{10} follow, according to the definitions previously given.

In the example of computation of r_1, column (2) shows the mean rainfall as observed on Watershed *A* at the stations named on page 150 and shown on Plate 7; column (3) the mean temperature, *t*; column (4) the observed mean vapor-pressure; column (5) the saturation vapor pressure corresponding to *t*; column (6), the *e* of equation (65); column (7), 0.319 *e*, or that part of the evaporation from a water surface due to the saturation deficit alone; column (8), the mean wind movement over the watershed; column (9), the estimated evaporation from land by use of equation (65); column (10), the observed run-off in c.f.s., reduced to 0.01 inch of

Example of observation equations for normal stream-flow. Solution M, Stream A, June 1913

Date	Terms of equation												
	$+1(R)$	$r_1(R_1)$	$r_2(R_2)$	$r_3(R_3)$	$r_4(R_4)$	$r_5(R_5)$	$r_6(R_6)$	$r_7(R_7)$	$r_8(R_8)$	$r_{9a}(R_{9a})$	$r_{9b}(R_{9b})$	$r_{10}(R_{10})$	$-D=v$
1913													
June 1	$+1\ (R)$	$-16\ (R_1)$	$-18\ (R_2)$	$-18\ (R_3)$	$-26\ (R_4)$	$-3\ (R_5)$	$-12\ (R_6)$	$-22\ (R_7)$	$+65\ (R_8)$	$-3\ (R_9)$	$-10\ (R_{10})$	$-5\ (R_{10})$	$-113=v_1$
2	+1	−18	−16	−18	−36	− 2	−12	−22	+61	− 1	− 9	−5	$-118=v_2$
3	+1	−20	−18	−16	−36	− 3	−12	−22	+52	+ 6	− 9	−5	$-122=v_3$
4	+1	−16	−20	−18	−34	− 3	−12	−22	+43	+14	− 9	−5	$-127=v_4$
5	+1	−11	−16	−20	−34	− 6	− 9	−23	+35	+22	− 9	−5	$-128=v_5$
6	+1	−11	−11	−16	−38	− 7	− 9	−23	+31	+25	− 9	−5	$-127=v_6$
7	+1	+40	−11	−11	−36	− 7	− 9	−23	+29	+26	− 9	−5	$-129=v_7$
8	+1	+16	+40	−11	−27	− 7	−10	−23	+28	+26	− 9	−5	$-129=v_8$
9	+1	+29	+16	+40	−22	− 7	−10	−23	+22	+31	− 9	−4	$-117=v_9$
10	+1	+56	+29	+16	+29	− 6	− 9	−24	+13	+39	− 9	−4	$-145=v_{10}$
11	+1	− 1	+56	+29	+56	− 6	−10	−24	+ 9	+42	− 9	−4	$-155=v_{11}$
12	+1	+32	− 1	+56	+45	0	−10	−25	+ 7	+42	− 8	−4	$-155=v_{12}$
13	+1	−11	+32	− 1	+85	+ 3	−13	−21	+ 6	+42	− 8	−4	$-151=v_{13}$
14	+1	−16	−11	+32	+55	+ 7	−14	−20	+ 4	+42	− 8	−4	$-148=v_{14}$
15	+1	−18	−16	−11	+31	+14	−13	−20	+ 2	+42	− 7	−4	$-146=v_{15}$
16	+1	−15	−18	−16	+21	+10	− 7	−20	− 3	+46	− 7	−4	$-146=v_{16}$
17	+1	+ 7	−15	−18	−27	+12	− 4	−21	−10	+51	− 6	−4	$-152=v_{17}$
18	+1	+14	+ 7	−15	−34	+ 8	+ 1	−21	−17	+56	− 6	−4	$-152=v_{18}$
19	+1	−10	+14	+ 7	−33	0	+ 8	−21	−23	+62	− 7	−5	$-154=v_{19}$
20	+1	−14	−10	+14	− 8	− 1	+10	−22	−27	+65	− 7	−5	$-146=v_{20}$
21	+1	−11	−14	−10	+21	− 6	+15	−22	−29	+67	− 7	−5	$-141=v_{21}$
22	+1	− 9	−11	−14	+ 4	− 4	+15	−22	−32	+69	− 7	−5	$-136=v_{22}$
23	+1	+ 5	− 9	−11	−24	− 1	+14	−22	−35	+69	− 7	−5	$-135=v_{23}$
24	+1	−11	+ 5	− 9	−25	0	+ 9	−17	−36	+69	− 6	−5	$-132=v_{24}$
25	+1	−20	−11	+ 5	−20	0	+ 6	−13	−39	+70	− 6	−5	$-130=v_{25}$
26	+1	−25	−20	−11	− 4	− 2	+ 3	− 8	−40	+70	− 6	−5	$-128=v_{26}$
27	+1	−15	−25	−20	− 6	− 4	− 1	− 1	−41	+67	− 4	−5	$-127=v_{27}$
28	+1	+ 9	−15	−25	−31	− 3	− 2	0	−42	+66	− 4	−5	$-130=v_{28}$
29	+1	−11	+ 9	−15	−45	− 3	− 6	+ 2	−40	+66	− 4	−5	$-121=v_{29}$
30	+1	−19	−11	+ 9	−40	− 4	− 6	+ 2	−40	+66	− 3	−5	$-119=v_{30}$

Example of computation of $-D$. *Solution M, Stream A, June 1913*

Date	R''	$r_1R''_1$	$r_2R''_2$	$r_3R''_3$	$r_4R''_4$	$r_5R''_5$	$r_6R''_6$	$r_7R''_7$	$r_8R''_8$	$r_{9a}R''_{9a}$	$r_{9b}R''_{9b}$	$r_{10}R''_{10}$	$-D'$	$-D$
1913 June 1	+61	−16 (0.26)	−18 (0.23)	−18 (0.15)	−26 (0.17)	− 3 (1.13)	−12 (1.05)	−22 (0.83)	+65 (0.87)	− 3 (0.314)	−10 (0.058)	−5 (0.58)	−179	= −113
2	+61	−18	−16	−18	−36	− 2	−12	−22	+61	− 1	− 9	−5	−177	= −118
3	+61	−20	−18	−16	−36	− 3	−12	−22	+52	+ 6	− 9	−5	−175	= −122
4	+61	−16	−20	−18	−34	− 3	−12	−22	+43	+14	− 9	−5	−173	= −127
5	+61	−11	−16	−20	−34	− 6	− 9	−23	+35	+22	− 9	−5	−171	= −128
6	+61	−11	−11	−16	−38	− 7	− 9	−23	+31	+25	− 9	−5	−169	= −127
7	+61	+40	−11	−11	−36	− 7	− 9	−23	+29	+26	− 9	−5	−182	= −129
8	+61	+16	+40	−11	−27	− 7	−10	−23	+28	+26	− 9	−5	−187	= −129
9	+61	+29	+16	+40	−22	− 7	−10	−23	+22	+31	− 9	−4	−181	= −117
10	+61	+56	+29	+16	+29	− 6	− 9	−24	+13	+39	− 9	−4	−218	= −145
11	+61	− 1	+56	+29	+56	− 6	−10	−24	+ 9	+42	− 9	−4	−224	= −155
12	+61	+32	− 1	+56	+45	0	−10	−25	+ 7	+42	− 8	−4	−226	= −155
13	+61	−11	+32	− 1	+85	+ 3	−13	−21	+ 6	+42	− 8	−4	−218	= −151
14	+61	−16	−11	+32	+55	+ 7	−14	−20	+ 4	+42	− 8	−4	−206	= −148
15	+61	−18	−16	−11	+31	+14	−13	−20	+ 2	+42	− 7	−4	−199	= −146
16	+61	−15	−18	−16	+21	+10	− 7	−20	− 3	+46	− 7	−4	−197	= −146
17	+61	+ 7	−15	−18	−27	+12	− 4	−21	−10	+51	− 6	−4	−202	= −152
18	+61	+14	+ 7	−15	−34	+ 8	+ 1	−21	−17	+56	− 6	−4	−205	= −152
19	+61	−10	+14	+ 7	−33	0	+ 8	−21	−23	+62	− 7	−5	−197	= −154
20	+61	−14	−10	+14	− 8	− 1	+10	−22	−27	+65	− 7	−5	−187	= −146
21	+61	−11	−14	−10	+21	− 6	+15	−22	−29	+67	− 7	−5	−182	= −141
22	+61	− 9	−11	−14	+ 4	− 4	+15	−22	−32	+69	− 7	−5	−175	= −136
23	+61	+ 5	− 9	−11	−24	− 1	+14	−22	−35	+69	− 7	−5	−174	= −135
24	+61	−11	+ 5	− 9	−25	0	+ 9	−17	−36	+69	− 6	−5	−169	= −132
25	+61	−20	−11	+ 5	−20	0	+ 6	−13	−39	+70	− 6	−5	−161	= −130
26	+61	−25	−20	−11	− 4	− 2	+ 3	− 8	−40	+70	− 6	−5	−154	= −128
27	+61	−15	−25	−20	− 6	− 4	− 1	− 1	−41	+67	− 4	−5	−150	= −127
28	+61	+ 9	−15	−25	−31	− 3	− 2	0	−42	+66	− 4	−5	−157	= −130
29	+61	−11	+ 9	−15	−45	− 3	− 6	+ 2	−40	+66	− 4	−5	−147	= −121
30	+61	−19	−11	+ 9	−40	− 4	− 6	+ 2	−40	+66	− 3	−5	−140	= −119

depth per day on the watershed by multiplying by 10.7; column (11), the total loss from watershed, or the sum of columns (9) and (10); column (12), r_1, or column (2) *minus* column (11). Column (12) appears in "Example of Observation Equations for Normal Stream-Flow," and also in the "Example of Computation of $-D$," already given.

The vapor-pressures and e are expressed in 0.01 inch of mercury; the wind velocity in miles of movement per day; t in degrees F; and all the other quantities in 0.01 inch of depth.

TABLE 41—*Example of computation of* r_1

Date	Observed mean rainfall	Observed mean air temperature	Observed mean vapor pressure	Saturation vapor pressure	e	$0.319e$	Mean wind movement	Evaporation from land, E_l	Observed run-off	Total loss	r_1
1913											
June 1	0	51	17	37	+20	6	77	14	2	16	−16
2	0	52	18	39	+21	7	86	16	2	18	−18
3	0	52	15	39	+24	8	71	18	2	20	−20
4	0	50	16	36	+20	6	63	14	2	16	−16
5	0	46	18	31	+13	4	70	9	2	11	−11
6	0	46	18	31	+13	4	65	9	2	11	−11
7	51	44	16	29	+13	4	68	9	2	11	+40
8	23	41	20	26	+ 6	2	55	5	2	7	+16
9	36	43	22	28	+ 6	2	37	5	2	7	+29
10	60	41	24	26	+ 2	1	60	2	2	4	+56
11	8	40	15	25	+10	3	58	7	2	9	− 1
12	39	36	14	21	+ 7	2	69	5	2	7	+32
13	0	44	17	29	+12	4	46	9	2	11	−11
14	0	50	17	36	+19	6	66	14	2	16	−16
15	0	53	18	40	+22	7	60	16	2	18	−18
16	1	51	18	37	+19	6	61	14	2	16	−15
17	16	46	23	31	+ 8	3	59	7	2	9	+ 7
18	16	47	31	32	+ 1	0	42	0	2	2	+14
19	1	51	25	37	+12	4	53	9	2	11	−10
20	0	51	21	37	+16	5	63	12	2	14	−14
21	0	50	24	36	+12	4	63	9	2	11	−11
22	0	47	23	32	+ 9	3	50	7	2	9	− 9
23	14	46	22	31	+ 9	3	62	7	2	9	+ 5
24	0	49	23	35	+12	4	54	9	2	11	−11
25	0	53	15	40	+25	8	60	18	2	20	−20
26	0	56	13	45	+32	10	75	23	2	25	−25
27	8	55	14	43	+29	9	78	21	2	23	−15
28	16	51	30	37	+ 7	2	68	5	2	7	+ 9
29	0	50	24	36	+12	4	60	9	2	11	−11
30	0	52	15	39	+24	8	67	18	1	19	−19

From the values of r_1 shown in column (12), and copied in as coefficients of R_1 in the sample of observation equations for normal stream-flow for June 1913, it is possible to verify the computations of certain of the other r's there shown according to the definitions previously given. As an illustration, take the observation equation for June 30, 1913. The computation of the r's is as follows:

$r_1 = -19$, computed as shown above.

$r_2 = r_1$ for June 29, $= -11$.

$r_3 = r_1$ for June 28, $= r_2$ for June 29 $= +9$.

$r_4 = \Sigma$ (r_1, for June 26 and 27) $= -25 - 15 = -40$.
$= \Sigma$ (r_3 for June 28 and 29).

$r_5 = \Sigma$ (r_1 for June 22 to 25 inclusive) $\div 10 = -4$ to the nearest unit.
$= \Sigma$ (r_4 on June 26 and 28) $\div 10$.

$r_6 = \Sigma$ (r_1 for June 14 to 21 inclusive) $\div 10 = -63 \div 10 = -6$ to nearest unit.
$= \Sigma$ (r_5 on June 22 and 26).

The remaining r's, r_7, r_8, r_9 and r_{10}, reach back in their derivation to dates preceding June 1913 and therefore their computation can not be illustrated. Note the division by 10 after r_4. A similar division by 100 was made after r_{9b}, the object in both cases being to make all the coefficients in the observation equations of approximately the same magnitude to facilitate general checking. These divisions also lessen the labor in forming the normal equations, and tend to give greater accuracy to the derived constants.

Referring to Plate 10, r_1 is shown plotted as a dotted line above or below the horizontal line of constant flow, S_c. In the manner indicated, the change in storage on the current day was computed for Stream *A* for the whole period 1911–1915. This is shown for 1912–1915 on Plates 9 to 12 inclusive. For the change in storage on Watershed *B* (see Plates 13 to 17 inclusive) the method of computation was the same except that different (better) values of *C*, *F* and *M* were used, and a different rule for fixing the last date of full net melting in the spring was adopted. These two changes, resulting in better results on Stream *B*, will be discussed more fully later.[1]

Attention is called to the fact that the wind movement (the w of equation (65)), in the example of computation of r_1, is so small that the expression in parenthesis in equation (65) was never positive on the dates given, hence the estimated evaporation from water is merely $0.319\ e$, and from land, $(0.319\ e)\ (2.3)$. In fact, in the whole five-year period on both watersheds *A* and *B*, w was rarely large enough to enter into the evaporation equation as derived from observations on the Great Lakes. This is evidently due to the difference in elevation of the anemometers above the ground (or lake) surfaces in the two cases. On the Great Lakes, as already stated, the average elevation of the anemometers was about 100 feet above ground surface, whereas on the Wagon Wheel Gap Watersheds they were about 5 feet above the ground surface. It is believed that this condition gave rise to too small an estimated evaporation from water at the Wagon Wheel Gap area, and accounts for the large value of $\frac{E_l}{E_w}$ of 2.3, in comparison with the 0.62 derived and used on the Great Lakes.

It should be noted further that in the example given of the computation of r_1, the net melting did not enter, because there was no snow or ice on the watershed available to be melted. All of the addition to storage in June 1913 occurred as rain. It is evident, however, from the definitions of the other r's, that net melting did enter into the computation of some of them. For example, some of the various r's in the equation for June 1, 1913, extended back, as follows:

On June 1, 1913, $r_{10} = \Sigma$ (r_1 for September 17, 1912, to January 22, 1913, inclusive);
. . . .
$r_8 = \Sigma$ (r_1 for March 29, 1913, to April 29, 1913, inclusive);
$r_7 = \Sigma$ (r_1 for April 30, 1913, to May 15, 1913, inclusive).

Now, the first date in the fall of 1912 on which snow fell was about October 30, and the last date in the spring of 1913 on which there was snow on the ground was about May 8. It is apparent, then, that every r for June 1, 1913, which reached back beyond May 8 was made up in part of net melting, which was, for Solution M, computed by equation (64). In order to apply equation (64) to any winter season to compute the net melting, it is necessary to know the limits of the freezing-melting period, and the law of decreased rate of melting.

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[1] See pages 157, 192 and 198.

FREEZING-MELTING PERIOD

When the temperature, t, of the air over a watershed begins to lower with the approach of the winter season, there is a tendency for some of the stored water (in the lakes and ponds, if any, and streams and ground) on the watershed to freeze. When it reaches the temperature T', there is no net change in storage on a day, due to freezing and melting, but as it goes below T', some of the water is held back from storage by freezing. As the spring season approaches, the temperature begins to rise and when it becomes greater than T' the storage in the watershed is augmented by the melting of the stored snow and ice on and in the ground. The "freezing-melting period," as used in this investigation, is defined as having its beginning when the temperature t first becomes and tends to remain continuously smaller than T' in the fall, and its ending when the last trace of snow has disappeared from the watershed. During the freezing-melting period, t may, and often does, rise above T' for short or long intervals. Throughout the freezing-melting period, with the exception to be stated in the following paragraph, that part of the change in storage due to melting or freezing is computed from the general expression, equation (36). In Solution M, the specific expression, equation (64), was used, as already mentioned. It is understood, from the definition of the r's, that if rain should fall during the freezing-melting period, it is added to the computed net melting to get the total addition to the storage for the day.

DECREASE IN RATE OF MELTING

The exception mentioned in the third preceding sentence is now to be stated. In the spring, toward the end of the freezing-melting period, the ground area, which might have been completely covered with snow several feet deep during the winter, becomes bare of it, rather rapidly at first, then more slowly, since the deep drifts offer more resistance to the melting influences than the lighter snows which cover most of the surface at a relatively uniform depth. After a while nothing remains but the scattered relics of the deep drifts, which yield smaller and smaller amounts of water to the underground storage by melting mainly around their fringes. From the definitions given after equation (36), in assumption No. (12), and elsewhere, and from the nature of the derivation of C, F and M as given later, the net melting can not be computed by equation (36) clear to the end of the freezing-melting period. After a certain part of the ground surface becomes bare, the decrease in rate of melting has been found to be approximated by the following expression:

$$\text{Decreased rate net melting of } = \left[+C + \left\{ \begin{matrix} -(T''-t)F \\ \text{or} \\ +(t-T'')M \end{matrix} \right\} \right] \frac{A-A_1}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

In this expression A is the total water equivalent of the snow on the ground at the beginning of the period during which melting is taking place at a decreased rate, and A_1 is the total computed amount of melting which has taken place, commencing the day before the beginning day of the period and up to, but not including, the current day. From the nature of the factor $\frac{A-A_1}{A}$ it will be seen that it decreases rapidly below unity at first, then more and more slowly, which corresponds with the rapid falling off at first in the yield of water from melting, and the gradual decrease in that yield as the snow disappears.

FIXING BEGINNING AND END OF THE PERIOD OF DECREASE IN RATE OF MELTING

It is highly important and relatively easy to fix the date on which the net melting begins to decrease in rate. It is relatively difficult, but fortunately unimportant, to fix the last date on which any addition to storage results from melting. On the Wagon Wheel Gap areas the date of the beginning of the period of decreased rate of melting could be estimated in four ways, as follows:

(*a*) by a study of the diurnal fluctuations in the stream-flow in conjunction with the air temperature;
(*b*) by a study of the record of the disappearance of the snow at the snow gages;
(*c*) by a preliminary substitution in observation equations;
(*d*) from the derived flood-flow formula.

The mean of one or more of the dates fixed by these methods was taken as the most probable date of beginning of disappearance of snow on Stream A. On Stream B the last-named method only was used. These methods will be briefly discussed in turn.

The observations of stream discharge were made hourly. Each discharge used in the observation equations was the mean of 24 hourly discharges. The stream-flow in the freezing-melting period during the daylight hours is greater than during the night hours, corresponding to fluctuations in the temperature as recorded on the thermographs. As the freezing-melting period approached its end, the diurnal fluctuations in stream-flow increase in amplitude, corresponding to a more or less parallel increase in amplitude in the temperature curve fluctuations. At the time when the melting begins to decrease in rate, the rough parallelism between the two curves disappears; that is, the temperature curve continues its diurnal fluctuations, but the fluctuations in the stream-flow curve begin to decrease in amplitude. At some point, the stream-flow continues to decrease both in the amplitude of its diurnal fluctuations and in absolute amount. This was normally taken as the beginning of the period of decreased rate of melting. This method was not always applicable. For instance, in 1913 on Stream A, it was impossible to estimate the beginning of the period of decreased rate of melting by examination of the diurnal fluctuations in stream-flow and temperature, as outlined above.

On the Wagon Wheel Gap areas, snow gagings were taken normally every three days. The record of mean water equivalent of the snow on the ground, and the number of gages at which snow appeared was available at the intervals stated. It was fairly easy from a study of such a record to decide when the melting began to diminish in rate.

The method of estimating the date of beginning of decrease in rate of melting by a preliminary substitution in observation equations is as follows: With approximate values of S_c, R'_1, R'_2, . . . R'_n, $\frac{E_l}{E_w}$ and C, F, M and T''' from previous solutions, the normal stream-flow is computed from equation (33), involving equations

(34) and (36). This computation should cover a period of about a month preceding the approximate date when the decrease in melting is assumed to take place, and extend past that date a week or more. The difference between the flow as computed and the observed flow gives a set of residuals which are mostly negative, since the observed flow during such a period is generally made up of surface as well as sub-surface flow, whereas equation (33) accounts only for the sub-surface flow. The date on which the first differences of the residuals changed sign, from a large negative to a large positive, is taken as the date of the beginning of the period of decreased rate of melting.

To the extent that assumption No. 10, page 140, is true, it is a simple matter to fix the beginning date of the period of decreased rate of melting from the flood flow formula, equation (37). From equation (37), the lag between a change in storage above ground surface and its maximum effect upon the stream-flow can be determined from the derived R'_f's. This lag amounts to three days, as a rule, as will be seen later from the numerical values of the R'_f's to be given. Therefore, by this method, three days preceding the date on which maximum discharge took place is the date on which the maximum amount of water was stored above the ground surface by melting, hence that is assumed to be the last date on which the melting took place at its maximum rate.

Having fixed the beginning of the period of decreased rate of melting and the assumed law of decrease in rate of melting, equation (67), the end of the freezing-melting period was taken as the date on which the melting as computed from equation (67) was for the last time in excess of 0.005 inch.

EXAMPLE OF ESTIMATE OF BEGINNING OF PERIOD OF DECREASED RATE OF MELTING

On Stream *A* in the spring of 1913 it was not possible, as already stated, to fix the beginning date of the period of decreased rate of melting by the first method stated above. The condition at the 19 snow gages on Watershed *A* was as follows on the various dates:

Date	No. of gages at which there was some snow	Water equivalent of the snow	
		Maximum at any gage	Mean
1913		*inches*	*inches*
Mar. 27	19	7.68	4.70
30	15	7.07	4.24
Apr. 2	15	7.13	4.05
5	15	8.40	4.36
8	17	8.74	4.41
11	15	9.20	4.59
14	13	9.07	3.97
17	12	8.70	3.13
20	10	7.66	2.49
23	9	6.61	1.83
26	7	6.48	1.71
29	5	5.04	1.08
May 2	4	2.52	0.44
5	1	0.97	0.02

From the above record, it was estimated that April 11 was the day of the beginning of the decrease of rate of melting, at which time the mean water equivalent of the snow on the ground—the *A* of equation (67)—was 459, in units of 0.01 inch.

At the time Solution *M* was made on Stream *A*, the flood-discharge formula, equation (37), had not yet been derived, hence it was not possible to fix the date of beginning the decrease of rate of melting by method (d), page 157, and, as it had been found in other cases that method (c) gave practically the same results as the other three, it was not used except in doubtful cases because of the labor involved. Hence on Watershed *A* for 1913, the date fixed by method (b) at April 11 was taken as the date of beginning of the period of decreased rate of melting.

TABLE 42—*Example of computation of net melting and of decreased rate of melting*

A numerical example from the computations of Solution *M* is presented illustrating the use of equations (64) and (67).

Date	$(t-28°)$	$(t-28°)\ 0.453$ or $(t-28°)\ 8.39$	C	Net melting	Reduced net melting	A_1	$A-A_1$	$\frac{A-A_1}{A}$
1913								
Apr. 1	+8	+67	+6	+73				
2	+4	+34	+6	+40				
3	−6	−3	+7	+4				
4	−2	−1	+6	+5				
5	+6	+50	+7	+57				
6	+10	+84	+6	+90				
7	+3	+25	+6	+31				
8	−3	−1	+7	+6				
9	−8	−4	+6	+2				
10	−7	−3	+7	+4				
11	+1	+8	+6	+14	+14	+14	+445	0.969
12	+5	+42	+6	+48	+47	+61	+398	.867
13	+9	+76	+7	+83	+72	+133	+326	.710
14	+10	+84	+6	+90	+64	+197	+262	.571
15	+12	+101	+7	+108	+62	+259	+200	.436
16	+12	+101	+6	+107	+47	+306	+153	.333
17	+10	+84	+6	+90	+30	+336	+123	.268
18	+10	+84	+7	+91	+24	+360	+99	.216
19	+9	+76	+6	+82	+18	+378	+81	.176
20	+9	+76	+7	+83	+15	+393	+66	.144
21	+11	+92	+6	+98	+14	+407	+52	.113
22	+7	+59	+6	+65	+7	+414	+45	.0980
23	−9	−4	+7	+3	0	+414	+45	.0980
24	−7	−3	+6	+3	0	+414	+45	.0980
25	+3	+25	+7	+32	+3	+417	+42	.0915
26	+9	+76	+6	+82	+8	+425	+34	.0741
27	+13	+109	+6	+115	+9	+434	+25	.0545
28	+13	+109	+7	+116	+6	+440	+19	.0414
29	+13	+109	+6	+115	+5	+445	+14	.0305
30	+15	+126	+7	+133	+4	+449	+10	.0218
May 1	+13	+109	+6	+115	+3	+452	+7	.0153
2	+5	+42	+6	+48	+1	+453	+6	.0131
3	+2	+17	+7	+24	0	+453	+6	.0131
4	+7	+59	+6	+65	+1	+454	+5	.0109
5	+14	+117	+7	+124	+1	+455	+4	.00871
6	+15	+126	+6	+132	+1	+456	+3	.00654
7	+15	+126	+6	+132	+1	+457	+2	.00436
8	+11	+92	+7	+99	0	+457	+2	.00436
9	+16	+134	+6	+140	+1	+458	+1	.00218
10	+17	+143	+7	+150	0	+458	+1	.00218
11	+18	+151	+6	+157	0	+458	+1	.00218

In Table 42, in column (2) are listed the differences between the mean air temperatures, *t*, over the watershed as computed from observations at the stations stated on page 150, and 28°. In column (3) are the products of the values in column (2) when negative by 0.453, and when positive, by 8.39, rounded off to the nearest even unit. In column (4) is written the value of the constant, C, of equation (64). In column (5) are written the sums of the values in columns (3) and (4), or the total net melting for the day as computed from equation (64), on the assumption that there is an abundant amount of snow and ice available for melting. The net melting was computed in this manner from the beginning of the freezing-melting period the preceding fall, hence many of the late r's in June 1913 in the example of observation equations already given, were made up in part by the net melting here referred to. In column (6) beginning April 11 is written the reduced net melting computed from equation (67), as follows: From the definitions of A and A_1 given after equation (67) it is evident that on April 10, the value of $\frac{A-A_1}{A}$ is unity. Hence the reduced net melting for April 11 is the same as the net melting. At the end of April 11, $A_1=14$, in the same units as A and as given by equation (64). Hence, $A-A_1=459-14=445$, making $(A-A_1)/A=0.969$, which, multiplied into the net melting for April 12, 48, gives the reduced net melting, 47, for April 12. This added to the reduced net melting of April 11, 14, gives $A_1=61$ for April 12, and so on. The computation is carried forward in this manner until the reduced net melting is for the last time in excess of 0.005 inch, or unity, in the units used. This is taken as the end of the period of reduced net melting, or the end of the freezing-melting period. It is to be noted that the reduced net melting may go below 0.005 inch several times before it finally goes and remains below that amount. The last date on which melting added anything to the ground-water storage, according to the computation, was May 9, 1913. On May 5, according to the snow-gage record there was a mean water equivalent of 0.02 inch of snow on the ground, and other observed evidence showed that the melting probably became zero on May 9.

The full net melting and the reduced net melting of Table 42 are shown plotted in the upper left-hand corner of Plate 22.

EXAMPLE OF COMPUTATION OF $\frac{E_l}{E_w}$, STREAM B

This example will be taken from the computations on Stream B. The values of C, F, M and T'' used were those shown in Table 49, page 192, as having been derived in Solution X. The daily values of all the quantities involved were computed (or observed), and these daily values combined into monthly totals. The total of all the values in 1911–12–13 was used in deriving $\frac{E_l}{E_w}$

The unit used (page 161) is 0.01 inch. Substituting the totals in equation (35a), page 137, there is obtained

$$\frac{E_l}{E_w}=\frac{3900+5646-1889}{2914}=2.6$$

Note that precipitation in the form of rain ordinarily fell in the months May to October, and in the frozen form from October to May. In seven of the 36 months

Date	Observed rainfall	Computed net melting	Computed E_v	Observed run-off
1911				
Jan.	0	+ 194	42	31
Feb.	0	− 9	23	28
Mar.	0	+ 448	63	29
Apr.	0	+1009	73	47
May	71	+1110	151	161
June	214	0	165	49
July	437	0	79	33
Aug.	311	0	129	31
Sept.	241	0	90	29
Oct.	410	+ 11	81	99
Nov.	0	+ 89	45	43
Dec.	0	− 74	35	36
1912				
Jan.	0	− 15	42	34
Feb.	0	+ 7	44	31
Mar.	0	+ 93	32	34
Apr.	0	+ 467	63	50
May	0	+ 927	149	294
June	203	0	126	96
July	403	0	123	50
Aug.	158	0	131	39
Sept.	37	0	128	34
Oct.	171	+ 6	75	38
Nov.	0	+ 76	69	35
Dec.	0	− 70	34	34
1913				
Jan.	0	− 65	33	33
Feb.	0	− 18	33	29
Mar.	0	+ 154	49	32
Apr.	0	+1087	85	57
May	39	+ 232	147	102
June	300	0	109	69
July	221	0	148	38
Aug.	224	0	113	27
Sept.	261	0	68	28
Oct.	91	0	73	31
Nov.	108	+ 68	41	29
Dec.	0	− 81	23	29
Totals 1911–1913..	3900	+5646	2914	1889

shown, the total computed net melting was negative; for instance, −0.09 inch in February 1911.

EXAMPLES OF NORMAL EQUATIONS FOR NORMAL STREAM-FLOW

There have been presented up to this point general definitions and specific illustrations of the meanings of all of the quantities which enter into the observation equations for determining the normal stream-flow.

In each least-square solution for determining the normal stream-flow equation, a set of normal equations was first formed in the usual way from the observation equations for each day. Then these sets of normal equations, one set for each month, were combined by addition to form the final set of normal equations for the whole solution.

The normal equations for June 1913, in Solution M, formed from the observation equations for that month, shown on page 152, are as follows:

Normal equations for June, 1913, Solution M, Stream A

$$\left.\begin{array}{l}
+30R-90R_1-89R_2-96R_3-239R_4-31R_5-109R_6-543R_7-47R_8+1354R_{9a}-218R_{9b}-140R_{10}-4059=0\\
-90R+11964R_1+5351R_2+4678R_3+2161R_4-638R_5-461R_6+191R_7+2821R_8-5236R_{9a}+383R_{9b}+527R_{10}+10585=0\\
-89R+5351R_1+11927R_2+5466R_3+7959R_4-612R_5-529R_6+464R_7+1190R_8-3967R_{9a}+388R_{9b}+524R_{10}+9400=0\\
-96R+4678R_1+5466R_2+12130R_3+11041R_4-102R_5-440R_6+587R_7-110R_8-3386R_{9a}+497R_{9b}+392R_{10}+10181=0\\
-239R+2161R_1+7959R_2+11041R_3+36921R_4+1913R_5-660R_6+2106R_7-1233R_8-8358R_{9a}+1406R_{9b}+1434R_{10}+24014=0\\
-31R-638R_1-612R_2-102R_3+1913R_4+1005R_5+9R_6+479R_7-912R_8-775R_{9a}+296R_{9b}+190R_{10}+3035=0\\
-109R-461R_1-529R_2-440R_3-660R_4+9R_5+2913R_6+2225R_7-6332R_8-348R_{9a}+1004R_{9b}+456R_{10}+14230=0\\
-543R+191R_1+464R_2+587R_3+2106R_4+479R_5+2225R_6+11789R_7-4010R_8-21753R_{9a}+4322R_{9b}+2496R_{10}+74595=0\\
-47R+2821R_1+1190R_2-110R_3-1233R_4-912R_5-6332R_6-4010R_7+32957R_8-24110R_{9a}-1200R_{9b}+268R_{10}+9554=0\\
+1354R-5236R_1-3967R_2-3386R_3-8358R_4-775R_5-348R_6-21753R_7-24110R_8+76056R_{9a}-8878R_{9b}-6337R_{10}-186005=0\\
-218R+383R_1+388R_2+497R_3+1406R_4+296R_5+1004R_6+4322R_7-1200R_8-8878R_{9a}+1682R_{9b}+1013R_{10}+29496=0\\
-140R+527R_1+524R_2+592R_3+1434R_4+190R_5+456R_6+2496R_7+268R_8-6337R_{9a}+1013R_{9b}+660R_{10}+18828=0
\end{array}\right\}\cdots\cdots\cdots\cdots(68)$$

The final normal equations for Solution M for eight months, formed by combining eight such sets of normal equations as shown in (68), are as follows:

Final normal equations for Solution M, Stream A

$$\left.\begin{array}{l}
-242R-1202R_1-1219R_2-1221R_3-2326R_4-433R_5-802R_6-1644R_7-3258R_8-1604R_{9a}+2160R_{9b}-71R_{10}-17656=0\\
+1202R+52242R_1+29802R_2+18073R_3+15358R_4+241R_5+717R_6+7899R_7+18964R_8+1353R_{9a}-13531R_{9b}-493R_{10}+89343=0\\
-1219R+29802R_1+52535R_2+29999R_3+29277R_4+543R_5+559R_6+7679R_7+17722R_8+1539R_{9a}-14215R_{9b}-244R_{10}+92353=0\\
-1221R+18073R_1+29999R_2+52597R_3+47302R_4+1781R_5+467R_6+7381R_7+16048R_8+2311R_{9a}-15325R_{9b}+10R_{10}+93970=0\\
-2326R+15358R_1+29277R_2+47302R_3+168880R_4+10381R_5+1888R_6+12113R_7+26822R_8+13546R_{9a}-28248R_{9b}+115R_{10}+180731=0\\
-433R+241R_1+543R_2+1781R_3+10381R_4+5125R_5+1531R_6+1826R_7+5185R_8+4237R_{9a}-4157R_{9b}-97R_{10}+33166=0\\
-802R+717R_1+559R_2+467R_3+1888R_4+1531R_5+13658R_6+2979R_7+5858R_8+11057R_{9a}-2951R_{9b}-297R_{10}+66047=0\\
-1644R+7899R_1+7679R_2+7381R_3+12113R_4+1826R_5+2979R_6+35922R_7+6902R_8-3858R_{9a}-1374R_{9b}+1220R_{10}+148394=0\\
-3258R+18964R_1+17722R_2+16048R_3+26822R_4+5185R_5+5858R_6+6902R_7+110088R_8-5577R_{9a}-36648R_{9b}+3318R_{10}+233818=0\\
-1604R+1353R_1+1539R_2+2311R_3+13546R_4+4237R_5+11057R_6-3858R_7-5577R_8+235012R_{9a}-112498R_{9b}-12150R_{10}-11578=0\\
+2160R-13531R_1-14215R_2-15325R_3-28248R_4-4157R_5-2951R_6-1374R_7-36648R_8-112498R_{9a}+437422R_{9b}-19503R_{10}-223218=0\\
-71R-493R_1-244R_2+10R_3+115R_4-97R_5-297R_6+1220R_7+3318R_8-12150R_{9a}-19503R_{9b}+5749R_{10}+15554=0
\end{array}\right\}\cdots\cdots\cdots\cdots(69)$$

This final set of normal equations for Solution M, (69) depends upon 242 observation equations for the eight months, February, August and September 1912, and June to September, inclusive, and December 1913.

The solution of the final normal equations for Solution M, (69) gives the following values for the unknowns:

$$\left.\begin{array}{lll} R\ =+66 & R_4=-0.091 & R_8\ =+0.054 \\ R_1=+\ 0.023 & R_5=-\ \ .630 & R_{9a}=+\ \ .832 \\ R_2=-\ \ \ .020 & R_6=-\ \ .950 & R_{9b}=+\ \ .453 \\ R_3=-\ \ \ .013 & R_7=-\ \ .430 & R_{10}=+1.586 \end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (70)$$

Combining (70) with (66) according to (61), there is obtained for the values of the unknowns:

$$\left.\begin{array}{lll} S_c\ =+127\ \ \ \ \ (\pm 4.0) & R'_4=+0.079(\pm 0.035) & R'_8\ =+0.92\ (\pm 0.03) \\ R'_1=+\ \ 0.28\ (\pm\ \ .05) & R'_5=+\ \ .50\ \ (\pm\ \ .17) & R'_{9a}=+1.15\ (\pm\ \ .15) \\ R'_2=+\ \ \ \ .21\ (\pm\ \ .06) & R'_6=+\ \ .10\ \ (\pm\ \ .11) & R'_{9b}=+0.51\ (\pm\ \ .15) \\ R'_3=+\ \ \ \ .14\ (\pm\ \ .06) & R'_7=+\ \ .40\ \ (\pm\ \ .09) & R'_{10}=+2.17\ (\pm\ \ .21) \end{array}\right\} \cdots \quad (71)$$

The probable errors shown in parentheses were not computed rigorously from the normal equations, (69), and from the residuals, v, of the 242 observation equations of Solution M, but were estimated on the basis of probable errors computed rigorously from normal equations like (69) in Solution L and the residuals of 183 observation equations.

In interpreting the above constants, (71), the units in which the r's were expressed should be borne in mind. There was a shift in decimal place to the left between r_4 and r_5 and again between r_{9b} and r_{10}, hence, to get the above constants into their true relationship with reference to each other, R'_5 to R'_{9b} inclusive should be divided by 10 and R'_{10} by 100. This will become clearer from the following paragraphs.

METHOD OF CONVERTING CONSTANTS OF STREAM-FLOW FORMULAS TO PERCENTAGES

From such constants, (71), it is possible to determine what percentage of the change in storage each day reaches the stream on a current or subsequent day and, from this, what percentage of the total change in storage on a day reaches the stream throughout the period covered by (71). For example, the derived fraction, R'_1, of the change in storage, r_1, reaches the point of measurement in the stream on the current day. Since the product, $R'_1\ r_1$, is in the same units as D, viz, 0.001 c.f.s., $(R'_1r_1)/1000$ c.f.s. is that part of the stream-flow above (or below) S_c the current day which is due to the change in storage, r_1, on the current day. To convert discharge on Stream A in c.f.s. to units of 0.01 inch of depth per day on the watershed one must multiply by 10.7, hence $\frac{R'_1r_1}{1000}\times 10.7$ of the change in storage the current day reaches the stream that day, in 0.01 inch of depth. Therefore $\frac{R'_1r_1}{1000}\times 10.7\times \frac{100}{r_1}$ or 1.07 R'_1 is the percentage of the total change in storage the current day, r_1, which is delivered on that day.

Now, assume that on a specific day there is rain or net melting sufficient to produce a gain in storage, and that thereafter for 256 days there is no change of storage on any day, the rainfall or net melting on each day being just sufficient to equal the evaporation plus stream flow. Then, according to the theory developed by this investigation, the influence of the specific day in increasing the flow during later days would be that indicated by the terms $r_1R'_1$, $r_2R'_2$, $r_3R'_3$, . . . $r_{10}R'_{10}$; the $+r_1$ (gain in storage) of the specific day becoming $+r_2$ the following day, $+r_3$ the next day following, and so on, all other r's on these days being zero, according to the assumption. From the preceding paragraph, 1.07 R'_1 per cent of the change in storage on the specific day will reach the stream on that day. Similarly, 1.07 R'_2 per cent of the change in storage on the specific day will reach the stream on the first day following (counting the specific day the 0th day); 1.07 R'_3 per cent will reach the stream on the second following day; 1.07 R'_4 per cent on *each* of the third and fourth following days; 0.107 R'_5 per cent on each of the fifth to eighth following days inclusive; 0.107 R'_6 per cent on each of the ninth to sixteenth following days inclusive; 0.107 R'_7 per cent on each of the seventeenth to thirty-second following days inclusive; 0.107 R'_8 per cent on each of the thirty-third to sixty-fourth following days inclusive; 0.107 R'_{9a} per cent on each of the sixty-fifth to ninety-sixth following days inclusive; 0.107 R'_{9b} per cent on each of the ninety-seventh to one hundred and twenty-eighth following days inclusive; and, 0.0107 R'_{10} per cent on each of the one hundred and twenty-ninth to two hundred and fifty-sixth following days inclusive. The division of the constant 1.07 by 10 after R'_4 and again after R'_{9b} is to account for the change in units in the computation of the r's.

Applying the above principles to (71), the results may be arranged as in the following table.

TABLE 43—*Conversion of normal-flow constants to percentages, Stream A*

Constant	Absolute value of constant from (71)	Factor to convert col. 2 to percentage of change in storage which is delivered on one day	Per cent of change in storage which reaches stream on one day [1] Col. 2×Col. 3	Number of days involved in each constant	Total per cent of change in storage on specific day which reaches stream in each time interval [1] Col. 4×Col. 5
R'_1	+0.28	1.07	0.30 (±0.05)	1	0.30
R'_2	+0.21	1.07	0.22 (±0.06)	1	0.22
R'_3	+0.14	1.07	0.15 (±0.06)	1	0.15
R'_4	+0.079	1.07	0.085(±0.037)	2	0.170
R'_5	+0.50	0.107	0.054(±0.018)	4	0.216
R'_6	+0.10	0.107	0.011(±0.012)	8	0.088
R'_7	+0.40	0.107	0.043(±0.010)	16	0.688
R'_8	+0.92	0.107	0.098(±0.003)	32	3.136
R'_{9a}	+1.15	0.107	0.123(±0.016)	32	3.936
R'_{9b}	+0.51	0.107	0.055(±0.016)	32	1.760
R'_{10}	+2.17	0.0107	0.023(±0.0022)	128	2.944
			Totals	257	13.608

In Table 43, column 4 shows the percentage of the change in storage on the specific day which reaches the stream *by underground travel* on each day of each time interval following that day which is involved in the definitions of the r's, assuming that no change in storage takes place after the specific, or 0th, day. Thus 0.30 per cent of the change in storage the current day reaches the stream on that day, or 0th day; 0.22 per cent reaches it the first following day; . . . . ; 0.023 per cent of it reaches the stream on each of the one hundred and twenty-ninth to two hundred and fifty-sixth days, inclusive, after the change occurred. The probable errors of these quantities are those of equation (71) reduced to percentages. The product of each quantity in column 4 by the time interval involved in the computation of the r's (shown in column 5) gives the total percentage of the change in storage on the current day which is delivered to the stream during the time interval in question. Thus, during the ninth to the sixteenth days, inclusive, after the change in storage occurs, a total of 0.088 per cent of it is delivered to the stream (shown in line 6, column 6). Evidently, totaling all the quantities in column 6 gives the total percentage of the change in storage, on the current or specific day, which is delivered to Stream *A* during a period of 257 days thereafter (including the current day). This is shown to be 13.608 per cent. In other words, according to Solution *M*, only 13.6 per cent of any change in storage in the drainage area of Stream *A* on a day is delivered to the stream after a lapse of 8.4 months by travel through the ground to the stream, if, after the current or 0th day, the storage is assumed constant; that is, if it is assumed that after the 0th day the evaporation and run-off are just equal to the rainfall plus net-melting on each day of the 8.4-month period. If the change in storage is great enough to cause surface travel also, it will be shown subsequently that somewhat more of it than the above will be delivered in 32 days, by surface travel alone.[1]

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[1] Assuming no change in storage takes place after the specific or current day.

SMOOTHING THE R''s

In column 4 of the preceding tabulation, for any time interval involving more than one day, the percentage of the change in storage on a day which reaches the stream subsequently is written as if it were a constant amount throughout the time interval involved, as it necessarily must from the nature of the definitions of the r's and the manner of derivation of the R''s. As a matter of fact this is probably not the case. Presumably there must be a gradual rise in the R''s expressed as percentages, from the beginning of the current day until a peak is reached, after which there must be a gradual decline. The values in column 4 are represented as having a peak on the current day, a subsequent decline to the end of the 16th day, a rise from then to the end of the 96th day, followed by a final decline to the end of the 257th day. These fluctuations are probably not real. The increase in storage on a specific day must certainly decrease continuously along a smooth curve after the maximum rate of delivery is reached, which for this case was on the current day. The fluctuations in column 4 are probably due, in the main, to errors in the computed net melting computed from equation (64). They may be due, also, in part, to approximations in the assumptions and to using too small a number of observations in their derivation. Whatever the causes, it was assumed in this investigation that the R''s expressed as percentages (column 4) should decrease along a smooth curve. Accordingly, column 4 and similar values for Stream *B* were smoothed off by taking successive means of three. The numerical work of that smoothing-off process for Stream *A* is shown in Table 44.

In Table 44, column 2 is copied from column 4, Table 43. It was assumed that R'_1, R'_2 and R'_3 should remain unchanged as they are apparently fairly well determined and fall on a smooth curve. It was assumed at this point also that R'_{10} should remain unchanged as it appeared to fall in a probable position, and that R'_{11},[2] the next value after R'_{10}, should be zero. With these assumptions, the smoothing-out process is shown in columns 3 to 9 inclusive. Thus in column 3, line 4, the smoothed-off value of R'_4, 0.096, is the mean of the three values 0.15, 0.085 and 0.054 opposite it in column 2. The final smoothed-off curve appears in column 9, and column 10 shows the R''s of column 9 converted back to their regular form as in (71) by dividing by 1.07, 0.107 and 0.0107. The smoothed-off R''s at this point give 10.9 per cent as the total part of the change in storage the current day which is delivered to the stream in 257 days, assuming constant storage after the current day, as compared with 13.6 per cent for the unsmoothed values.

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[1] See page 178.

[2] R'_{11} was not evaluated in this investigation.

The values in column 9 were taken as the best values of the R''s expressed as percentages which were obtainable from Solution *M*. These values (except R'_{10}) are shown plotted on Plate 8. On this plate, the abscissas represent time in days and the ordinates the percentage of the change in storage, r_1, on the current or 0th day, which reaches the stream on each day thereafter, assuming that no change in storage occurs after the current day. The horizontal scale is shown divided into time intervals corresponding to the various R''s, with the R''s indicated in the centers of their respective time intervals. The plotted points are shown connected by straight lines, although the actual facts presumably follow a smooth curve. Note that in plotting R'_{9a} and R'_{9b}, they were grouped together as R'_9 with the average value of 0.048 per cent. The small difference between R'_{9a} and R'_{9b} apparently does not justify their separate determination. In other words, the run-off curve at this point is probably so horizontal that little gain in accuracy is obtained by separating R'_9 into two parts.

TABLE 44—*Smoothing-off R''s of Solution M, Stream A*

Constant	R''s from Solution *M*	First smoothing by means of 3	Second smoothing	Third smoothing	Fourth smoothing	Fifth smoothing	Sixth smoothing	Seventh smoothing	Smoothed R''s
	p. ct.								
R'_1........	0.30	....	....	....	....	....	....	0.03	..+0.28
R'_2........	.22	....	....	....	....	....	....	.22	+ .21
R'_3........	.15	....	....	....	....	....	....	.15	+ .14
R'_4........	.085	0.096	0.099	0.103	0.107	0.111	0.114	.117	+ .109
R'_5........	.054	.050	.061	.069	.076	.081	.086	.090	+ .84
R'_6........	.011	.036	.046	.055	.061	.066	.070	.074	+ .69
R'_7........	.043	.051	.058	.060	.062	.064	.065	.066	+ .62
R'_8........	.098	.087	.076	.077	.068	.065	.063	.062	+ .58
R'_{9a}.......	.123	.091	.081	.073	.065	.061	.057	.054	+ .50
R'_{9b}.......	.055	.066	.060	.051	.049	.046	.043	.041	+ .38
R'_{10}........	.023	.023	.023	.023	.023	.023	.023	.023	+2.17
R'_{11}........	.000	.000	.000	.000	.000	.000	.000	.000	......

NECESSARY ADDITIONAL ADJUSTMENTS TO NORMAL-FLOW EQUATION

Obviously after smoothing off the constants, (71), in the manner described, it becomes necessary to make another least-square adjustment, using the same observations, for the purpose of compensating as much as possible for the changes made in that smoothing-off process. In this adjustment, which was designated as Solution *N*, all the errors introduced into (71) by the smoothing-off process were assumed to be thrown into the constant term, S_c, and the adjusted value of S_c was determined from the solution. The method of procedure was as follows:

Using the values shown in column 10, Table 44, a substitution in observation equations was made, using the r's from Solution M and leaving the S_c column blank. The equations had the following form:

$$0.28r_1+0.21r_2+0.14r_3+0.109r_4+0.84r_5+0.69r_6+0.62r_7+0.58r_8+0.50r_{9a}+0.38r_{9b}+2.17r_{10}-D'=v \qquad (72)$$

The right-hand member, v, of equation (72), in addition to containing the discrepancies between computed and observed stream-flow due to all the various causes, also contained the omitted constant part of the stream-flow, S_c. Hence the mean v of Solution N with its sign changed became the adjusted value of S_c of that solution. This value was $+116$; the change from $+127$, the S_c of (71), being attributable to the smoothing-off process. By this adjustment the probable error of a single observation was reduced from ± 12.4 in Solution M to ± 8.6 in Solution N.

With the new value of S_c, $+116$, and equation (72) as written above, a substitution in observation equations was made having the form

$$+116+0.28r_1+0.21r_2+0.14r_3+0.109r_4+0.84r_5+0.69r_6+0.62r_7+0.58r_8+0.50r_{9a}+0.38r_{9b}+2.17r_{10}-D'=v \qquad (73)$$

for the three-year period 1911 to 1913 inclusively. This computation gave a set of residuals, v, of the nature of those of equation (58), from which, during times of flood, F''s of equation (57) as well as values of G for various D''s could be obtained for determining the flood-flow formula. After the derivation of the flood-flow formula, the illustration of which will be shown later, and taken from the computations on Stream B,[1] a final adjustment was made to the normal flow formula for Stream A. This final adjustment became advisable from a study of the residuals of equation (73), in which it was found that large systematic errors existed principally because of the $+2.17r_{10}-$ term. The adjustment, in this case, was designated as Solution P and it had for its form of observation equation the following:

$$R+r_{10}R_{10}-L=v \qquad (74)$$

in which R is the desired correction to the S_c of equation (73) and R_{10} is the desired correction to the R'_{10} of equation (73). One observation equation of the form of equation (74) was written for each *month*. r_{10} in equation (74) was the *mean* value of r_{10} of Solution M for the month. $-L$ was the *mean* residual, v, of equation (73) for each month. In selecting the months to be used in Solution P, all of the months in 1911 to 1913, to which equation (73) had been applied, were selected which were complete as to the r_{10} value and apparently free from flood-flow. A month was not considered free from flood-flow unless every negative residual in it was numerically less than 3.5 times the probable error of a single observation of Solution N, or ($3.5\times 8.6=$) 30, which, in the adopted units, is 0.030 c.f.s.

EXAMPLE OF OBSERVATION EQUATIONS FOR ADJUSTING S_c AND R'_{10}

The following observation equations of the form of equation (74) were used in Solution P, from which to derive corrections to S_c and R'_{10}.

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[1] See page 172.

Month	Observation equation	Month	Observation equation
1911		1913	
July........	$+1R+2R_{10}+36=v$	Jan.........	$+1R-4R_{10}-7=v$
Aug.........	$+1R+9R_{10}+50=v$	Feb.........	$+1R-6R_{10}+0=v$
Sept........	$+1R+21R_{10}+44=v$	Aug.........	$+1R+1R_{10}-2=v$
Dec.........	$+1R+10R_{10}+24=v$	Sept........	$+1R+3R_{10}+1=v$
		Oct.........	$+1R+1R_{10}+1=v$
1912		Nov.........	$+1R+0R_{10}+13=v$
Jan.........	$+1R-3R_{10}+7=v$	Dec.........	$+1R-5R_{10}+11=v$
Feb.........	$+1R-1R_{10}+15=v$		
Mar.........	$+1R+1R_{10}+17=v$		
Aug.........	$+1R+0R_{10}-2=v$		
Sept........	$+1R+9R_{10}-2=v$		
Oct.........	$+1R+8R_{10}+1=v$		
Nov.........	$+1R+7R_{10}+8=v$	Sums.......	$+19R+58R_{10}+222=0$
Dec.........	$+1R+5R_{10}+7=v$		

The normal equations for Solution *P* formed from the observation equations shown above, are as follows:

$$\left.\begin{array}{l} +19R+\ \ 58R_{10}+\ \ 222=0 \\ +58R+944R_{10}+1732=0 \end{array}\right\}\qquad (75)$$

The solution of these equations give the following values for the two unknowns: $R=-7.53$ $\qquad R_{10}=-1.365$

Applying these values (corrections) to the S_c and R'_{10}, respectively, of Solution *N*, equation (73), there is obtained the corrected values of those unknowns as follows: $S_c=+116-8=+108$; $R'_{10}=+2.17-1.36=+0.81$

Writing in these values in equation (73) in place of the values there shown, and combining R'_{9a} and R'_{9b} into the single constant, R'_9, there is obtained for the final equation of normal-flow for Stream *A* the following:

$$+108+0.28r_1+0.21r_2+0.14r_3+0.109r_4+0.84r_5+0.69r_6+0.62r_7+0.58r_8+0.44r_9+0.81r_{10}=D' \qquad (76)$$

which is of the form of equation (33).

Converting the new value of R'_{10} into a percentage, there is obtained $(+0.81\times 0.0107=)$ 0.0087 per cent as that part of a change in storage on a specific day which is delivered to the stream on each of the 129th to 256th days thereafter, in place of the 0.023 per cent previously obtained. Multiplying this by 128, there is obtained 1.11 per cent as the total part of the change in storage on a specific day which reaches Stream *A* by underground travel during the subsequent 129th to 256th days thereafter, in place of the 2.94 per cent previously obtained. Using the new value of R'_{10} expressed as a percentage, 0.0087, in connection with the smoothed-off values of the other R''s of equation (73), shown in column 9 of Table 44, there is obtained 9.0 per cent, in place of the preceding values of 13.6 and 10.9 per cent, as the total part of the change in storage on a specific day on Stream *A* which is delivered to the stream by underground travel during a period of 257 days thereafter (and including the specific day) if during the 256-day period following the current day the storage is held constant. Note that it is the above value of R'_{10}, expressed as a percentage which is shown in the curve on Plate 8. Note also that the R_{10} of equation (74) is a second correction to the assumed value of R'_{10}, or R''_{10}, of equation (66). Thus, the final corrected value of $R'_{10}=+0.58+1.586-1.36=+0.81$.

EXAMPLE OF SUBSTITUTION IN OBSERVATION EQUATIONS FOR NORMAL STREAM-FLOW

For the months used in Solution *M*, from which most of the constants in equation (76) were derived, the substitution in observation equations was made in that solution by using the derived values of the corrections from that solution shown in equation (70). In Solution *N*, the substitution in observation equations was made, using the smoothed-off values of the constants, (71), shown in (73). In Solution *P*, the substitution was made using the final constants of equation (76), except that R'_{9a} and R'_{9b} were used separately, to correspond with the method of computing r_{9a} and r_{9b} used in the months of these solutions. It should be evident that in each case of these substitutions in observation equations, a set of residuals was obtained like those in the right-hand members of the sample of observation equations shown on page 152. The smoothed-off values used in Solution *N* gave smaller residuals than were those of Solution *M*. Another reduction in the size of the residuals was effected by using the more correct values of the unknowns of Solution *P*. As it is the purpose here to show the final residuals obtained by use of the best derived normal-flow equation for Stream *A*, equation (76), the sample of observation equations shown on page 152 will be changed to the form of equation (58), which is a combination of equations (59) and (60).

Substitution in observation equations for June 1913, Stream A
The unit is 0.001 cubic foot per second

Date	S_e	$r_1R'_1$	$r_2R'_2$	$r_3R'_3$	$r_4R'_4$	$r_5R'_5$	$r_6R'_6$	$r_7R'_7$	$r_8R'_8$	$r_{9a}R'_{9a}$	$r_{9b}R'_{9b}$	$r_{10}R'_{10}$	$-D'$	v
1913														
June 1	+108	− 4	− 4	−3	−3	− 3	− 8	−14	+38	− 2	−4	−4	−179	− 82
2	+108	− 5	− 3	−3	−4	− 2	− 8	−14	+35	0	−3	−4	−177	− 80
3	+108	− 6	− 4	−2	−4	− 2	− 8	−14	+30	+ 3	−3	−4	−175	− 82
4	+108	− 4	− 4	−3	−4	− 3	− 8	−14	+25	+ 7	−3	−4	−173	− 80
5	+108	− 3	− 3	−3	−4	− 5	− 6	−14	+20	+11	−3	−4	−171	− 77
6	+108	− 3	− 2	−2	−4	− 6	− 6	−14	+18	+12	−3	−4	−169	− 75
7	+108	+11	− 2	−2	−4	− 6	− 6	−14	+17	+13	−3	−4	−182	− 74
8	+108	+ 4	+ 8	−2	−3	− 6	− 7	−14	+16	+13	−3	−4	−187	− 77
9	+108	+ 8	+ 3	+6	−2	− 6	− 7	−14	+13	+16	−3	−3	−181	− 62
10	+108	+16	+ 6	+2	+3	− 5	− 6	−15	+ 8	+20	−3	−3	−218	− 87
11	+108	0	+12	+4	+6	− 5	− 7	−15	+ 5	+21	−3	−3	−224	−101
12	+108	+ 9	0	+8	+5	0	− 7	−16	+ 4	+21	−3	−3	−226	−100
13	+108	− 3	+ 7	0	+9	+ 3	− 9	−13	+ 3	+21	−3	−3	−218	− 98
14	+108	− 4	− 2	+4	+6	+ 6	−10	−12	+ 2	+21	−3	−3	−206	− 93
15	+108	− 5	− 3	−2	+3	+12	− 9	−12	+ 1	+21	−3	−3	−199	− 91
16	+108	− 4	− 4	−2	+2	+ 8	− 5	−12	− 2	+23	−3	−3	−197	− 91
17	+108	+ 2	− 3	−3	−3	+10	− 3	−13	− 6	+26	−2	−3	−202	− 92
18	+108	+ 4	+ 1	−2	−4	+ 7	+ 1	−13	−10	+28	−2	−3	−205	− 90
19	+108	− 3	+ 3	+1	−4	0	+ 6	−13	−13	+31	−3	−4	−197	− 88
20	+108	− 4	− 2	+2	−1	− 1	+ 7	−14	−16	+32	−3	−4	−187	− 83
21	+108	− 3	− 3	−1	+2	− 5	+10	−14	−17	+34	−3	−4	−182	− 78
22	+108	− 3	− 2	−2	0	− 3	+10	−14	−19	+34	−3	−4	−175	− 73
23	+108	+ 1	− 2	−2	−3	− 1	+10	−14	−20	+34	−3	−4	−174	− 70
24	+108	− 3	+ 1	−1	−3	0	+ 6	−11	−21	+34	−2	−4	−169	− 65
25	+108	− 6	− 2	+1	−2	0	+ 4	− 8	−23	+35	−2	−4	−161	− 60
26	+108	− 7	− 4	−2	0	− 2	+ 2	− 5	−23	+35	−2	−4	−154	− 58
27	+108	− 4	− 5	−3	−1	− 3	− 1	− 1	−24	+34	−2	−4	−150	− 56
28	+108	+ 3	− 3	−4	−3	− 3	− 1	0	−24	+33	−2	−4	−157	− 57
29	+108	− 3	+ 2	−2	−5	− 3	− 4	+ 1	−23	+33	−2	−4	−147	− 49
30	+108	− 5	− 2	+1	−4	− 3	− 4	+ 1	−23	+33	−1	−4	−140	− 43

In the above tabular arrangement of the substitution in observation equations, the heading on each column identifies the term of the observation equations as shown on page 152, except that, instead of using the headings there shown, viz, R, r_1R_1, r_2R_2, . . . $r_{10}R_{10}$ and $-D$, there has been used S_e, $r_1R'_1$, $r_2R'_2$, . . . $r_{10}R'_{10}$

and $-D'$. As a specific example, note that the value given in the R'_5—column for June 17, namely, $+10$, is the product of the quantity $+12$ in the June 17 observation equations, which is the coefficient of R'_5, times the final value of R'_5, namely, $+0.84$. The specific meaning of the $+10$ is that the total addition to storage on the watershed due to the rains of June 9, 10, 11 and 12 (Table 41) increased the stream-flow 0.010 c.f.s. above S_c on June 17. The total water which passed through the ground to the stream on June 17 is given by the sum of the columns S_c to $r_{10}R'_{10}$ inclusive. This is found to be 0.110 c.f.s. The observed stream-flow on that date was 0.202 c.f.s., consequently the difference $(0.110-0.202=)-0.092$ c.f.s. is unaccounted for by equation (76). This 0.092 c.f.s., contains, in part the flood-flow for that date and all of the errors involved in the theory and the assumptions combined. Using the adopted criterion for flood-flow previously mentioned, viz, 0.030 c.f.s. (page 167), the residuals of June 1913 show that on every day of that month part of the flow of the stream was delivered by surface travel or flood-flow.

Referring again to Plate 10, the normal flow of the stream as computed from equation (76) is shown as a dash-dot-dash line. S_c or 108, the constant part of the flow, is shown as a light, solid, horizontal line, from which as a base line the r_1's are plotted as a dotted line. The residuals, v, of the substitution in observation equations for June 1913 are shown on Plate 10, as the difference between the observed flow (heavy solid line) and the computed normal flow (dash-dot-dash) line. The light dashed line (with triangles between the dashes) between the curve of observed flow and computed normal flow represents the total computed stream-flow as computed from equation (76), and the flood-flow formula of the form of equation (37), an illustration of the derivation of the constants of which will presently be given.

EQUATION OF NORMAL FLOW OF STREAM *B*

By methods already described on Stream *A*, the values in column 2 of Table 45 of the constants in the normal-flow formula, equation (33), were derived from observations on Stream *B* in 1911 to 1913.

Table 45 is comparable with Table 43, except that R'_9 is not divided into two parts, and that, since the unit used in computing r_9 was 1.0 inch of depth, the factor 0.0119 for converting R'_9 to a percentage is the same as for R'_{10}. In this table 1.19

TABLE 45—*Conversion of normal-flow constants to percentages, Stream B*

Constant	Absolute value of constant	Factor to convert col. 2 to percentage of change in storage which is delivered in one day	Per cent of change in storage which reaches stream on one day (col. 2×col. 3)	No. of days or time interval involved in each constant	Total per cent of change in storage current day which reaches stream in each time interval (col. 4×col. 5)
S_c	+96.0 (±4)				
R'_1	+ 0.060±0.021	1.19	0.071±0.025	1	0.071
R'_2	+ .071± .024	1.19	.084± .029	1	.084
R'_3	+ .082± .031	1.19	.098± .037	1	.098
R'_4	+ .079± .017	1.19	.094± .020	2	.188
R'_5	+ .790± .088	0.119	.094± .010	4	.376
R'_6	+ .715± .054	.119	.085± .006	8	.680
R'_7	+ .496± .029	.119	.059± .003	16	.944
R'_8	+ .269 —	.119	.032 —	32	1.024
R'_9	+ .403 —	.0119	.0048 —	64	.307
R'_{10}	+ .210 —	.0119	.0025 —	128	.320
				257	4.092

is the factor for converting run-off from Watershed B in c.f.s. to units of 0.01 inch of depth per day on that watershed. The values in column 2 are the derived values after one smoothing by taking means of three, holding R'_1 constant and assuming R'_{11} constant at zero.

Column 4 of Table 45 appears plotted on Plate 8.

The probable errors of the constants R'_1, R'_2, . . . R'_7 were computed rigorously from the normal equations similar to equations (69), from which the constants were derived, and from the residuals, v, of 193 observation equations of Solution CC, Stream B. The probable error of S_c, ±4.0, was estimated. The probable errors of R'_8, R'_9 and R'_{10} were not computed, but are believed to be smaller than those of the corresponding unknowns derived on Stream A, viz, ±0.03, ±0.015 and ±0.21, respectively. (Note the division of the probable error of R'_9 of Stream A by 10 to account for the different unit used in Stream B.)

From Table 45 only 4.09 per cent of any change in storage in the drainage area of Stream B on a day is delivered to the stream after a lapse of 257 days (including the day) by travel through the ground to the stream, if the storage remains constant after the first of the 257 days, which is less than half the corresponding value for Stream A.

For purposes of discussion it is convenient to write out the equation of normal flow of Stream B thus:

$$+96+0.060r_1+0.071r_2+0.082r_3+0.079r_4+0.790r_5+0.715r_6+0.496r_7+0.269r_8 \\ +0.403r_9+0.210r_{10}=-D' \ldots\ldots\ldots\ldots (77)$$

which is of the form of equation (33) and corresponds to equation (76) for Stream A.

The normal flow of Stream B was computed from equation (77) for the whole period March 1911 to December 1915. It is shown on Plates 13 to 17 inclusive.

VALUES OF *G* FOR STREAMS *A* AND *B*

From the computation of the normal stream-flow on Streams A and B in the summer months of 1911, 1912 and 1913, in comparison with the observed stream-flow, and using the criterion for flood-flows previously explained, values of G were obtained for various discharges as shown in Table 46, from which it appears that

TABLE 46—*Values of G for Streams A and B*

Stream-flow on preceding day D' (0.001 c.f.s.)	Value of r_1 the current day required to glut ground G (0.01″)	Stream-flow on preceding day D' (0.001 c.f.s.)	Value of r_1 the current day required to glut ground G (0.01″)
40	66	120	38
50	65	130	36
60	63	140	34
..	..	150	28
70	60	160	25
..	..	170	24
80	55	180	23
..	..	190	23
90	49	200	23
..	..	210	23
100	43	250	22
...	..	300	21
110	40	350	21
...	..	400 and over	20

when the stream-flow is as low as 0.040 c.f.s., an addition to the storage on the watershed of 0.66 inch is necessary to saturate the ground before any of it goes to the stream as surface run-off. On the other extreme, when the stream-flow is 0.400 c.f.s. or more, only 0.20 inch increase in storage is necessary to saturate the ground before surface run-off begins. These values are shown plotted in the upper right-hand corner of Plate 22.

EXAMPLE OF OBSERVATION EQUATIONS FOR FLOOD-FLOW, STREAM *B*

One form of observation equation to determine flood-flow is equation (57). This was the first form used in the investigation, before any idea at all was obtained as to how large the R'_f's would be. If it is possible to estimate values of these constants, it is generally more advantageous to use the following form of observation equation for determining flood-flows:

$$r_{f1}R_{f1}+r_{f2}R_{f2}+r_{f3}R_{f3}+r_{f4}R_{f4}+r_{f5}R_{f5}+r_{f6}R_{f6}+r_{f7}R_{f7}-F=v \ldots\ldots\ldots\ldots (78)$$

In which F is computed from the following:

$$r_{f1}R''_{f1}+r_{f2}R''_{f2}+r_{f3}R''_{f3}+r_{f4}R''_{f4}+r_{f5}R''_{f5}+r_{f6}R''_{f6}+r_{f7}R''_{f7}-F'=-F \ldots\ldots\ldots (79)$$

In equation (79) R''_{f1}, R''_{f2}, R''_{f3}, . . . R''_{f7} are assumed values of R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7}, respectively. $-F$ is that part of the flood-flow not accounted for in terms of $r_{f1}R''_{f1}$, $r_{f2}R''_{f2}$, $r_{f3}R''_{f3}$, . . . $r_{f7}R''_{f7}$. It represents the total error resulting from the use of the R''_f's. The corrections to the R''_f's to get the R'_f's are the R_f's of equation (78); that is,

$$\left.\begin{array}{ll} R'_{f1}=R''_{f1}+R_{f1} & R'_{f5}=R''_{f5}+R_{f5} \\ R'_{f2}=R''_{f2}+R_{f2} & R'_{f6}=R''_{f6}+R_{f6} \\ R'_{f3}=R''_{f3}+R_{f3} & R'_{f7}=R''_{f7}+R_{f7} \\ R'_{f4}=R''_{f4}+R_{f4} & \end{array}\right\} \ldots\ldots\ldots\ldots (80)$$

Observation equations for flood-flow, Solution K, Stream B, October 1911

Date	Terms of equation							
	$+r_{f1}R_{f1}$	$+r_{f2}R_{f2}$	$+r_{f3}R_{f3}$	$+r_{f4}R_{f4}$	$+r_{f5}R_{f5}$	$+r_{f6}R_{f6}$	$+r_{f7}R_{f7}$	$-F=v$
Oct. 4...	$+15R_{f1}$							$+6=v_1$
5...	+197	$+15R_{f2}$						$-272=v_2$
6...		+197	$+15R_{f3}$					$-247=v_3$
7...			+197	$+15R_{f4}$				$-189=v_4$
8...				+212				$-247=v_5$
9...				+197	$+2R_{f5}$			$-303=v_6$
10...					+21			$-275=v_7$
11...					+21			$-214=v_8$
12...					+21			$-149=v_9$
13...					+20	$+2R_{f6}$		$-105=v_{10}$
14...						+21		$-119=v_{11}$
15...						+21		$-88=v_{12}$
16...						+21		$-58=v_{13}$
17...						+21		$-41=v_{14}$
18...						+21		$-26=v_{15}$
19...						+21		$-13=v_{16}$
20...						+21		$+0=v_{17}$
21...						+20	$+2R_{f7}$	$+12=v_{18}$
22...							+21	$-36=v_{19}$
23...							+21	$-27=v_{20}$
24...							+21	$-24=v_{21}$
25...							+21	$-14=v_{22}$
26...							+21	$-9=v_{23}$

The similarity between equations (78), (79) and (80), and equations (60), (59) and (61) should be evident.

The observation equations (page 172) are written in the form of equation (78). The unknowns R_{f1}, R_{f2}, R_{f3}, . . . at the beginning of each column should be considered as repeated down the column. The method of computing $-F$ will next be illustrated and thereafter an illustration will be given of the method of computing r_{f1}, r_{f2}, r_{f3}, . . .

The computation of $-F$, the absolute term in the above observation equations, was effected by the use of equation (79) in which the following values of the unknowns were used:

$$\begin{array}{ll} R''_{f1}=+0.067 & R''_{f5}=+6.81 \\ R''_{f2}=+\ .244 & R''_{f6}=+4.54 \\ R''_{f3}=+\ .571 & R''_{f7}=+2.02 \\ R''_{f4}=+\ .723 & \end{array} \Bigg\} \quad \ldots\ldots\ldots\ldots (81)$$

These values were taken directly from preliminary results obtained on Stream *A*, in which the form of observation equation used was equation (57). In other words, the R''_f's in equation (81) for Stream *B* were the R'_f's of equation (57) previously derived on Stream *A* as in step (2), page 147.

Example of computation of $-F$

Solution K, Stream *B*, October 1911

Date	$r_{f1}R''_{f1}$	$r_{f2}R''_{f2}$	$r_{f3}R''_{f3}$	$r_{f4}R''_{f4}$	$r_{f5}R''_{f5}$	$r_{f6}R''_{f6}$	$r_{f7}R''_{f7}$	$-F'$	$-F$
1911									
Oct. 4	+ 15(0.067)							+ 5	+ 6
5	+197	+ 15(0.244)						−289	−272
6		+197	+ 15(0.571)					−304	−247
7			+197	+ 15(0.723)				−312	−189
8				+212				−400	−247
9				+197	+ 2(6.81)			−459	−303
10					+21			−418	−275
11					+21			−357	−214
12					+21			−292	−149
13					+20	+ 2(4.54)		−250	−105
14						+21		−214	−119
15						+21		−183	− 88
16						+21		−153	− 58
17						+21		−136	− 41
18						+21		−121	− 26
19						+21		−108	− 13
20						+21		− 95	0
21						+20	+ 2(2.02)	− 83	+ 12
22							+21	− 78	− 36
23							+21	− 69	− 27
24							+21	− 66	− 24
25							+21	− 56	− 14
26							+21	− 51	− 9

In the above example of computation of $-F$, the quantities in parentheses at the beginning of each column except the last two are the R''_f's of equation (81). They should be considered as repeated down the column. The other quantities, except the last two columns, are the r_f's of equation (78) and (79). The column headed $-F'$ is the difference between the normal stream-flow of Stream *B* as computed from equation (77) and the observed flow, D'. (See Plate 13.) The column headed $-F$ is the algebraic sum of the preceding eight columns. It is that part of the flood-flow not accounted for by $r_{f1}R''_{f1}+r_{f2}R''_{f2}+r_{f3}R''_{f3}+\ .\ .\ .$, which is to be accounted for by $r_{f1}R_{f1}+r_{f2}R_{f2}+r_{f3}R_{f3}+\ .\ .\ .\ .$ These values of F are shown in the observation equations on page 172.

The following is an example of the computation of r_{f1}, from which r_{f2}, r_{f3}, . . . r_{f7} follow according to the definitions previously given.

TABLE 47—*Example of computation of* r_{f1}, r_{f2}, r_{f3}, . . *Solution K, Stream B, October 1911*

Date	r_1	G	r_{f1}
1911			
Oct. 4	+ 62	47	+ 15
5	+243	46	+197
6	− 13	20	.....
7	− 10	20	.....
8	− 10	20	.....
9	− 15	20	.....
	etc.		

In the second column of Table 47 are shown values of r_1 computed as already explained in connection with Stream *A*, pages 151 and 154. The unit is 0.01 inch.

Values of G in the third column were selected from the tabulated values in Table 46 in connection with the observed stream-flow. For example, on October 3, 1911 (Plate 13), the observed stream-flow is shown to be 93, which, by interpolation in Table 46, gives a value of G of 47 for October 4. On October 4, the observed stream-flow was 97, which gives a value of G of 46 for October 5. On October 5, the observed stream-flow was 398, which gives a value of G of 20 for October 6, and so on.

In the fourth column are shown values of r_{f1}, which are obtained by subtracting values of G from values of r_1 in the two preceding columns.

Preceding October 4, 1911, and throughout the remainder of October, after the 5th, no other values of r_1 were great enough to produce flood-flows (see Plate 13) hence the whole of the computation of r_{f1}, r_{f2}, r_{f3}, . . . r_{f7} for October 1911 results from the two values +15 and +197 shown above. As an illustration (refer to *Example of Computation of* −*F* and *Observation Equation for Flood-Flow*) take the observation equation for October 9, 1911. For that date there are only two r_f's as shown, viz, $r_{f4}=\Sigma(r_{f1}$ for October 5 and 6$)=+197$. $r_{f5}=\Sigma(r_{f1}$ for October 1 to 4 inclusive$)\div 10=15\div 10=2$ to the nearest even unit. The other r_f's for October 9 are blank, because the r_f's involved in their determination are all blank,

The computation of all the other r_f's shown can be verified according to the definitions previously given.

EXAMPLE OF NORMAL EQUATIONS FOR FLOOD-FLOW, STREAM *B*

In each least-square solution for determining the flood-flow equation, a set of normal equations was first formed in the usual way from the observation equations for each day. Then these sets of normal equations, one set for each group of days involved in a flood and its after-effects, were combined by addition to form the final set of normal equations for the whole solution.

The normal equations for October 4 to November 6, 1911, in Solution *K*, Stream *B*, formed from the observation equations for that group of days partly shown on page 172, are as follows:

Normal equations for Oct. 4–Nov. 6, 1911. Solution K, Stream B

$$\left.\begin{array}{rrrrrrrr}
+39{,}034R_{f1} & +\ 2{,}955R_{f2} & & & & & & -\ 53{,}494=0\\
+\ 2{,}955R_{f1} & +39{,}034R_{f2} & +\ 2{,}955R_{f3} & & & & & -\ 52{,}739=0\\
 & +\ 2{,}955R_{f2} & +39{,}034R_{f3} & +\ 2{,}955R_{f4} & & & & -\ 40{,}938=0\\
 & & +\ 2{,}955R_{f3} & +83{,}978R_{f4} & +\ 394R_{f5} & & & -114{,}890=0\\
 & & & +\ 394R_{f4} & +1{,}727R_{f5} & +\ 40R_{f6} & & -\ 16{,}104=0\\
 & & & & +\ 40R_{f5} & +3{,}491R_{f5} & +\ 40R_{f7} & -\ 7{,}215=0\\
 & & & & & +\ 40R_{f6} & +7{,}019R_{f7} & +\ 464=0
\end{array}\right\}(82)$$

Attention is called here to the particular group of days embodied in the normal equations (82). The rain on October 4 started a flood-flow which was accentuated by the heavy rain on October 5. In the observation equation (78), seven r_fR_f terms only are included which cover a total time interval of 33 days. Since there was no other r_{f1} later than October 5 in this time interval, the last r_{f7} which can be computed is on November 6—33 days after October 5, inclusive.

The final normal equations for Solution K, formed by combining four such sets of normal equations as are shown in (82), are as follows:

Final normal equations for Solution K, Stream B

$$\left.\begin{array}{rrrrrrrr}
+44{,}070R_{f1} & +\ 3{,}501R_{f2} & +\ 660R_{f3} & +\ 792R_{f4} & +\ 18R_{f5} & +\ 0R_{f5} & +\ 0R_{f7} & -\ 64{,}680=0\\
+\ 3{,}501R_{f1} & +44{,}070R_{f2} & +\ 3{,}501R_{f3} & +\ 1{,}452R_{f4} & +\ 18R_{f5} & & +\ 0R_{f7} & -\ 62{,}866=0\\
+\ 660R_{f1} & +\ 3{,}501R_{f2} & +44{,}070R_{f3} & +\ 4{,}161R_{f4} & +\ 78R_{f5} & +\ 6R_{f6} & +\ 0R_{f7} & -\ 49{,}113=0\\
+\ 792R_{f1} & +\ 1{,}452R_{f2} & +\ 4{,}161R_{f3} & +95{,}142R_{f4} & +\ 726R_{f5} & +\ 24R_{f5} & +\ 0R_{f7} & -128{,}568=0\\
+\ 18R_{f1} & +\ 18R_{f2} & +\ 78R_{f3} & +\ 726R_{f4} & +2{,}003R_{f5} & +\ 77R_{f6} & +\ 0R_{f7} & -\ 18{,}239=0\\
+\ 0R_{f1} & & 6R_{f3} & +\ 24R_{f4} & +\ 77R_{f5} & +4{,}157R_{f5} & +\ 78R_{f7} & -\ 10{,}981=0\\
+\ 0R_{f1} & 0R_{f2} & +\ 0R_{f3} & +\ 0R_{f4} & +\ 0R_{f5} & +\ 78R_{f5} & +8{,}469R_{f7} & -\ 3{,}150=0
\end{array}\right\}(83)$$

The final set of normal equations for Solution K (83), depends upon 166 observation equations covering the flood-flows produced by the rains during the summer months of 1911, 1912 and 1913.

The solution of the final normal equations for Solution K (83) gives the following values for the unknowns:

$$\left.\begin{array}{ll}
R_{f1}=+1.34 & R_{f5}=+8.50\\
R_{f2}=+1.21 & R_{f6}=+2.48\\
R_{f3}=+0.87 & R_{f7}=+0.35\\
R_{f4}=+1.23 &
\end{array}\right\}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(84)$$

The probable error of a single observation for Solution K, computed rigorously from the 166 residuals and from the normal equations, was ±20.0.

Combining (84) with (81) according to (80), there is obtained for the values of the unknowns:

$$\left.\begin{array}{ll}
R'_{f1}=+1.41 & R'_{f5}=+15.31\\
R'_{f2}=+1.45 & R'_{f6}=+\ 7.02\\
R'_{f3}=+1.44 & R'_{f7}=+\ 2.37\\
R'_{f4}=+1.95 &
\end{array}\right\}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(85)$$

In interpreting the constants (85), it is to be remembered that in the computation of r_{f1}, r_{f2}, r_{f3}, . . . r_{f7}, the values after r_{f4} were divided by 10, hence to get the above constants into their true relationship with reference to each other, R'_{f5}, R'_{f6} and R'_{f7} should be divided by 10.

The constants (85) may be converted into percentages in the same manner and by the same conversion factors used on the normal-flow equations. In Table 48 the process of conversion to percentages is similar to that previously given.

In Table 48, column 2 shows the constants (85); column 3 the conversion factor for converting the stream-flow of Stream *B* in c.f.s. to units of 0.01 inch or 0.1 inch of depth per day on Watershed *B*; column 4, the per cent of that part of the change in storage above the ground surface on the current day which reaches the stream by *surface travel* on each day of each time interval involved in the definitions of the r_f's.[1] Thus 1.67 per cent of the change in storage above the ground surface on the current day is delivered to Stream *B* by surface travel on that day, or 0th day; 1.73 per cent of the change the current day is delivered to the stream the following day[1]; . . . 0.282 per cent of the change in storage above ground the current day is delivered on *each* of the seventeenth to thirty-second days (inclusive) following.[1] The product of each quantity in column 4 by the time interval in column 5 gives the total percentage of the change in storage above ground surface on the current day which is delivered to the stream in the whole of the time interval in question.[1] Thus, during the ninth to sixteenth days, inclusive, after the change in storage occurs, a total of 6.68 per cent (col. 6, line 6) of it is delivered over the surface of the ground to Stream *B*.[1] Totaling all the quantities in column 6, there is obtained 28.2 per cent as the total part of the change in storage on the current day which is delivered to Stream *B* by surface travel during a period of 33 days following (and including) the current day.[1]

TABLE 48—*Conversion of flood-flow constants to percentages, Stream B*

Constant	Absolute value of constant from (85)	Factor to convert col. 2 to percentage of change in storage which is delivered in one day	Per cent of change in storage which reaches stream in one day[1] (col. 2×col. 3)	No. of days or time interval involved in each constant	Total per cent of change in storage above ground surface on current day which is delivered to Stream *B* in each time interval[1]	Col. 4 smoothed once by means of 3
R'_{f1}.......	+ 1.41	1.19	1.67	1	1.67	1.67
R'_{f2}.......	+ 1.45	1.19	1.73	1	1.73	1.70
R'_{f3}.......	+ 1.44	1.19	1.71	1	1.71	1.92
R'_{f4}.......	+ 1.95	1.19	2.32	2	4.64	1.95
R'_{f5}.......	+15.31	0.119	1.82	4	7.28	1.66
R'_{f6}.......	+ 7.02	0.119	0.835	8	6.68	0.98
R'_{f7}.......	+ 2.37	0.119	0.282	16	4.51	0.37
			Totals.......	33 days	28.22 *p. ct.*	29.6 *p. ct.*

The percentages shown in column 4 were smoothed once by taking successive means of three values in that column, and assuming R'_{f8}, the next constant after R'_{f7}, zero. The results of this smoothing is shown in column 7, with the total of 29.6 per cent instead of 28.2 per cent as the total part of the current day change in storage reaching the stream in 33 days.[1]

---

[1] Assuming the water table to be at the level of the ground surface on each day of the 32-day period after the current day.

By dividing column 7 by 1.19, 1.19, . . . 0.119, the percentage values are converted back to absolute values as in column 2. This gives the following flood-flow formula for Stream *B*:

$$+1.40r_{f1}+1.43r_{f2}+1.161r_{f3}+1.64r_{f4}+14.0r_{f5}+8.24r_{f6}+3.11r_{f7}=\text{Flood-flow of Stream } B \ldots\ldots (86)$$

EXAMPLE OF SUBSTITUTION IN OBSERVATION EQUATIONS FOR FLOOD-FLOW, STREAM *B*

For the months used in Solution *K*, using the constants of that solution (84), a substitution in observation equations was made which served to determine the residuals, *v*, of equation (78), for that solution. These residuals are the discrepancies between the total stream-flow (normal-flow+flood-flow) as computed from the theory on the one hand and the observed flow on the other hand.

The following example covers the same days used in the observation equations shown on page 172.

Substitution in observation equations for flood-flow, Stream B

Date	$r_{f1}R_{f1}$	$r_{f2}R_{f2}$	$r_{f3}R_{f3}$	$r_{f4}R_{f4}$	$r_{f5}R_{f5}$	$r_{f6}R_{f6}$	$r_{f7}R_{f7}$	$-F$	v
1911									
Oct. 4	+ 15 (1.34)							+ 6	+26
5	+197	+ 15 (1.21)						−272	+10
6		+197	+ 15 (0.86)					−247	+ 4
7			+197	+ 15 (1.23)				−189	0
8				+212				−247	+14
9				+197	+ 2 (8.50)			−303	−44
10					+21			−275	−97
11					+21			−214	−36
12					+21			−149	+29
13					+20	+ 2 (2.48)		−105	+70
14						+21		−119	−67
15						+21		− 88	−36
16						+21		− 58	− 6
17						+21		− 41	+11
18						+21		− 26	+26
19						+21		− 13	+39
20						+21		0	+52
21						+20	+ 2 (0.349)	+ 12	+63
22							+21	− 36	−29
23							+21	− 27	−20
24							+21	− 24	−17
25							+21	− 14	− 7
26							+21	− 9	− 2

The quantities in parentheses at the beginnings of the second to eighth columns are the R_f's. They should be considered as repeated down the column. The other values in these columns are the r_f's, as shown at the heads of the columns. The other two columns are $-F$ and *v*, respectively, as shown. The sum of the products in the second to eighth columns, inclusive, represents the correction to the computed flood-flow, or the correction to the computed part of the water which reaches Stream *B* by surface travel. For example, on October 9, the observed flow was 0.586 c.f.s. (see Plate 13), which was the peak of this flood. Of that flow, only 0.127 c.f.s. is accounted for by equation (77) as having reached the stream by percolation. This leaves 0.459 c.f.s. to be accounted for as surface travel or flood-flow. Of this 0.459 c.f.s., 0.156 c.f.s. is estimated by the assumed constants (81), leaving 0.303 c.f.s. to be accounted for by the corrections to those constants (84), which accounts for $197(1.23)+2(8.50)=0.259$ c.f.s., leaving a residual of 0.044 c.f.s. That is, of the flood peak on October 9 of 0.586 c.f.s., 0.127 c.f.s. reached the stream by percolation, and 0.415 c.f.s. reached it by surface travel, making the total computed flow for that day 0.542 c.f.s., which is 0.044 c.f.s. or 7.5 per cent smaller than the observed flow.

Referring to the October flood, Plate 13, it is seen that the curve of total computed flow does not rise, and particularly does not fall, as smoothly as the curve of observed flow. Instead of falling along a smooth curve, the computed-flow curve falls by steps. This is also seen in the residuals of the preceding tabulation. It should be evident that the cause of this is peculiar to the nature of the form of equation used, in which a single constant is made to account for the flow over an interval of several days. Obviously if the equation were lengthened out so as to account for each day's flow by one constant, it would require an equation of 30-odd terms to represent the flood-flows on this stream alone, which would be practically prohibitive, if not theoretically so.

FLOOD-FLOW EQUATION OF STREAM *A*

By the same process explained in detail on Stream *B*, resulting in equation (86), the following flood-flow equation for Stream *A* was derived:

$$+1.6r_{f1}+1.7r_{f2}+1.9r_{f3}+1.6r_{f4}+8.9r_{f5}+4.5r_{f6}+1.3r_{f7}=\text{Flood-flow of Stream } A \ldots\text{(87)}$$

in which the r_f's are defined the same as in equation (86); that is, the same values of G may be used.

The above constants, converted into percentages, give a total of 19.0 per cent as that part of the change in storage above the ground surface on a day which is delivered to Stream *A* by surface travel during a period of 33 days thereafter, and including the current day.[1] This is much smaller than the 29.6 per cent obtained for Stream *B*.

RUN-OFF CURVES OF STREAMS *A* AND *B*

The constants in equations (86) and (87) converted to percentages are shown plotted on Plate 8 above the normal run-off curves plotted from the constants of equations (76) and (77) converted to percentages and to a different scale on the left. In plotting the constants of equations (86) and (87), the last value plotted, R'_{f7}, falls in the center of the 16-day interval covered by R'_{f7}. This is the end in each case of the known part of the flood-flow curve, as determined from the least-square computations. Evidently if these curves could be extended they would ultimately join the normal-flow curves at some point, which, for these two streams, is not less than 32 days after the current day. The curves are shown extended beyond R'_{f7} as dotted lines. Similarly, the normal-flow run-off curves are shown extended beyond R'_{10} as dotted lines to the end of the 256-day interval.

The characteristics of the two streams as shown by the run-off curves are considerably different. Stream *A* responds to both normal and flood-flows more quickly and to a greater extent at first than Stream *B*. This might be expected from a comparison of the shapes and gradients of the watersheds (Plate 7) and from the dimensions (Table 40). Stream *A* is long and narrow in comparison with Stream *B*. The mean gradient of Stream *A* is slightly less than that of Stream *B*. In the case of normal-flows, after about the thirtieth day the rate of delivery, expressed in percentages, to Stream A is larger than that to Stream B for many days thereafter. In the case of flood-flows the rate of delivery to Stream B is larger than that to Stream A after about the third day after a large addition to storage.

---

[1] Assuming the water table to be at ground surface level on each day of the 32-day period after the current day.

If the run-off curves are reduced to the same basis by taking account of the difference in drainage areas, they represent the comparative *flows* of the two streams as affected by changes in storage in their respective drainage areas. Since, however, the difference in drainage areas is only about 10 per cent, the curves may, as they are, be taken for approximate comparative flow characteristics.

EXAMPLES OF COMPUTATIONS USED IN THE DETERMINATION OF C, F, M AND T'''

Further comments upon the comparative flow characteristics of the two streams will be reserved for a later place. It is now proposed to present sample computations from Solution AA, Stream A, which was the final solution on that stream which served to fix final values of C, F, M and T'''. These specific illustrations will serve to clarify the theory presented in general terms on pages 140 to 143, and represents an illustration of some multiple of step (6) on page 147.

EXAMPLE OF COMPUTATION OF D_n, SOLUTION AA, STREAM A, MARCH TO APRIL 1913

In the computation of D_n, equation (43) is used, substituting therein the best values of S_c, R'_1, R'_2, . . . R'_{10} determined to date, and the computed values of n_1, n_2, n_3, . . . n_{10} as computed from equation (40) in exactly the same manner as r_1, r_2, r_3, . . . r_{10} are computed from equation (41) and as illustrated on pages 154 and 155 except that in the computation of n_1, n_2, n_3, . . . n_{10} the net melting is not known and therefore does not enter the computation. The only precipitation which enters the computation of n_1, n_2, n_3, . . . n_{10} is in the form of rain. In the computation of n_1, n_2, n_3, . . . for Solution AA, the evaporation from land was estimated from equation (34) in which $\frac{E_l}{E_w}$ was taken as 2.6 instead of 2.3 used in Solution M, the value 2.6 being based upon computations made subsequent to Solution M, and probably more exact. Because of the similarity between the computation of r_1, r_2, r_3, . . . and n_1, n_2, n_3 . . . , and since the former has already been illustrated, the latter will be assumed to be known in this illustration.

The best values of S_c, R'_1, R'_2, . . . R'_{10} determined at the time Solution AA was made were those of equation (76). Using those values, the computation of D_n is shown on page 180.

The values in parentheses at the tops of the columns are the R''s of equation (76) except that R'_9 is multiplied by 10 because the unit in which n_9 was computed was 1.0 inch of depth instead of 0.1 inch of depth as in the case of r_9. These values should be considered as repeated down the column.

The other values except in the first, second and last columns are n_1, n_2, . . . n_9; in the following units: n_1 to n_4, 0.01 inch of depth; n_5 to n_8, 0.1 inch of depth; n_9, 1.0 inch of depth.

The products, nR', and S_c and D_n are in 0.001 c.f.s.

In column 2 is S_c, the constant part of the stream-flow.

The sum of S_c and the nR'-products gives D_n.

It is to be noted that the n's are all negative. The reason is that there was no addition to storage in the form of rain in the period shown, and none, or not enough as rain, in the earlier periods into which the late n's reached back to make them positive.

Example of computation of D_n, Solution AA, Stream A

Date	S_c	$n_1R'_1$	$n_2R'_2$	$n_3R'_3$	$n_4R'_4$	$n_5R'_5$	$n_6R'_6$	$n_7R'_7$	$n_8R'_8$	$n_9R'_9$	D_n
1913											
Mar. 14	+108	−4(0.28)	−1(0.21)	−1(0.14)	−10(0.109)	−2(0.84)	−4(0.69)	−8(0.62)	−15(0.58)	−4(4.4)	+69
15	+108	−4	−4	−1	−5	−2	−4	−8	−15	−4	+68
16	+108	−6	−4	−4	−2	−2	−4	−7	−16	−3	+73
17	+108	−9	−6	−4	−5	−2	−5	−7	−16	−3	+71
18	+108	−11	−9	−6	−8	−1	−5	−7	−16	−3	+71
19	+108	−4	−11	−9	−10	−1	−5	−7	−16	−3	+73
20	+108	−4	−4	−11	−15	−1	−4	−7	−16	−3	+72
21	+108	−4	−4	−4	−20	−2	−4	−7	−17	−3	+71
22	+108	−6	−4	−4	−15	−2	−3	−7	−17	−3	+71
23	+108	−4	−6	−4	−8	−3	−3	−7	−18	−3	+72
24	+108	−4	−4	−6	−8	−3	−3	−6	−18	−3	+72
25	+108	−1	−4	−4	−10	−3	−3	−7	−17	−3	+73
26	+108	−4	−1	−4	−10	−2	−4	−7	−17	−3	+73
27	+108	−6	−4	−1	−8	−2	−4	−8	−17	−3	+71
28	+108	−9	−6	−4	−5	−2	−4	−8	−17	−3	+69
29	+108	−6	−9	−6	−5	−2	−4	−7	−17	−3	+70
30	+108	−11	−6	−9	−10	−2	−5	−7	−16	−3	+71
31	+108	−12	−11	−6	−15	−1	−5	−7	−16	−3	+70
Apr. 1	+108	−7	−12	−11	−15	−2	−5	−8	−16	−3	+67
...	.....	....	....	....	....	....	....	....	....	....	.....
4	+108	−10	−1	−7	−19	−3	−3	−9	−15	−3	+69
5	+108	−12	−10	−1	−14	−4	−3	−8	−15	−3	+69
6	+108	−7	−12	−10	−8	−4	−4	−8	−15	−3	+68
7	+108	−7	−7	−12	−11	−4	−4	−8	−16	−3	+69
8	+108	−2	−7	−7	−22	−3	−4	−8	−16	−3	+70
9	+108	−5	−2	−7	−19	−2	−5	−8	−16	−3	+72
10	+108	−6	−5	−2	−14	−3	−6	−8	−16	−3	+69
11	+108	−9	−6	−5	−9	−3	−6	−7	−16	−3	+69
12	+108	−12	−9	−6	−7	−4	−6	−7	−16	−3	+68
...	.....	....	....	....	....	....	....	....	....	....	.....
15	+108	−20	−17	−13	−21	−2	−7	−9	−15	−3	+59
16	+108	−17	−20	−17	−25	−2	−6	−9	−15	−3	+60

Note also that the term $n_{10}R'_{10}$ has been omitted from the above computation, for the reason that the n_{10}'s, if computed, would reach back into the preceding fall before the beginning of the freezing-melting period. It is to be remembered that the purpose of Solution *AA* is to derive better values for *C*, *F*, *M* and T'' than those from previous solutions, constants for expressing the rate of change in storage per day per degree *F*, due to freezing and melting, hence observations outside the freezing-melting period can not be used. This omission of the $n_{10}R'_{10}$—term introduces an error which is nearly constant, hence is possibly largely thrown into the derived value of *C*.

EXAMPLE OF COMPUTATION OF D_f AND $-D$, SOLUTION *AA*, STREAM *A*

In the computation of D_f, equation (44) is used, substituting therein the best values of R'_{f1}, R'_{f2}, R'_{f3} . . . and *G* determined to date, and the computed values of n_1, n_2, n_3 . . . as computed from equation (40) and used in the preceding illustration. The computation of $-D$ is effected by use of equation (45), in which D_n is that already illustrated as computed from equation (43), and D' is the observed flow.

The best values of R'_{f1}, R'_{f2}, R'_{f3} . . . and of *G* available at the time of this solution were those of equation (87) and Table 46, respectively. Using those values, the computation of D_f and $-D$ is as follows:

Example of computation of D_f and $-D$, Solution AA, Stream A

Date	R'_{f1} (n_1-G_1)	R'_{f2} (n_2-G_2)	R'_{f3} (n_3-G_3)	R'_{f4} (n_4-G_4)	R'_{f5} (n_5-G_5)	R'_{f6} (n_6-G_6)	R'_{f7} (n_7-G_7)	D_f	D_f+D_n	$-D'$	$-D$
1913											
Mar. 31	1.6 (−46)							−74	−4	−182	−186
Apr. 1	1.6 (−30)	1.7 (−46)						−126	−59	−181	−240
......											
4				1.6 (−76)				−122	−53	−145	−198
5				1.6 (−30)	8.9 (−5)			−92	−23	−190	−213
6	1.6 (−30)				8.9 (−8)			−119	−51	−230	−281
7		1.7 (−30)			8.9 (−8)			−122	−53	−225	−278
8			1.9 (−30)		8.9 (−8)			−128	−58	−187	−245
9				1.6 (−30)	8.9 (−3)	4.5 (−5)		−97	−25	−154	−179
10				1.6 (−30)		4.5 (−8)		−84	−15	−133	−148
11					8.9 (−3)	4.5 (−8)		−63	+6	−128	−122
12					8.9 (−3)	4.5 (−8)		−63	+5	−174	−169
......											
15	1.6 (−41)	1.7 (−39)	1.9 (−37)			4.5 (−11)		−252	−193	−339	−532
16	1.6 (−38)	1.7 (−41)	1.9 (−39)	1.6 (−37)		4.5 (−11)		−314	−254	−379	−633

The values in parentheses are values of $(n-G)$, the illustration of the computation of which has yet to be given. The unit is 0.01 inch of depth to (n_4-G_4) inclusive, and 0.1 inch of depth thereafter. The coefficients of these quantities are the R'_f's of equation (87). The sum of the products, $R'_f\ (n-G)$, which are in 0.001 c.f.s., gives D_f. D_f added to D_n, computed on page 180 and their sum added to $-D'$ gives $-D$. The various quantities can be identified by the column headings.

The above computation of D_f and $-D$ is given as if beginning on March 31. Earlier than March 31 the mean air temperature on the watershed does not rise high enough to produce a large enough amount of melting to create a flood-flow, hence the R'_f's and the G's do not enter into the computation of $-D$ before that date. Preceding March 31, $-D$ is merely the sum of D_n, computed on page 180 and $-D'$; that is, preceding March 31, D_f is zero because there were no flood-flows due either to melting or to rain. The computation of $-D$ for the dates preceding March 31, which will appear later in the example of observation equations for those dates, page 191, can be verified by adding the D_n's of page 180 to $-D'$ scaled from Plate 10.

ESTIMATING DATES TO USE IN COMPUTATION OF D_f AND OF $(n-G)$

Only certain dates enter in the computation of D_f, as shown in the particular illustration. These are dates on which it is believed that there was sufficient melting to create a flood-flow. In order to decide which dates to use, it is necessary to assume values of C, M and T'', and compute r_1 for the stretch of dates near the beginning of the period of rapid melting. Whenever these estimated values of r_1 exceed G, the date is included in the computation of D_f.

Since Solution *AA* on Stream *A* was made after Solution *M*, the values of C, M and T'' used in Solution *M* could be used in estimating the dates to use in the computation of D_f. There were two modifications made in these values, however, the necessity for which became apparent after the completion of Solution *M* and its successors on Stream *A* up to Solution *AA*, and after completing Solution *X* on Stream *B*. The first modification consisted in reducing M from $+8.39$ to $+5.00$. The evidence in favor of a reduction in the size of that M was clear, and was of the following kinds: The values given in (62) were used in the computation of the stream-flow for Stream *A* which is shown on Plates 9 to 12, inclusive. In a study of these graphs, in connection with the computations in the least-square solutions, clear evidence was found that $M=+8.39$ is too large. Such evidence was of three kinds, as follows: (*a*) the rate of melting near the close of the season—during the supposed period of decrease of melting—as computed appeared to be too rapid to be plausible. In some cases it appeared to wipe out the whole of the remaining snow in a single day, whereas it was known to last a week or more; (*b*), the observed curve of discharge is much less sensitive to winter thaws and early spring thaws than the computed curve of discharge based upon $M=+8.39$. This can be verified from Plates 9 to 12, inclusive; (*c*), in the derivation of the above value of M the flood-flows were not taken into account by the flood coefficients, R'_f. On days of flood-flow the M was forced to do double duty, express the rate of melting and also the flood-flow. Hence its derived value came out too large.

It may be worth while to discuss the evidence mentioned in (*b*) more in detail. Attention is directed to the computed and observed stream-flow for the period November to February of each winter of 1912–13 to 1914–15 shown plotted on Plates 9 to 12 inclusive. On November 5 to 9 and 19 to 22, 1915 (Plate 12) the response of the observed discharge to temperature was very small, less than 0.005 c.f.s., and on the computed was about 0.030 c.f.s. This is in the first month of the freezing-melting period. There was over 1.0 inch (of water equivalent) of snow on the ground. In September and October the two responses corresponded well.

In November to December 1914 (Plate 11) the observed discharge shows more response than the computed, though the two responses corresponded fairly well in October 1914. The long decided freeze, November 28 to December 18, with a minimum temperature of +3° F. on December 12, produced a decided drop in the observed discharge, down to a minimum of 0.041 c.f.s. on December 16. It produced only a very slight drop in the computed discharge—perhaps one-fifth as large—down to a minimum of 0.087 c.f.s. on December 13. At this time, up to December 10, there was less than 0.5 inch of snow on the ground. A similar freeze in December–January 1915 (Plate 12) produced about equal moderate responses in the observed and computed discharge.

In November–December 1913 (Plate 10), and January–February 1914 (Plate 11) the response of the observed discharge is clearly less than that of the computed discharge. This also seems to be true, to a less extent possibly, of November 1912 to January 1913, and January–February 1912.

The proper conclusions from this evidence seems to be: (1) that in November to February, a steady freezing period, the observed discharge is somewhat less responsive in general than the computed discharge, and (2) that the observed discharge, very low, in December 15, 16, 17, 1914, is an abnormal, non-typical case, which has no counterpart on Stream *B* (Plate 16).

This evidence is clearly in favor of a reduction in the derived value of *M*, +8.39.

The decision to reduce *M* from +8.39 to +5.00 as based upon the evidence (*a*), (*b*) and (*c*) was made from a study of the computations and graphs on Stream *A*. Subsequent to that decision, a value of +5.04 was obtained in Solution *X* on Stream *B* in which the flood-coefficients referred to in (*c*) were used. This fourth piece of evidence, not connected in any way with that from Stream *A*, was a strong corroboration of the conclusions reached on Stream *A*.

In Solution *X* on Stream *B*, the derived value of *C* was +3.97, which was much less than the value +6.40 used in Solution *M*. It was believed at that time that this reduction in *C* was caused by the introduction of the flood-coefficients into Solution *X*, which would theoretically make it nearer the truth. Hence in estimating the dates to use in the computation of D_f in Solution *AA*, Stream *A*, *C* was assumed equal to +4.0 and *M*, +5.0. Using these values and the best value of T'', viz, 28° F., obtained up to that time, a preliminary computation of r_1, and from that, $(n_1 - G_1)$, $(n_2 - G_2)$, . . . , was made as shown on page 184.

This computation was begun on March 29 because previous to that time $(t - T'')$ was largely negative, hence no flood-flow could be produced. The values in the second to fifth columns were computed in a manner analogous to that already described in detail in the illustration of the computation of net melting and reduced rate of melting, pages 159 and 160, except that *F* was not used, inasmuch as a flood-flow can not be initiated by melting when $t < T''$. The unit in all but the second and last columns is 0.01 inch of depth. The last column is in 0.1 inch of depth, and the second column is in °F. The values of r_1 in the sixth column are the sums of the values of net melting in the fifth column and the values of n_1 shown in the computation of D_n, page 180,[1] and *G* is obtained from the tabulation in Table 46 in con-

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[1] By Eq. (41), page 141.

Example of computation of (n_1-G_1), (n_2-G_2), ... *Solution AA, Stream A*

Date	$(t-T'')$	$(t-T'')M$	C	Estimated net melting	r_1	G	r_{f1}	(n_1-G_1)	(n_2-G_2)	(n_3-G_3)	(n_4-G_4)	(n_5-G_6)
1913												
Mar. 29	+ 3	+15	+4	+19	+13	51	.....	....	....	....	....	....
30	+ 9	+45	+4	+49	+38	42	.....	....	....	....	....	....
31	+10	+50	+4	+54	+42	34	+ 8	−46	....	....	....	....
Apr. 1	+ 8	+40	+4	+44	+37	23	+14	−30	−46	....	....	....
2	+ 4	+20	+4	+24	+17	23	.....	....	−30	−46	....	....
3	− 6	....	....	....	.....	....	.....	....	....	−30	−46	....
4	− 2	....	....	....	.....	....	.....	....	....	....	−76	....
5	+ 6	+30	+4	+34	+22	31	.....	....	....	....	−30	−4.6
6	+10	+50	+4	+54	+47	23	+14	−30	....	....	....	−7.6
7	+ 3	+15	+4	+19	+12	22	.....	....	−30	....	....	−7.6
8	− 3	....	....	....	.....	....	.....	....	....	−30	....	−7.6
9	− 8	....	....	....	.....	....	.....	....	....	....	−30	−3.0
10	− 7	....	....	....	.....	....	.....	....	....	....	−30	....
11	+ 1	+ 5	+4	+ 9	0	35	.....	....	....	....	....	−3.0
12	+ 5	+25	+4	+29	+17	36	.....	....	....	....	....	−3.0
13	+ 9	+45	+4	+49	+36	24	+12	−37	....	....	....	−3.0
14	+10	+50	+4	+54	+37	22	+15	−39	−37	....	....	−3.0
15	+12	+60	+4	+64	+44	21	+23	−41	−39	−37	....	....
16	+12	+60	+4	+64	+47	21	+26	−38	−41	−39	−37	....

nection with the observed flow, D'. These can be verified from Plate 10. Thus on March 30, D' was 138, which gives a value of 34 for G on March 31. Since the estimated r_1, +42, for that date is greater than G, the melting was large enough to initiate a flood-flow. Hence March 31 becomes one date to use in the computation of D_f, and the value of (n_1-G_1) for that date, −46, is the sum of n_1 shown on page 180, −12, and the value of $-G$ estimated above, −34. This becomes the value of (n_2-G_2) for the following date, (n_3-G_3) for the next following date, and enters into the values of (n_4-G_4), (n_6-G_5), . . . for the succeeding dates according to the grouping of days in the definitions of n_1, n_2, n_3, . . . and r_1, r_2, r_3, ...

Each date on which the estimated value of r_1 is greater than G for that date is used in the computation of (n_1-G_1), (n_2-G_2), . . . which values are shown in the computation of D_f, page 181.

It is important to note that the assumed values of C, M and T'' enter only *indirectly* into the computations of this Solution *AA*. They serve merely to fix certain dates to be used in that solution. If, after the solution is completed, the derived values of C, M and T'' from it check the dates estimated as above, no error whatsoever is introduced into the derived C, F, M and T'' from the solution as a result of the assumed values of C, F, M and T''. If they do not verify those dates, it becomes necessary to estimate the error produced by using the wrong dates, and, if large, to repeat the process until the desired degree of accuracy is attained.

Up to this point in the illustrations of the computations used in Solution *AA*, there has been shown the method of computing $-D$, the absolute term of the observation equation (46). Illustrations of the method of computing the other known quantities x, y and z will now be given.

EXAMPLE OF COMPUTATION OF x_n SOLUTION *AA*, STREAM *A*

The definition of x_n is given by equation (50). It consists of the known quantities, the R''s and the time intervals involved in those R''s. Substituting in equation (50) the known values of R' from equation (76) and the known time intervals involved, there is obtained:

$$\left.\begin{aligned} x_n C &= +(0.28)(1)(C)+(0.21)(1)(C)+(0.14)(1)(C)+ \\ &\quad +(0.109)(2)(C)+(0.84)(0.4)(C)+(0.69)(0.8)(C)+ \\ &\quad +(+0.62)(1.6)(C)+(0.58)(3.2)(C)+(4.40)(0.64)(C) \\ &= (+0.28+0.21+0.14+0.218+0.336+0.552+ \\ &\quad +0.992+1.856+2.816)\ C \\ &= +7.40\ C, \text{ or} \end{aligned}\right\} \tag{88}$$

$$x_n = +7.40, \text{ retaining three significant figures} \tag{89}$$

The unit of $R'\Sigma(C)$ is 0.001 c.f.s.

In the computation of x_n the term $R'_{10}\Sigma(C)$ or (0.81)(1.28)(C) is neglected for the reason, as before, that this term reached back into the preceding summer and fall of 1912, before the beginning of the freezing-melting period. For the nine terms involved in the computation of $x_n = +7.40$, there was an abundance of ice and snow available for melting on every day concerned, even in the $R'_9\Sigma_9(C)$-term, with the possible negligible exception of some days in November 1912, hence the value of x_n may be taken as a constant for all of the observation equations involved in Solution *AA*.

EXAMPLE OF COMPUTATION OF x_f AND OF x, SOLUTION *AA*, STREAM *A*

The definition of x_f is given by equation (51). It consists of the known quantities, the R'_f's and the time intervals involved in them. The values of R'_{f1}, R'_{f2}, R'_{f3} . . . are given in equation (87). Before it is possible to substitute these and their time intervals in equation (51), it is necessary to compute the $\Sigma(C)$'s of that equation. These will not be the same for every day, as was the case with x_n, because equation (51) is applicable only on days on which flood-flows were initiated. The method of evaluating these $\Sigma(C)$'s will be evident from the following tabulation:

Example of computation of $\Sigma_1(C)$, $\Sigma_2(C)$, $\Sigma_3(C)$, . . .

Date	$\Sigma_1(C)$	$\Sigma_2(C)$	$\Sigma_3(C)$	$\Sigma_4(C)$	$\Sigma_5(C)$	$\Sigma_6(C)$	$\Sigma_7(C)$
1913							
Mar. 31	1	....	....	....	....	....	....
Apr. 1	1	1	....	....	....	....	....
2	....	1	1	....	....	....	....
3	....	....	1	1	....	....	....
4	....	....	....	2	....	....	....
5	....	....	....	1	0.1	....	....
6	1	....	....	....	0.2	....	....
7	....	1	....	....	0.2	....	....
8	....	....	1	....	0.2	....	....
9	....	....	....	1	0.1	0.1	....
10	....	....	....	1	....	0.2	....
11	....	....	....	....	0.1	0.2	....
12	....	....	....	....	0.1	0.2	....
13	1	....	....	....	0.1	0.2	....
14	1	1	....	....	0.1	0.2	....
15	1	1	1	....	....	0.3	....
16	1	1	1	1	....	0.3	....

The dates used in this computation are the same as those used in the computation of D_f. The unit is 1° F. for $\Sigma_1(C)$ to $\Sigma_4(C)$ inclusive, and 10° F. thereafter. The method of computing the $\Sigma(C)$'s for flood dates is apparent. It consists of summing up the coefficients of C in the various terms, with proper attention to the change in decimal place after $\Sigma_4(C)$. Into the tabular values above, the R'_f's of equation (87) are multiplied, giving the coefficients of C in equation (51). This computation is carried out below.

Example of computation of x_f and x

Date	$R'_f\Sigma_1(c)$	$R'_{f2}\Sigma_2(c)$	$R'_{f3}\Sigma_3(c)$	$R'_{f4}\Sigma_4(c)$	$R'_{f5}\Sigma_5(c)$	$R'_{f6}\Sigma_6(c)$	$R'_{f7}\Sigma_7(c)$	x_f	x_n	x
1913	+1.6	....	....	....	....	....	....	+1.60	+7.40	+ 9.00
Mar. 31	+1.6	+1.7	....	....	....	....	....	+3.30	+7.40	+10.70
Apr. 1	....	+1.7	+1.9	....	....	....	....	+3.60	+7.40	+11.00
2	....	....	+1.9	+1.6	....	....	....	+3.50	+7.40	+10.90
3	....	....	....	+3.2	....	....	....	+3.20	+7.40	+10.60
4	....	....	....	+1.6	+0.89	....	....	+2.49	+7.40	+ 9.89
5	+1.6	....	....	....	+1.78	....	....	+3.38	+7.40	+10.78
6	....	+1.7	....	....	+1.78	....	....	+3.48	+7.40	+10.88
7	....	....	+1.9	....	+1.78	....	....	+3.68	+7.40	+11.08
8	....	....	....	+1.6	+ .89	+0.45	....	+2.94	+7.40	+10.34
9	....	....	....	+1.6	....	+ .90	....	+2.50	+7.40	+ 9.90
10	....	....	....	....	+ .89	+ .90	....	+1.79	+7.40	+ 9.19
11	....	....	....	....	+ .89	+ .90	....	+1.79	+7.40	+ 9.19
12	+1.6	....	....	....	+ .89	+ .90	....	+3.39	+7.40	+10.79
13	+1.6	+1.7	....	....	+ .89	+ .90	....	+5.09	+7.40	+12.49
14	+1.6	+1.7	+1.9	....	....	+1.35	....	+6.55	+7.40	+13.95
15	+1.6	+1.7	+1.9	+1.6	....	+1.35	....	+8.15	+7.40	+15.55
16										

In the second to eighth columns, inclusive, are written the products $R'_f\Sigma(C)$ as shown in the headings, divided by C. The unit of the products $R'_f\Sigma(C)$ is 0.001 c.f.s. The sums of these values gives x_f, which, added to x_n of equation (89) in accordance with equation (47), give x, the coefficient of C in the observation equation (46).

Note that preceding the first day of flood-flow, March 31, $x_f=0$, therefore for those dates $x=x_n=+7.40$, and constant.

EXAMPLE OF COMPUTATION OF y_n, SOLUTION *AA*, STREAM *A*

The definition of y_n is given by equation (52). It consists of the known quantities R'_1, R'_2, R'_3, . . . and $(t-T''')$, $\Sigma_2(t-T''')$, $\Sigma_3(t-T''')$, . . . in which $t<T'''$. These last quantities are first computed from the mean observed air temperature, t, and the best value to date of T''', viz, 28° F., and then multiplied by R'_1, R'_2, R'_3, . . . of equation (76). The sum of the products gives y_n. This is illustrated as shown on the facing page.

The quantities in parentheses are values of $(t-T''')$, $\Sigma_2(t-T''')$, $\Sigma_3(t-T''')$, . . . The unit is 1° F. in columns 2 to 5 inclusive; 10° F. in columns 6 to 9 inclusive; and 100° F. in column 10. After computing the first column, $(t-T''')$, the others are derived from it in the same manner as in the case of the r's, the n's, the $(n-G)$'s, etc. The computation is restricted to negative values of $(t-T''')$.

The quantities not in parentheses at the head of each column except the last column are the R''s of equation (76). They should be considered as repeated down the column. The sum of the products $\Sigma(t-T''')R'$ gives y_n. The unit of the products $\Sigma(t-T''')R'$ is 0.001 c.f.s. In taking the products $\Sigma(t-T''')R'$, the value of $\Sigma(t-T''')$ was first rounded off to the nearest even unit before multiplying by R'. The individual products were then rounded off to the nearest even unit before taking their sums.

Since $y_f=0$ (because a flood-flow can not be started when $t<T''$), $y_n=y$, the coefficient of F in the observation equation (46).

Example of computation of $(t-T'')$, $\Sigma_2(t-T'')$, $\Sigma_3(t-T'')$, . . . *and of* y_n

Date	$(t-T'')R'_1$	$\Sigma_2(t-T'')R'_2$	$\Sigma_3(t-T'')R'_3$	$\Sigma_4(t-T'')R'_4$	$\Sigma_5(t-T'')R'_5$	$\Sigma_6(t-T'')R'_6$	$\Sigma_7(t-T'')R'_7$	$\Sigma_8(t-T'')R'_8$	$\Sigma_9(t-T'')R'_9$	y_n
1913										
Mar. 14.....	(−23) 0.28	(−17) 0.21	(− 8) 0.14	(− 6) 0.109	(− 1) 0.85	(− 8) 0.69	(−18) 0.62	(−38) 0.58	(− 7) 4.4	−83
15.....	(−18)	(−23)	(−17)	(−11)	(− 1)	(− 8)	(−17)	(−38)	(− 7)	−84
16.....	(−11)	(−18)	(−23)	(−25)	(− 1)	(− 6)	(−17)	(−37)	(− 7)	−81
17.....	(− 2)	(−11)	(−18)	(−40)	(− 2)	(− 5)	(−18)	(−36)	(− 7)	−78
18.....	....	(− 2)	(−11)	(−41)	(− 3)	(− 4)	(−18)	(−35)	(− 8)	−78
19.....	(− 6)	....	(− 2)	(−29)	(− 5)	(− 3)	(−18)	(−34)	(− 8)	−77
20.....	(− 9)	(− 6)	....	(−13)	(− 7)	(− 3)	(−19)	(−33)	(− 8)	−79
21.....	(− 6)	(− 9)	(− 6)	(− 2)	(− 7)	(− 3)	(−19)	(−33)	(− 8)	−79
22.....	....	(− 6)	(− 9)	(− 6)	(− 5)	(− 4)	(−19)	(−32)	(− 8)	−76
23.....	(− 9)	....	(− 6)	(−15)	(− 3)	(− 6)	(−20)	(−31)	(− 8)	−78
24.....	(−16)	(− 9)	....	(−15)	(− 2)	(− 8)	(−19)	(−32)	(− 8)	−82
25.....	(−21)	(−16)	(− 9)	(− 6)	(− 2)	(− 9)	(−17)	(−33)	(− 8)	−84
26.....	(−18)	(−21)	(−16)	(− 9)	(− 2)	(− 8)	(−15)	(−33)	(− 8)	−83
27.....	(− 8)	(−18)	(−21)	(−25)	(− 2)	(− 8)	(−13)	(−34)	(− 9)	−88
28.....	(− 3)	(− 8)	(−18)	(−37)	(− 2)	(− 8)	(−11)	(−34)	(− 9)	−85
29.....	....	(− 3)	(− 8)	(−39)	(− 3)	(− 9)	(−11)	(−34)	(− 9)	−82
30.....	....	....	(− 3)	(−26)	(− 5)	(− 8)	(−12)	(−33)	(− 9)	−79
31.....	....	....	....	(−11)	(− 6)	(− 5)	(−14)	(−34)	(− 9)	−78
Apr. 1.....	....	....	....	(− 3)	(− 6)	(− 4)	(−14)	(−34)	(− 9)	−77
2.....	....	....	....	....	(− 5)	(− 5)	(−13)	(−35)	(− 9)	−75
3.....	(− 6)	....	....	....	(− 3)	(− 7)	(−12)	(−36)	(− 9)	−78
4.....	(− 2)	(− 6)	....	....	(− 1)	(− 9)	(−12)	(−36)	(− 9)	−77
5.....	....	(− 2)	(− 6)	....	(0)	(− 9)	(−11)	(−36)	(− 9)	−75
6.....	....	....	(− 2)	(− 6)	....	(− 8)	(−12)	(−35)	(− 9)	−74
7.....	....	....	....	(− 8)	....	(− 8)	(−12)	(−34)	(− 9)	−74
8.....	(− 3)	....	....	(− 2)	(− 1)	(− 8)	(−11)	(−33)	(− 9)	−74
9.....	(− 8)	(− 3)	....	....	(− 1)	(− 7)	(−12)	(−31)	(−10)	−78
10.....	(− 7)	(− 8)	(− 3)	....	(− 1)	(− 5)	(−13)	(−30)	(−10)	−77
11.....	....	(− 7)	(− 8)	(− 3)	(− 1)	(− 3)	(−15)	(−30)	(−10)	−75
12.....	....	....	(− 7)	(−11)	(0)	(− 2)	(−17)	(−29)	(− 9)	−71
13.....	....	....	....	(−15)	(0)	(− 1)	(−17)	(−29)	(− 9)	−71
14.....	....	....	....	(− 7)	(− 1)	(− 1)	(−17)	(−29)	(− 9)	−71
15.....	....	....	....	....	(− 2)	(− 1)	(−15)	(−30)	(− 9)	−69
16.....	....	....	....	....	(− 2)	(− 1)	(−13)	(−31)	(− 9)	−69

EXAMPLE OF COMPUTATION OF z_n, SOLUTION AA, STREAM A

The definition of z_n is given by equation (54). It consists of the known quantities R'_1, R'_2, R'_3, . . . and $(t-T'')$, $\Sigma_2(t-T'')$, $\Sigma_3(t-T'')$ . . . , in which $t>T''$ and there is an abundance of snow and ice on and in the ground of the watershed available for melting. The last quantities are first computed from the mean observed air temperature over the watershed, t, and $T''=28°$ F., and then multiplied by the R''s of equation (76). The sum of the products gives z_n. This is illustrated as shown on page 189.

The quantities in parentheses are values of $(t-T'')$, $\Sigma_2(t-T'')$, $\Sigma_3(t-T'')$ . . . , restricted to days on which $t>T''$, and on which there is an abundance of ice and snow on and in the ground available for melting. The units are the same as in the computation of y_n. After computing $(t-T'')$, the other parenthetical values were computed from it as in previous analogous computations. The quantities not in parentheses, at the heads of the columns are the R''s of equation (76). They should be considered as repeated down the column. The sum of the products $\Sigma(t-T'')R'$ gives z_n. The unit of the products $R'\Sigma(t-T'')M$ is in 0.001 c.f.s. The rounding off of the numerical quantities is done as in the case of y_n.

Previous to March 31, there being no flood-flows, $z_n=z$, the coefficient of M in equation (46). From March 31 on, $z=z_n+z_f$. The illustration of the computation of z_f follows.

EXAMPLE OF COMPUTATION OF z_f AND z, SOLUTION AA, STREAM A

The definition of z_f is given by equation (55). It consists of the known quantities R'_{f1}, R'_{f2}, R'_{f3}, . . . and $(t-T'')$, $\Sigma_2(t-T'')$, $\Sigma_3(t-T'')$ . . . , confined to days affected by flood-flows, as in the case of x_f, and days on which there is an abundant supply of ice and snow on and in the ground available for melting. The quantities $(t-T'')$ and their sums are first computed for the same days decided upon in the computation of (n_1-G_1) and their sums, and then multiplied by the R'_f's of equation (87). This is illustrated on page 190.

The quantities in parentheses are values of $(t-T'')$ and their sums, restricted to days affected by flood-flows and on which there is an abundance of ice and snow available for melting. The units are the same as in the computations of y_n and z_n. The quantities not in parentheses, at the heads of the columns, except the last three columns, are the R'_f's of equation (87). The sum of the products $\Sigma(t-T'')R'_f$ gives z_f. In evaluating z_f, the quantities are rounded off as previously described for y_n. The unit of the products $R'_f\Sigma(t-T'')M$ is 0.001 c.f.s. The last column but one contains z_n copied from page 189. The last column, z, is the sum of the two preceding ones.[1] It is the coefficient of M in equation (46) on days affected by flood-flow.

This completes the illustrations of the computations of all the known quantities in equation (42) and the combining and grouping of them as far as possible to get them into the form of equation (46) according to equations (43) to (45) and (47) to (55) inclusive. The next step is to write out the observation equations for the

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[1] By Eq. (49), page 143.

Example of computation of z_n

Date	$(t-T'')R'_1$	$\Sigma_2(t-T'')R'_2$	$\Sigma_3(t-T'')R'_3$	$\Sigma_4(t-T'')R'_4$	$\Sigma_5(t-T'')R'_5$	$\Sigma_6(t-T'')R'_6$	$\Sigma_7(t-T'')R'_7$	$\Sigma_8(t-T'')R'_8$	$\Sigma_9(t-T'')R'_9$	z_n
1913										
Mar. 14.....	...	...	...	...	...	...	...	...	(+0.31)4.4	0
15.....	...	...	...	...	...	...	...	...	(+0.31)	0
16.....	...	...	...	...	...	...	...	...	(+0.25)	0
17.....	...	...	...	...	...	...	...	...	(+0.20)	0
18.....	(+ 1) 0.28	...	...	...	...	...	...	...	(+0.11)	0
19.....	...	(+ 1) 0.21	...	...	...	...	...	...	(+0.04)	0
20.....	...	...	(+ 1) 0.14	...	...	...	...	...	(+0.04)	0
21.....	...	...	...	(+ 1) 0.109	...	...	...	...	(+0.04)	0
22.....	(+ 2)	...	...	(+ 1)	...	...	...	...	(+0.04)	+ 1
23.....	...	(+ 2)	...	...	(+0.1) 0.85	...	...	...	(+0.04)	0
24.....	...	...	(+ 2)	...	(+0.1)	...	...	...	(+0.04)	0
25.....	...	...	...	(+ 2)	(+0.1)	...	...	...	(+0.04)	0
26.....	...	...	...	(+ 2)	(+0.1)	...	...	...	(+0.04)	0
27.....	...	...	...	...	(+0.2)	(+0.1) 0.69	...	...	(+0.04)	0
28.....	...	...	...	...	(+0.2)	(+0.1)	...	...	(+0.04)	0
29.....	(+ 3)	...	...	...	(+0.2)	(+0.1)	...	...	(+0.04)	+ 1
30.....	(+ 9)	(+ 3)	...	...	(+0.2)	(+0.1)	...	...	(+0.04)	+ 4
31.....	(+10)	(+ 9)	(+ 3)	...	...	(+0.3)	...	...	(+0.04)	+ 5
Apr. 1.....	(+ 8)	(+10)	(+ 9)	(+ 3)	...	(+0.3)	...	...	...	+ 5
2.....	(+ 4)	(+ 8)	(+10)	(+12)	...	(+0.3)	...	...	...	+ 5
3.....	...	(+ 4)	(+ 8)	(+19)	(+0.3)	(+0.3)	...	...	...	+ 4
4.....	...	...	(+ 4)	(+18)	(+1.2)	(+0.2)	(+0.1) 0.62	...	...	+ 4
5.....	(+ 6)	...	...	(+12)	(+2.2)	(+0.2)	(+0.1)	...	...	+ 5
6.....	(+10)	(+ 6)	...	(+ 4)	(+3.0)	(+0.2)	(+0.1)	...	...	+ 7
7.....	(+ 3)	(+10)	(+ 6)	...	(+3.1)	(+0.5)	(+0.1)	...	...	+ 7
8.....	...	(+ 3)	(+10)	(+ 6)	(+2.2)	(+1.2)	(+0.3)	...	...	+ 6
9.....	...	...	(+ 3)	(+16)	(+1.2)	(+2.2)	(+0.3)	...	...	+ 4
10.....	...	...	...	(+13)	(+1.0)	(+3.0)	(+0.3)	...	...	+ 4
11.....	(+ 1)	...	...	(+ 3)	(+1.6)	(+3.4)	(+0.3)	...	...	+ 4
12.....	(+ 5)	(+ 1)	...	...	(+1.9)	(+3.4)	(+0.3)	...	...	+ 5
13.....	(+ 9)	(+ 5)	(+ 1)	...	(+1.9)	(+3.4)	(+0.3)	...	...	+ 8
14.....	(+10)	(+ 9)	(+ 5)	(+ 1)	(+1.3)	(+4.0)	(+0.3)	...	...	+10
15.....	(+12)	(+10)	(+ 9)	(+ 6)	(+0.3)	(+4.7)	(+0.6)	...	...	+11
16.....	(+12)	(+12)	(+10)	(+14)	(+0.1)	(+4.1)	(+1.5)	...	...	+13

Example of computation of z_f, (and z)

Date	$(t-T'')R'_{f1}$	$\Sigma_2(t-T'')R'_{f2}$	$\Sigma_3(t-T'')R'_{f3}$	$\Sigma_4(t-T'')R'_{f4}$	$\Sigma_5(t-T'')R'_{f5}$	$\Sigma_6(t-T'')R'_{f6}$	$\Sigma_7(t-T'')R'_{f7}$	z_f	z_n	z
1913										
Mar. 31...	(+10)1.6	....	....	....	....	....	....	+16	+ 5	+21
Apr. 1...	(+ 8)	(+10)1.7	....	....	....	....	....	+30	+ 5	+35
2...	....	(+ 8)	(+10)1.9	....	....	....	....	+33	+ 5	+38
3...	....	....	(+ 8)	(+10)1.6	....	....	....	+31	+ 4	+35
4...	....	....	....	(+18)	....	....	....	+29	+ 4	+33
5...	....	....	....	(+ 8)	(+1.0)8.9	....	....	+22	+ 5	+27
6...	(+10)	....	....	....	(+1.8)	....	....	+34	+ 7	+41
7...	....	(+10)	....	....	(+1.8)	....	....	+35	+ 7	+42
8...	....	....	(+10)	....	(+1.8)	....	....	+37	+ 6	+43
9...	....	....	....	(+10)	(+0.8)	(+1.0)4.5	....	+29	+ 4	+33
10...	....	....	....	(+10)	....	(+1.8)	....	+25	+ 4	+29
11...	....	....	....	....	(+1.0)	(+1.8)	....	+18	+ 4	+22
12...	....	....	....	....	(+1.0)	(+1.8)	....	+18	+ 5	+23
13...	(+ 9)	....	....	....	(+1.0)	(+1.8)	....	+32	+ 8	+40
14...	(+10)	(+ 9)	....	....	(+1.0)	(+1.8)	....	+49	+10	+59
15...	(+12)	(+10)	(+ 9)	....	....	(+2.8)	....	+67	+11	+78
16...	(+12)	(+12)	(+10)	(+ 9)	....	(+2.8)	....	+86	+13	+99

group of dates used in the illustrations. They are written out as shown in the following tabulation.

Example of observation equations for determining C, F, M, and T″, Solution AA, Stream A

Date	Observation equation	Date	Observation equation
1913		1913	
Mar. 14........	$+8C-83F+0M-10=v_1$	Apr. 1........	$+11C-77F+35M-240=v_{19}$
15........	$+7C-84F+0M-9=v_2$	2........	$+11C-75F+38M-210=v_{20}$
16........	$+8C-81F+0M-4=v_3$	3........	$+11C-78F+35M-184=v_{21}$
17........	$+7C-78F+0M-7=v_4$	4........	$+9^*C-77F+33M-198=v_{22}$
18........	$+7C-78F+0M-9=v_5$	5........	$+8^*C-75F+27M-213=v_{23}$
19........	$+8C-77F+0M-8=v_6$	6........	$+10^*C-74F+41M-281=v_{24}$
20........	$+7C-79F+0M-10=v_7$	7........	$+10^*C-74F+42M-278=v_{25}$
21........	$+8C-79F+0M-8=v_8$	8........	$+10^*C-74F+43M-245=v_{26}$
22........	$+7C-76F+1M-13=v_9$	9........	$+9^*C-78F+33M-179=v_{27}$
23........	$+7C-78F+0M-13=v_{10}$	10........	$+9^*C-77F+29M-148=v_{28}$
24........	$+8C-82F+0M-11=v_{11}$	11........	$+9C-75F+22M-122=v_{29}$
25........	$+7C-84F+0M-8=v_{12}$	12........	$+9^*C-71F+23M-169=v_{30}$
26........	$+8C-83F+0M-8=v_{13}$	....	....
27........	$+7C-88F+0M-10=v_{14}$	15........	$+14C-69F+78M-532=v_{31}$
28........	$+7C-85F+0M-19=v_{15}$	16........	$+15^*C-69F+99M-633=v_{32}$
29........	$+8C-82F+1M-36=v_{16}$		
30........	$+7C-79F+4M-67=v_{17}$		
31........	$+9C-78F+21M-186=v_{18}$		

*These values differ slightly from those computed on page 186 because of a numerical error discovered in the computations after the completion of Solution AA. The computations on page 186 are correct.

These equations are written out in the form of equation (46). The coefficients of C, F and M or x, y and z, respectively, and the absolute term, $-D$, are copied from the computations of those quantities on pages 179 to 190, inclusive.

The 32 equations are part of the larger group of 137 equations used in Solution AA, Stream A, which served to fix the values of C, F, M and T'' derived from observations on that stream.

The observation equations were normally written in groups of a month apiece, and the normal equations formed for each group in the usual way. These normal equations were then combined by addition into a final set of normal equations.

FINAL NORMAL EQUATIONS, SOLUTION *AA*, STREAM *A*

The final normal equations for Solution AA for 137 days, formed from 137 observation equations like and including the above 32, are as follows:

$$\left.\begin{array}{l} +\ 9{,}119\,C-\ 76{,}890\,F+13{,}996\,M-105{,}228=0 \\ -76{,}890\,C+705{,}001\,F-69{,}758\,M+585{,}775=0 \\ +13{,}996\,C-\ 69{,}758\,F+72{,}774\,M-474{,}246=0 \end{array}\right\}\dots\dots\dots\dots(90)$$

This final set of normal equations for Solution AA (90) depends upon 137 observation equations on the following dates: February 14—April 19, 13—May 1, 3–6, 1912, and February 14–April 1, 4–12, 15–16, 1913, inclusive.

The solution of the final normal equations for Solution AA, (90), gives the following values for the unknowns:

$$C=+8.59\pm1.16 \qquad F=+0.647\pm0.117 \qquad M=+5.49\pm0.123$$

The probable errors shown were computed rigorously from the normal equations (90), and from the residuals, v, of the 137 observation equations of Solution AA.

THE TWO FINAL SOLUTIONS FOR DETERMINING C, F, M AND T''

In this stream-flow investigation, made by methods as outlined in steps (1) to (6) on pages 146 and 147, several separate least-square solutions were made on the basis of each group of observational data. The form of each successive solution from one group of data was based upon all the information available up to the time that that form was adopted, including the information from earlier solutions based upon the same data.

There was a total of 34 least-square solutions made for determining the values of C, F, M and T'', 18 on Stream A and 16 on Stream B. The two final solutions for determining numerical values of these unknowns were those designated as Solution AA on Stream A, and Solution X on Stream B. Each of these solutions was the culminating one on each stream based upon the same, or nearly the same, group of observational data in each case. The principal facts from these two solutions are shown in Table 49.

TABLE 49—*Principal facts from the two final freezing-melting solutions*

	Solution AA—Stream A	Solution X—Stream B
No. of days of observation used in the solution	137	151
No. of observation equations	137	151
Computed value of C	+ 8.59	+ 3.97
Computed probable error of C	± 1.16	± 0.71
Computed value of F	+ 0.647	+ 0.403
Computed probable error of F	± 0.117	± 0.062
Computed value of M	+ 5.49	+ 5.04
Computed probable error of M	± 0.123	± 0.083
Assumed value of T''	28° F	28° F
Probable error of a single observation	± 12.4	± 6.3

The probable errors of C, F and M as shown were computed rigorously from the normal equations and the residuals of the observation equations.

The probable error of a single observation as shown above is the probable error, computed rigorously from the normal equations and the residuals, of the computed average stream-flow of the stream at the point of measurement during one day.

Table 50 shows the best values of C, F and M determinable from the observations on Streams A and B in the winters of 1912 and 1913. The values of C, F and M and their probable errors in this table are the same as in Table 49. From these are computed the weights assigned to each unknown, the weighted mean value and its probable error, and the residuals of each value from its own weighted mean. The assigned weights are inversely proportional to the squares of the respective probable errors, corresponding to the assumption that all errors in the computed values of these constants are of the accidental character. Unit weight corresponds to a probable error squared of 0.1.

The residual of the value of C in Solution AA, viz, -3.36, is over five times the probable error of the weighted mean C from both solutions, $(5\times0.61=3.0)$, therefore that value of C, $+8.59\pm1.16$, is rejected.

With the rejection stated in the preceding paragraph, the following adopted values of C, F, and M and T'' are the best that can be derived from the present investigation:

$$\left.\begin{array}{l} C=+3.97 \quad \pm 0.71 \\ F=+0.457 \pm 0.055 \\ M=+5.19 \quad \pm 0.07 \\ T''=28^\circ \text{ F} \end{array}\right\} \dots\dots\dots\dots (91)$$

These final values agree so closely with the values used in estimating the dates to use in computing D_f, that little or no change in those dates would be caused by using these more exact values. Hence the only assumptions on which these values depend directly are Assumptions Nos. (9), (10), (11) and (12).

The freezing-melting curve, based on these values, is shown plotted in the lower left-hand corner of Plate 22.

The unknown T'' can not be determined directly from the observations. It can be determined only by trial. The value at first was assumed at 30° F., with which value the derived value of C was positive and real, hence in the next least-square solution, the corrected value of $T''=T''-\frac{C}{M}$, which came out 29° F., was used. With this value of T'', C still came out positive on both streams and, after a careful study of the residuals, proof was obtained that $T''=29^\circ$ F. was too large. The final reduction to 28° F. was accordingly made. The evidence as to whether $T''=28^\circ$ F. is still too large is weak and inconclusive. Note the rather poorly determined value of C above. The true value of T'' may be 27° F. The true value would make C zero, or it would come out poorly determined—that is, with a large probable error, which would give it the character of an accidental error.

TABLE 50—*Best values of C, F and M*

	Probable error	Assigned weight	Value of—	Residual	Sum of weights	Weighted mean	Probable error of weighted mean
Best value of C							
Solution *AA*, Stream *A*.....	±1.16	0.0741	C+8.59*	−3.36	0.271	+5.23	±0.61
Solution *X*, Stream *B*......	±0.712	0.197	+3.97†	+1.26			
Best value of F							
Solution *AA*, Stream *A*.....	±0.117	7.30	F +0.647	−0.190	33.3	+0.457	±0.055
Solution *X*, Stream *B*......	±0.062	26.04	+0.403	+0.054			
Best value of M							
Solution *AA*, Stream *A*.....	±0.123	6.62	M+5.49	−0.30	21.1	+5.19	±0.07
Solution *X*, Stream *B*......	±0.083	14.51	+5.04	+0.15			

* Rejected. † Adopted best value of *C*.

The evidence external to the least-square solutions indicated that the true value of T'' is about 28° F. This consisted of a study of the daily and hourly temperatures and the daily and hourly discharges for February to May 1912 and in the spring of 1913 on Stream *B*.

Using the adopted values (91), equation (36), expressing the net melting on any day in terms of the mean air temperature, may be written

$$\left.\begin{array}{l}\text{Net melting on any}\\ \text{day in 0.01}\\ \text{inch of depth}\end{array}\right\}=+3.97+\left\{\begin{array}{c}-(28^\circ-t)\ 0.457\\ \text{or}\\ +(t-28^\circ)\ 5.19\end{array}\right\} \dots\dots\dots\dots (92)$$

from which it is seen that when the mean temperature is 28° F., and there is an abundant amount of snow and ice on and in the ground available for melting, there is a constant addition to the storage of 0.0397 inch of depth per day. For each day for each degree that the mean temperature for the day is less than 28° F., there is a subtraction of water from storage due to freezing at the rate of 0.00457 inch of depth per day. For each day for each degree that the mean temperature for the day is greater than 28° F., there is an addition to storage by melting at the rate of 0.0519 inch of depth per day. The rate of addition to storage by melting is therefore about 11 times greater than the rate of subtraction therefrom by freezing $\left(\frac{5.19}{0.457}=11.4\right)$.

The critical temperature $T'''=28°$ F. is less than 32° F., the freezing point, by somewhat less than the daily range of temperature, because a rapid melting during a few hours must be offset by slow freezing during many night hours if the net melting is to be zero for the 24 hours.

Subsequent to the derivation of the above values, it was interesting to find that a value for M had been obtained by Mr. Robert E. Horton (*The Melting of Snow*, Weather Review, vol. 43, 599) by an entirely different method. Horton melted snow in the laboratory and measured the rate of accumulation of melted snow at the base of the cylinder. He concludes that the rate of melting is from 0.04 to 0.06 inch per 24-hours per degree F. Incidentally, the fact that Horton's value for the rate of melting—obtained by direct observation—agrees well with the value obtained by the methods used in this investigation, tends to vindicate the use of assumption No. (10), especially, since upon the validity of that assumption, more than any other, rests the determinability of M by this method.

From the nature of the derivation of equation (92), and because of the units in which it is expressed, it should apply to any watershed anywhere in the world, hence in applying this method to other streams, the labor involved in deriving equation (92) probably need not be repeated.

SUMMARY OF CONSTANTS DERIVED IN STREAM-FLOW INVESTIGATION

For convenience in reference for further discussion, all of the constants derived on the stream-flow investigation on Streams A and B up to this point are summarized in Table 51. These represent all the constants derived in the stream-flow study except the frequency-ratio constants shown in the last column of Table 66, page 218.

ACCURACY OF COMPUTED NORMAL STREAM-FLOW

In Table 51 are shown the various normal-flow constants S_c, R'_1, R'_2, R'_3 . . . R'_{10} for Streams A and B, both in absolute amount and converted to percentages, with the probable errors. These probable errors are a measure of the accuracy which is the best that can be obtained, provided the errors in the derived constants are all accidental in character. Assuming that the errors are all accidental, that is, that there are no systematic or constant errors affecting the results, it is an even chance that the actual error in any constant is greater than or less than its computed probable error as shown. For example, on Stream A, $R'_{10}=0.0087$, expressed in per cent, and its probable error is ± 0.0022. This means that the chances are even for and against the proposition that the true value of R'_{10} for Stream A lies within

TABLE 51—*Summary of constants*

Constant	Normal flow constants				Constant	Flood flow constants			
	Stream *A*		Stream *B*			Stream *A*		Stream *B*	
	Absolute value	Expressed as percentage	Absolute value	Expressed as percentage		Absolute value	Expressed as percentage	Absolute value	Expressed as percentage
S_c	+108 ±4.0	....	+96 ±4.0	....	....	.....	....	....	....
R'_1	+ 0.28 ± .05	0.30 ±0.05	+ 0.060 ± .021	0.071 ±0.025	R'_{f1}	+1.6	1.7	+1.40	1.67
R'_2	+ 0.21 ± .06	0.22 ± .06	+ 0.071 ± .024	0.084 ± .029	R'_{f2}	+1.7	1.8	+1.43	1.70
R'_3	+ 0.14 ± .06	0.15 ± .06	+ 0.082 ± .031	0.098 ± .037	R'_{f3}	+1.9	2.0	+1.61	1.92
R'_4	+ 0.109 ± .035	0.117 ± .037	+ 0.079 ± .017	0.094 ± .020	R'_{f4}	+1.6	1.7	+1.64	1.95
R'_5	+ 0.84 ± .17	0.090 ± .018	+ 0.790 ± .088	0.094 ± .010	R'_{f5}	+8.9	1.0	+14.0	1.66
R'_6	+ 0.69 ± .11	0.074 ± .012	+ 0.715 ± .054	0.085 ± .006	R'_{f6}	+4.5	0.48	+8.24	0.98
R'_7	+ 0.62 ± .09	0.066 ± .010	+ 0.496 ± .029	0.059 ± .003	R'_{f7}	+1.3	0.14	+3.11	0.37
R'_8	+ 0.58 ± .03	0.062 ± .003	+ 0.269 —	0.032 —	....	.....	....	....	....
R'_9	+ 4.44 ±1.5	0.047 ± .015	+ 0.403 —	0.0048 —	....	.....	....	....	....
R'_{10}	+ 0.81 ± .21	0.0087 ± .0022	+ 0 210 —	0.0025 —	....	.....	....	....	....
Total	....	9.0 *p. ct.*	....	4.1 *p. ct.*		....	19.0 *p. ct.*	....	29.6 *p. ct.*

Freezing-melting constants	Values of G on current day for discharges, D', on preceding day			
	Discharge, D', preceding day (Unit=0.001c.f.s.)	G on current day (Unit=0.01 in.)	Discharge, D', preceding day (Unit=0.001c.f.s.)	G on current day (Unit=0.01 in.)
$C=+3.97 \pm 0.71$ $F=+0.457 \pm 0.055$ $M=+5.19 \pm 0.07$ $T''=28°$ F. $\frac{E_l}{E_w}=2.6$	40	66	150	28
	50	65	160	25
	60	63	170	24
	70	60	180	23
	80	55	190	23
	90	49	200	23
	100	43	210	23
	110	40	250	22
	120	38	300	21
	130	36	350	21
	140	34	400 and over	20

0.0022 of 0.0087. In other words, there is one chance in two that the value 0.0087 is correct within one-fourth of itself $\left(\frac{0.0022}{.0087}=\frac{1}{4}\right)$.

Of the constants of Stream *A*, three are unreal in the sense that they are less than 3.5 times their own probable errors, viz, R'_3, R'_4, and R'_9. Of the constants of Stream *B*, R'_1, R'_2, and R'_3 are similarly unreal. The extreme uncertainty in any constant for which the probable error has been computed or estimated, judged by the probable errors, occurs in the constant R'_3 for Stream *B*, of which the value is 0.098 expressed as a percentage, only 2.65 times its own probable error, ±0.037. On the supposition that all the errors are accidental in character, there is about one chance in thirteen that this derived value, 0.098 for R'_3 for Stream *B*, is entirely fictitious.

The most accurately determined of all the constants is M of which the value is +5.19, with a probable error of ±0.07, only 1/74th or 1.3 per cent of itself. This means that on any day on which the mean air temperature on the watershed is greater than 28° F., and there is an abundance of snow and ice available for melting, there is one chance in two that the computed rate of addition to the stored water in the ground due to melting is correct within 1.3 per cent, if one regard the observed mean temperature as exact and assume that all the errors are accidental in character.

Suppose one assumed that on a given day there is a rain or net melting sufficient to produce a gain in storage, and that thereafter for 256 days there is no change of storage on any day, the rainfall or net melting on each day being just sufficient to equal the evaporation plus stream-flow. For Stream A, referring to Table 51, the maximum part of the change in storage on the current day which reaches the stream subsequently in any time interval involved in the definitions of the r's is 3.0 per cent for the 64 days included in the 65th to 128th days, inclusive, after the change in storage takes place $(0.047 \times 64 = 3.0)$. This is represented by the R'_9 constant of which the probable error is ±0.016 or ±34 per cent, hence it may be said that the total computed extra stream-flow during that time interval is in error ±34 per cent. This would be true if the error persisted with one sign through that 64-day interval. It is extremely unlikely that it would. It would tend to obey the law of accidental distribution of errors, hence the error would be much less than ±34 per cent. The phrase "extra stream-flow" is intended to mean that part of the flow which is an excess above the constant part of the flow, S_c.

On Stream B, still assuming a gain in storage followed by no change for 256 days, the maximum part of the gain in storage on the current day which reaches the stream subsequently in any time interval involved in the definitions of the r's is 1.0 per cent for the 32 days included in the 33d to 64th days, inclusive, after the change in storage takes place $(0.032 \times 32 = 1.0)$. This is represented by the R'_8 constant of which the probable error may be assumed the same as for Stream A, viz, ±0.003, or ±9 per cent, which would be the error in the total computed excess stream-flow during this time interval under the assumed conditions if the error persisted throughout the interval with the same sign, which would be unlikely, causing the actual error to be less.

In the actual case in nature in which a gain or loss in storage is not followed by constant conditions thereafter, the probable error of the computed excess flow on any day would be a complicated function of the probable errors of the separate constants. As a measure of the over-all accuracy of the total computed flow, the excess flow plus the constant part of the flow, the probable error of a single observation may be used, which for Stream A is ±8.6, and for Stream B, ±8.4. These errors are ±8.0 per cent and ±8.7 per cent of the constant part of the flow for each stream, respectively $\left(\frac{8.6}{108} = 0.080. \quad \frac{8.4}{96} = 0.087\right)$. Therefore, according to this test, on any day on which the total stream-flow is at or near the constant part of its flow, there is one chance in two that the error in the total computed flow of Stream A is less than 8 per cent and of Stream B 9 per cent.

The most convincing test of the accuracy of the computed normal stream-flow is given subsequently in tabular form and by graphs.

ACCURACY OF THE COMPUTED FLOOD-FLOW

The flood-flow constants of Streams *A* and *B* are shown summarized in the right-hand side of Table 51. The probable errors for these constants were not computed. During times of flood, the changes in stream-flow are large and rapid, as contrasted with the small changes in rate of flow which normally take place during flow fed by percolation only. For this reason it is certain that the accuracy of the derived flood-flow constants is of a much higher order than that of the normal flow constants. For instance, if one assume that on Watershed *B* a rain heavy enough to supersaturate the ground occurs, followed by no rain large enough to supersaturate the ground for a period of 32 days thereafter, 0.37 per cent of the gain in storage produced by this rain will be delivered to Stream *B* on each of the 17th to 32d days, inclusive, thereafter. This is represented by the constant $R'_{f7} = +3.11$, expressed as a percentage. The error in this 0.37 per cent is probably much less than ±5 per cent, the error in the corresponding normal-flow constant, R'_7 $\left(\frac{0.003}{.059} = 0.05\right)$.

As another test of the accuracy of the computed flood-flow, consider the probable error of a single observation. On Stream *B*, the probable error of a single observation, computed rigorously from the normal equations and from the residuals of the 166 observation equations from which the flood-flow constants in Table 51 were derived, was ±20.0. This includes the probable error of a single observation of the normal stream-flow, ±8.4. This means that on any day picked at random, on which there is a flood-flow, the chances are equal for and against the proposition that the total computed flow (normal+flood) will be in error by less than 20.0 or 0.020 c.f.s.

The probable errors used as a basis for the foregoing discussion of the accuracy of the computed normal stream-flow and of the computed flood-flow are obviously too small to represent the truth. The approximate theory based upon the approximate assumptions would necessarily introduce systematic errors into the computations, the effects of which, upon the probable errors, would be difficult to estimate. It will be shown later in the discussion of rainfall, net melting, evaporation and run-off on Stream *B*, that the indicated accuracy of determination of the melting constant, *M*, is probably fictitious to some extent.

TESTS OF OVERALL ACCURACY OF COMPUTED STREAM-FLOW

If one insert the constants S_c, R'_1, R'_2, R'_3, . . . R'_{10}, R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7}, C, F, M, T''' and $\frac{E_l}{E_w}$ as derived separately for Streams *A* and *B* into the general formulas, equations (33), (34), (36) and (37), the resulting specific formulas are those for which it is claimed that the discharges of Streams *A* and *B* may be computed for any day provided the necessary meteorological observations are available. As a test of the theory, and of the accuracy of the derived constants, the discharges on Streams *A* and *B* have been computed from the formulas for each day from March 1, 1911, to December 31, 1915, and the computed stream-flow for each day has been compared with the observed stream-flow.

The comparison of computed stream-flow with observed stream-flow will be given in such form as to help the reader to reach his own conclusions on certain pertinent questions. Are the formulas with the constants in them mere interpolation formulas which reproduce fairly well the observed daily discharges of the period used in computing the constants, or, on the other hand, do these formulas represent the facts of nature and are the derived constants *real physical constants* such that for these streams they express the relations actually existing at all times between the meteorological facts and the discharge of the stream?

Obviously in making these comparisons one can and should distinguish between the months of observations of stream-flow that were used in computing the constants on the one hand, and the other months of observations which did not enter into the computations at all on the other hand.

For Stream *A*, the computed constants R'_1, R'_2, R'_3, . . . R'_9 depend upon the 8 months, August to September 1912, June to September 1913.

S_c to R'_{10} depend upon the 19 months July to December 1911, January to March and August to December 1912, and January to February and August to December 1913.

R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7} depend on little more than two months, October 4 to November 4, 1911, just after the greatest October rain that occurred within the period of observation, and April 8 to May 9, 1912, in the early part of a spring flood.[1]

C, *F* and *M* depend upon the six months, February to April 1912, and February to April 1913. In computing them, observations on Stream *B* were used still more than those on Stream *A*.[2]

G depends upon an inspection of all the observations on Stream *A* in the three years, 1911–12–13.[3]

E_l/E_w depends upon the three years 1911–12–13. The value used on Stream *A* was 2.3, instead of the more correct value 2.6, derived later.

For Stream *B*, the computed constants R'_1, R'_2, R'_3, . . . R'_7 depend upon 193 selected observations in the 18 months March, April, June to October 1911, May, July, August 1912, and April to November 1913.

S_c, R'_8, R'_9 and R'_{10} depend upon the 31 months March 1911 to December 1913 except for May 1911 and May and June 1912.

R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7} depend upon 166 days of observation in the following months: October to November 1911, June to August 1912, June to August 1913.

C, *F* and *M* depend upon 151 days of observation in February to May 1912 and February to April 1913. These were the values of Solution *X*, Stream *B*,

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[1] The values of R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7} here referred to are not those shown in Table 51. The values in Table 51 are believed to be much closer to the truth than the values here referred to. The values here referred to were the ones actually used in computing the flood-flow on Stream *A*, and are as follows:

$R'_{f1} = +0.82$ $R'_{f4} = +0.85$ $R'_{f7} = +1.87$
$R'_{f2} = +1.00$ $R'_{f5} = +6.17$
$R'_{f3} = +1.04$ $R'_{f6} = +3.83$

Compare these values with those for Stream *A* in Table 51. There is considerable difference, which is believed to be due to the use of the approximate and large value of the melting constant $M = +8.39$ instead of the latest and best value derived, +5.19.

[2] The values here referred to are those in equations (62) and (63). It is certain that the accuracy on Stream *A* would be much greater if the final values, equation (91), were used in connection with the R'_f's of Table 51.

[3] The values here referred to are not the values of *G* in Table 51. Those values were obtained from an inspection of all the observations on both Stream *A* and *B* in the three years 1911–12–13. The values referred to above, obtained on Stream *A* alone, are as follows:

D' on preceding day	*G*
85 or less	.60
86 to 95	.50
96 to 105	.40
106 to 135	.30
136 and more	.20

shown in Table 49. Note the close correspondence of these values with the final values (91).

G depends upon all the observations on both streams in 1911–12–13.

$\frac{E_l}{E_w}=2.6$, depends upon all of the months from March 1911 to December 1913 inclusive.

Thus in each case, the constants depend upon the observations of 1911–12–13. The observations of 1914 and 1915 were not used in determining them.

ACCURACY AS TESTED BY GRAPHS. EXPLANATION OF PLATES 9 TO 19, INCLUSIVE

The comparison of observed stream-flow and computed stream-flow on Stream *A* for each day of 4 years is shown on Plates 9 to 12 inclusive. On Stream *B* the comparison is shown for 5 years on Plates 13 to 17 inclusive.

The heavy solid line is the observed stream-flow. The parts of the high peaks which would fall beyond the upper edge of the sheet are plotted below on a reduced scale one-tenth as large as the scale of the main drawing.

The horizontal, light, solid line is the constant part of the stream-flow, S_c, 0.108 cubic foot per second for Stream *A*, and 0.096 cubic foot per second for Stream *B*.

The dotted line is plotted from the S_c line as a zero and represents the change in storage in the drainage area, both above and below ground, in each day.

The dash-dot-dash line is the computed normal stream-flow, without flood-flow. The dash-dot-dash line (with dots enclosed in small triangles), where it occurs, is the total computed flow—the sum of the computed normal flow and the computed flood-flow. Where no dash-dot-dash line (with dots enclosed in triangles) occurs, there was no computed flood-flow. The highest dash-dot-dash line, with dots not enclosed in triangles in ordinary times and with dots enclosed in triangles in times of flood-flow, is everywhere the total computed flow.

The line made up of long dashes and extending from the upper left-hand corner of the lower right-hand corner is the duration curve of observed stream-flow. The dash-two-dot-dash line is the duration curve of total computed stream-flow.

If the theory were perfect, if the formulas as written represented accurately all of the relations between the meteorological elements and the flows of these streams, if the adopted constants were the exact physical constants, and if no errors or approximations were made in the computations, the computed stream-flow would agree exactly with the observed stream-flow and the highest dash-dot-dash line would everywhere coincide with the solid black line, and the duration curves would coincide. To the extent that there is an approach to perfection in the various respects indicated the highest dash-dot-dash curve of the hydrographs should be near the heavy solid curve and should be of the same shape. Similarly the dash-two-dot-dash duration curve should be near the dashed duration curve, and of the same shape. Detailed comparison of the curves is invited. They furnish convincing indication of a close approximation to a satisfactory degree of accuracy. The agreement is not poorer in the last two years, 1914–15, than in the first two or three. It is doubtful if anyone could tell simply from the examinations of the curves which three years out of the five were used in determining the constants.

On Plates 18 and 19, respectively, are shown the mean duration curves for Streams *A* and *B* for the period 1912–1915 inclusive. The mean duration curve of observed flow is a solid line and of computed flow it is a dashed line. Each of these curves is the mean of the four[1] curves shown on Plates 9 to 17 inclusive, of the kind of flow on the stream to which it pertains.

It is desirable to discuss in more detail the evidence as presented in graphic form on Plates 9 to 12 on Stream *A* and on Plates 13 to 17 on Stream *B*. In either case, a cursory examination shows that a large percentage of the discrepancies between the two curves, computed and observed flow, are under 20 per cent. On Stream *A* the principal discrepancy between the two curves is that the summit and center of gravity of the computed spring flow comes before that of the observed flood in every case. It is about 13 days before in 1911 (not shown on plates), 12 days before in 1912, 10 days before in 1913, 18 days before in 1914, and 23 days before in 1915.

This displacement in time on the Stream *A* spring floods is the principal discrepancy as a rule between the computed and observed flood-flows.[2]

The flood summits, observed and computed, are nearly the same height on both streams in 1911, 1913, 1914 and 1915. In 1912 the observed summit of the spring flood is about three times as high as the computed, on both streams. In 1912, the spring flood was the largest that occurred within the period 1911–1915.

On Stream *B* the discrepancy in time between the computed and observed spring flood peak is small. In this connection it should be remembered that the flood coefficients used, R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7}, as well as the values of G, were derived from observations in the *summer* months only. The use of these constants to compute the stream-flow observations in the winter months, with the degree of agreement shown, is a strong indication that assumption No. 10, especially, on which the laws of freezing and melting mostly depend, is not far from the truth. In this connection it is interesting to note that Horton also reached a conclusion (The *Melting of Snow*) that would justify assumption No. 10; viz, that under suitable conditions snow behaves like any other permeable medium, such as porous soil, as regards the percolation of water through it and capillary retention of water in the interstices of the medium.

A comparison of the extreme values on Stream *A* is as follows:

The lowest daily discharge observed was 0.041 c.f.s. on December 16, 1914. The computed discharge on that day was 0.089 c.f.s., in error by 117 per cent or 0.048 c.f.s.

The lowest daily discharge computed was 0.048 c.f.s. on September 22–23, 1915, in error by 38 per cent, the observed discharge being 0.078 c.f.s. and differing by only 0.007 c.f.s. or 17 per cent from the lowest observed discharge.

The highest daily discharge observed was 1.645 c.f.s. on May 20, 1912. The computed discharge on that day was 0.350 c.f.s., in error by 79 per cent.

The highest daily discharge computed was 0.845 c.f.s. on April 28, 1911. The observed discharge on that day was 0.328 c.f.s. and the computed discharge was therefore in error by 0.517 c.f.s. or 160 per cent. The highest computed discharge was $(1.645-0.845=)$ 0.800 c.f.s. less than the highest observed discharge, or 49 per cent less.

The greatest absolute error occurred on May 20, 1912, when the computed discharge was only 0.353 c.f.s., and the observed was 1.642 c.f.s.

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[1] Five curves for Stream *B*.

[2] This displacement was largely removed on Stream *B* as a result of improvement in the rule for determining the last day of full net melting in the spring.

The greatest percentage error occurred on December 15, 1914, when the computed discharge was 0.072 c.f.s., larger than the observed of 0.042 c.f.s., an error of 71 per cent.

A similar comparison of the extreme values on Stream B shows the following:

The lowest daily discharge observed was 0.057 c.f.s. on September 12, 1915. The computed discharge on that day was 0.073 c.f.s., in error by 28 per cent or 0.016 c.f.s.

The lowest daily discharge computed was 0.064 c.f.s. on October 2, 1912. The observed discharge on that day was 0.100 c.f.s., in error by 36 per cent or 0.036 c.f.s. The lowest daily discharge computed was 0.007 c.f.s. greater (12 per cent) than the lowest daily discharge observed.

The highest daily discharge observed was 1.966 c.f.s. on May 22, 1912. The computed discharge on that day was 0.565 c.f.s., in error by 71 per cent or 1.401 c.f.s.

The highest daily discharge computed was 0.974 c.f.s. on May 15, 1911. The observed discharge on that day was 0.563 c.f.s., making the computed discharge in error by 73 per cent or 0.411 c.f.s. The highest computed discharge was $(1.966 - 0.974 =)$ 0.922 c.f.s. less than the highest observed discharge, or 50 per cent less.

The greatest absolute error occurred on May 22, 1912, when the computed discharge was only 0.565 c.f.s., and the observed was 1.966 c.f.s.

The greatest percentage error occurred on May 9, 1911, when the computed discharge was 0.520 c.f.s. larger than the observed of 0.366 c.f.s., an error of 142 per cent.

These comparisons of extreme values shows a considerably greater degree of accuracy on Stream B than on Stream A, as is to be expected.

If the constants in these formulas were not true constants of nature, there would presumably tend to be a systematic change in sign of the residuals with the seasons. The residuals here referred to are the final ones, represented by the difference (total computed)—(observed) stream-flow. As a test of any seasonal changes in the difference "$C-O$" (computed)—(observed) flow, Tables 52 and 53 have been prepared, representing the distribution of the residuals $C-O$. In these tables, for each year, there are two columns headed $C-O+$ and $C-O-$. Under each is indicated by double and single vertical lines the time intervals during which the residuals were + and −, respectively. For instance, referring to Plate 10, the computed stream-flow was less than the observed from April 19 to August 3, 1913. This is shown in Table 52 in the $C-O-$ column for 1913 as a single vertical line. By glancing across the table, it is possible to detect any tendency for the residuals to be of the same sign at the same season of the year. An examination of the distribution of the residuals as shown in Tables 52 and 53 shows that there is little systematic change, except (*a*) that during the greater part of May in each year $C-O$ is negative, the change from + to − always occurring on the downward slope of the computed curve of discharge for the spring flood; (*b*) that in August of each year there is a decided tendency of $C-O$ to be +. In Table 53, comment (*a*) above is not so applicable. The change mentioned occurs less regularly and has shifted to a later time.

Note that on Stream A, the cause of the systematic change (*a*) is the disagreement in time of the computed and observed spring-flood peaks and centers of gravity. On Stream B this was removed, partly by having more accurate values of C, F and M, and R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7}, and partly by estimating the last

TABLE 52—*Distribution of residuals on Stream A*

	1911		1912		1913		1914		1915	
	C-O+	C-O-	C-O+	C-O-	C-C+	C-C-	C-O+	C-C-	C-C+	C-O-
Jan.			Jan 7 Jan 14	Jan 8 Jan 13					Jan 4 Jan 14 Jan 19	Jan 5 Jan 13 Jan 20
Feb.					Feb 5 Feb 9 Feb 10 Feb 12 Feb 22 Feb 24	Feb 4 Feb 6 Feb 8 Feb 11 Feb 23			Feb 4 Feb 8 Feb 11	Feb 3 Feb 9 Feb 10
Mar.	Mar 1		Mar 5 Mar 7 Mar 17 Mar 19	Mar 6 Mar 18	Mar 9 Mar 12 Mar 27 Mar 30	Mar 10 Mar 11 Mar 28 Mar 29				
Apr.					Apr 13 Apr 15 Apr 17 Apr 18	Apr 14 Apr 16 Apr 19				
May	May 7	May 7	May 7	May 8			May 8	May 9	May 12	May 13
June	Jun 16	Jun 16					Jun 17 Jun 19 Jun 21 Jun 23 Jun 26	Jun 16 Jun 20 Jun 24 Jun 25		
July			Jul 26	Jul 25					Jul 6	Jul 5
Aug.					Aug 4 Aug 13 Aug 15	Aug 3 Aug 5 Aug 12 Aug 16				
Sep.	Sep 30		Sep 5	Sep 6					Sep 2 Sep 26 Sep 27	Sep 3 Sep 25 Sep 28
Oct.	Oct 2 Oct 4	Oct 1 Oct 5								
Nov.	Nov 3 Nov 6 Nov 10	Nov 3 Nov 7 Nov 9			Nov 6 Nov 7 Nov 10	Nov 5 Nov 8 Nov 9			Nov 9 Nov 11 Nov 14	Nov 8 Nov 12 Nov 13
Dec.										

TABLE 53—*Distribution of residuals on Stream B*

	1911		1912		1913		1914		1915	
	C-O+	C-O-	C-O+	C-O-	C-C+	C-O-	C-O+	C-O-	C-O+	C-O-
Jan.										
Feb.										
									Feb 24	Feb 23
	Mar 1									
							Mar 17	Mar 18		
Mar.							Mar 21	Mar 20		
	Mar 28	Mar 29					Mar 25	Mar 26		
	Apr 6	Apr 5			Apr 9	Apr 8				
Apr.	Apr 9	Apr 10			Apr 12	Apr 13				
	Apr 12	Apr 11			Apr 16	Apr 15	Apr 15	Apr 14		
May					May 14	May 15	May 15	May 16		
									May 25	May 26
June	Jun 15	Jun 16					Jun 15	Jun 14		
	Jun 25	Jun 24								
	Jul 2	Jul 3					Jul 1	Jul 2		
	Jul 4						Jul 7	Jul 6		
July										
			Jul 26	Jul 25	Jul 25	Jul 24			Jul 26	Jul 25
			Aug 11	Aug 12						
Aug.			Aug 14	Aug 13						
	Aug 21	Aug 22								
	Aug 24	Aug 23	Aug 27	Aug 28						
Sep.										
	Sep 30									
	Oct 2	Oct 1					Oct 3	Oct 4		
	Oct 7	Oct 8					Oct 6	Oct 5		
	Oct 12	Oct 11								
	Oct 13	Oct 14								
Oct.	Oct 16	Oct 15								
	Oct 21	Oct 22					Oct 19	Oct 20		
	Oct 23						Oct 28	Oct 27	Oct 27	Oct 28
	Nov 6	Nov 7					Nov 4	Nov 5		
									Nov 12	Nov 11
Nov.										
							Nov 27	Nov 26		
	Nov 30	Nov 29					Nov 30	Nov 29		
							Dec 1			
	Dec 2	Dec 3					Dec 3	Dec 2		
Dec.										
							Dec 23	Dec 24		

date of full net melting by method (*d*), page 157, only. Thus, on Stream *B*, the maximum $R'_{/}$ was $R'_{/4} = +1.64$, hence, 3½ days preceding the day of maximum flow in the spring was the day of maximum addition to storage by melting, which was normally taken as the last day of full net melting.

The systematic change noted in comment (*b*) above is a possible indication of a divergence of the facts from the simple assumptions stated as Assumptions Nos. 6 and 7. These are indications that equation (34) does not represent the facts as closely as they might be represented. For example, consider equation (34) with reference to Assumptions Nos. 6 and 7, and comment (*b*). From the equation

$$r = (\text{rainfall}) - (E_w)\left(\frac{E_l}{E_w}\right) - (\text{run-off}) \ldots\ldots\ldots\ldots (93)$$

it appears that if the abstract number $\frac{E_l}{E_w}$ is made larger in July and August than the assumed constant value 2.6, r will be smaller, the computed stream-flow will be smaller and $C-O$ will be a smaller $+$. That is, the evidence of a systematic distribution of the residuals with the summer season suggests that $\frac{E_l}{E_w}$ is not a constant but a variable whose value is larger in summer than in winter. On some streams a further refinement may be necessary. It may be proved that $\frac{E_l}{E_w}$ varies with time between rains, being a maximum at the time of a rain, and diminishing to a minimum just before the next rain. This last variation would simply mean that the rate of evaporation is a maximum when the land surface is thoroughly wet, and diminishes as the water disappears under the ground surface. The increase of $\frac{E_l}{E_w}$ in the spring and summer would mean that the loss of water by evaporation may be enhanced because (*a*) of the larger exposed evaporation surfaces produced by the growing vegetation, and (*b*) because of the loss of water by that utilized in the plants themselves, and by transpiration. The increase in loss due to (*a*) will be offset partially or wholly by the decrease in loss from the land surface which will be partially shaded by the growing vegetation.

Consider the distribution of the residuals in Tables 52 and 53 in connection with Assumption No. 8, page 137. If the temperature of the surface of the snow and ice is generally warmer than the surrounding air, the evaporation from land will be actually larger than as figured on the basis of equality of temperature, and vice versa. Hence the general presence of positive residuals in the winter months November to February, inclusive, is an indication that the surface of the snow and ice is, in general, warmer than the surrounding air. Similarly, the general presence of negative residuals in those months is an indication that the surface of the snow and ice is generally colder than the surrounding air. The residuals on Stream *A* appear to be generally positive in the winter months mentioned, whereas, on Stream *B* they appear to be generally negative. This evidence is, therefore, inconclusive and contradictory. The proper conclusion seems to be that, insofar as can be determined from this evidence, the temperature of the snow and ice surfaces and that of the air are about the same, and Assumption No. 8, insofar as it depends upon equality of these temperatures, is as near the truth as can be at present established.

The evidence of a systematic change in the sign of the residuals in the summer season is too weak to be given much weight. It is evidently masked by other errors which are not small in comparison with the error introduced by the assumption of a constant $\frac{E_i}{E_w}$. It is to be noted in this connection that (I) the distribution of precipitation on these areas is fairly uniform throughout the year; and (II), that the vegetation is of a kind which does not change its foliage considerably from season to season. That is, most of the trees are evergreens and, except for the grass, the change in loss of water from the ground supply from season to season due to absorption by the growing vegetation would presumably be small. Both these conditions would favor a constant $\frac{E_i}{E_w}$. On the other hand, a watershed subject to large variations in the annual rainfall and large variations in vegetal cover from season to season due to change in foliage and crop harvesting might require a variable $\frac{E_i}{E_w}$.

ACCURACY AS TESTED BY DETAILED NUMERICAL COMPARISON—COMPARISON OF DAILY OBSERVED AND COMPUTED FLOWS

The daily numerical values of the computed stream-flow have been compared in detail with the daily observed stream-flow for the whole 58 months, March 1, 1911, to December 31, 1915, on both streams. This comparison is shown in Table 54.

TABLE 54—*Comparison of daily observed and computed flows, all days*

	Percentage of days on which computed flow agrees with observed		
	Within 10 per cent	Within 20 per cent	Within 50 per cent
Stream *A*	27	54	87
Stream *B*	28	62	97

COMPARISON OF FLOOD WITH NON-FLOOD MONTHS

Even a casual examination shows that the agreement of computed with observed stream flow is much better in non-flood months than in months in which there is flood-flow. Using the criterion of flood-flow already explained, each month in which there is a day of computed flood-flow as great as 0.030 cubic foot per second

TABLE 55—*Comparison of daily computed and observed flows in non-flood against flood months*

	Percentages of days on which computed flow agrees with observed		
	Within 10 per cent	Within 20 per cent	Within 50 per cent
Stream *A*			
Non-flood months	37	67	99
Flood months	17	30	72
Stream *B*			
Non-flood months	46	81	100
Flood months	24	43	85

on Stream *A* and 0.029 cubic foot per second on Stream *B* (28 and 30 per cent of the constant part of the flow, respectively) is classed as a flood month. On this basis it appears that there are 26 flood months among the 58 on Stream *A* and 24 on Stream *B*. The agreement between the computed and observed daily flow in non-flood and flood months is shown in Table 55.

It is clear that the flood-flows are much less accurately computed than the normal flows.

COMPARISON OF 1914-15 WITH 1911-12-13

Is the agreement between computed and observed stream-flow much poorer during the years 1914 and 1915, which were not considered in the computations, than during the years 1911–12–13, from which the constants were computed? This agreement is shown in Table 56.

TABLE 56—*Comparison of daily computed and observed flows in 1911–12–13 against 1914–15*

	Percentage of days on which computed flow agrees with observed flow		
	Within 10 per cent	Within 20 per cent	Within 50 per cent
Stream *A*			
1911–12–13	37	64	89
1914–15	17	40	81
Stream *B*			
1911–12–13	29	63	94
1914–15	48	68	93

The comparison on Stream *A* is against the period 1914–15. Some of the comparisons to be given later show closer agreement in 1914–15 on both streams than in the earlier period from which the constants were computed.

COMPARISON BY MONTHS

For many purposes it is more important to compute the mean flow accurately for a month than to compute it accurately for *each* day. A comparison has, therefore, been made between the computed and the observed values of the mean stream-flow in each month. This is shown in Table 57.

TABLE 57—*Comparison of computed and observed flows by months*

	Percentage of months in which computed flow agrees with observed		
	Within 10 per cent	Within 20 per cent	Within 50 per cent
Stream *A*	31	62	88
Stream *B*	28	62	97

SPRING FLOOD COMPARISONS

Even a casual examination shows that the larger monthly discrepancies usually occur at the spring flood-flows. This naturally leads one to a special study of the spring floods. The displacement in time between the computed and observed spring flood peaks has already been noted. The comparison of total computed with total observed spring flood for each year and for the whole 5-year period is shown in Table 58.

TABLE 58—*Comparison of total computed and observed spring floods*

Calendar year	Percentage error in total computed spring flood	
	Stream *A*	Stream *B*
1911..........	+31	+48
1912..........	−31	−38
1913..........	−15	+ 7
1914..........	+13	+ 9
1915..........	+12	+10

The aggregate computed flood-flow for these spring-flood periods for all five years was 1 per cent less on Stream *A* and 0.3 per cent less on Stream *B* than the observed, respectively.

COMPARISON BY YEARS

How close is the agreement between the aggregate stream-flow as computed for each year and that observed for the year? This is shown in Table 59*a*.

TABLE 59*a*—*Comparison of annual aggregate computed with observed stream-flow*

Calendar year	Percentage error in total computed annual flow	
	Stream *A*	Stream *B*
1911..........	+20	+26
1912..........	−26	−31
1913..........	−12	+ 5
1914..........	+16	+ 8
1915..........	+10	+ 5

For the whole 58 months on each stream treated as a single group, the aggregate computed flow is exactly equal to the observed flow.

Note that the agreement on both streams for 1914–15 is better than for the years 1911–13, from which the constants were computed.

RAINFALL, NET MELTING, EVAPORATION AND RUN-OFF ON WATERSHED *B*

In Table 59*b* are shown the rainfall, net melting, evaporation and run-off on Watershed *B* in units of 0.01 inch of depth for four years beginning October 1, 1911.

The net melting was computed from equation (36) in which were substituted values of C, F, M and T'' shown in Table 49 for Solution X, Stream B. It will be noted that the resulting specific formula differs but little from the final adopted one, equation (92).

The computed evaporation from water was computed by equation (23), in which, as already mentioned on page 155, the wind term entered very rarely because of the low altitude and sheltered locations of the anemometers.

TABLE 59b—*Rainfall, net melting, evaporation and run-off on watershed B (unit = 0.01 inch)*

Date	Observed rainfall	Computed net melting	Computed evaporation from water	Observed run-off	Computed evaporation from land
1911					
Oct.	410	+11	81	99	186
Nov.	0	+89	45	43	104
Dec.	0	−74	35	36	80
1912					
Jan.	0	−15	42	34	97
Feb.	0	+7	44	31	101
Mar.	0	+93	32	34	74
Apr.	0	+467	63	50	145
May	0	+927	149	294	343
June	203	0	126	96	290
July	403	0	123	50	283
Aug.	158	0	131	39	301
Sept.	37	0	128	34	294
Total	1,211	+1,505	999	840	2,298
1912					
Oct.	171	+6	75	38	172
Nov.	0	+76	69	35	159
Dec.	0	−70	34	34	78
1913					
Jan.	0	−65	33	33	76
Feb.	0	−18	33	29	76
Mar.	0	+154	49	32	113
Apr.	0	+1,087	85	57	196
May	39	+232	147	102	338
June	300	0	109	69	251
July	221	0	148	38	340
Aug.	224	0	113	27	260
Sept.	261	0	68	28	156
Total	1,216	+1,402	963	522	2,215
1913					
Oct.	91	0	73	31	168
Nov.	108	+68	41	29	94
Dec.	0	−81	23	29	53
1914					
Jan.	0	+25	37	28	85
Feb.	0	−27	31	25	71
Mar.	0	+145	62	30	143
Apr.	0	+849	62	46	143
May	147	+785	94	174	216
June	164	0	145	69	334
July	511	0	62	38	143
Aug.	199	0	98	29	225
Sept.	140	0	105	28	242
Total	1,360	+1,764	833	556	1,917
1914					
Oct.	143	0	69	32	159
Nov.	0	+62	89	29	205
Dec.	0	−70	21	29	48
1915					
Jan.	0	−77	32	30	74
Feb.	0	−16	30	26	69
Mar.	0	+124	49	28	113
Apr.	0	+1,072	48	44	110
May	3	+824	95	158	219
June	46	0	206	81	474
July	209	0	180	34	414
Aug.	177	0	128	26	294
Sept.	281	0	94	25	216
Total	859	1,919	1,041	542	2,395
Totals 1911–1915	4,646	6,590	3,836	2,460	8,825
Mean Annual	1,162	1,648	959	615	2,206
Mean Monthly	....	....	80	51	184
Mean Daily	....	....	2.6	1.7	6.0

The evaporation from land was computed from equation (34) in which the value of E_l/E_w used was 2.3, which was computed from the totals in Table 59*b*. This value is smaller than the value, 2.6, computed from the calendar years 1911–13 in the illustration of computation of E_l/E_w on pages 160 and 161. Also this ratio computed from the totals for the calendar years 1911–15 is 2.6.

The principal facts brought out in Table 59*b* are that the computed net melting is possibly too large, and the computed evaporation from water is probably too small. The latter resulted in a ratio $E_l/E_w=2.6$, which is undoubtedly too large; that is, the true evaporation from land is probably not 2.6 times as great as that from a water surface.

In Table 36, Monthly Weather Review Supplement No. 17, the precipitation and run-off for the years beginning October 1 are as shown in the accompanying tabulation in inches of depth on Watershed *B*:

Water year	Precipitation	Run-off
1911–12........	21.49	8.38
1912–13........	19.66	5.21
1913–14........	21.84	5.55
1914–15........	19.85	5.40
Total..........	82.84	24.53
Mean Annual..	20.71	6.13

The precipitation is said to be (in Supplement No. 17) 52 per cent in the form of rain and 48 per cent in the form of snow. Applying these averages to the mean annual values for the four years, the average precipitation as rain appears to be 10.77 inches (20.71×0.52), and as snow 9.94 inches (20.71×0.48).

From this it appears that the mean annual rainfall shown in Table 59*b* is $\left(\frac{11.62}{10.77}=1.08\right)$ 8 per cent larger than that given in Supplement 17, the mean annual estimated net melting is $\left(\frac{16.48}{9.94}=1.66\right)$ 66 per cent larger, and the total precipitation estimated in this investigation is 36 per cent larger $\left(\frac{11.62+16.48}{20.71}=1.36\right)$ than the total precipitation shown in Table No. 36, Supplement 17.

The estimate of rainfall in this investigation was made from the original record of observations at Wagon Wheel Gap, and the distinction between snow and rain was made according to notations shown in that record. The above 8 per cent discrepancy in rainfall can be attributable to different interpretations of those records.

The computed net melting is possibly too high and casts suspicion upon the derived freezing-melting constants, C, F, M and T''. This external evidence shows that the small probable error of M (page 195), especially, is probably fictitious to some extent, due probably to the rejection of too many observations in the spring months used in the observation equations of Solution X. Since the values from Solution X are very close to the best values obtainable to date in this investigation (91), the latter possibly give a net melting which is too large.

The whole of the error between the observed precipitation, 20.71 inches, and the computed precipitation, 28.10 inches, however, should not be attributable to the constants C, F, M and T''. Part of the error may be due to the errors in the decision as to when the freezing-melting period begins in the fall. It appears that the beginning of the freezing-melting period was taken too early rather than too late. In such a case it can be shown that the C-term would tend to cause too large an estimate of the computed net melting, not necessarily because C is itself in error, but primarily because it was entered into the estimate on too many days.

The derivation of the constants C, F, M and T'' was made from observations in two winters only, 1911–12 and 1912–13, in order that they could be used to estimate the net melting in the winters 1913–14 and 1914–15, not used in their derivation, as a test of their validity. These two winters 1911–12 and 1912–13 happened to be winters of extremely heavy and extremely light snowfall, respectively. Using these extreme observations was ideal in the sense that the constants derived from such observations would represent the whole range of observations used. On the other hand, the number of observations used were so few, the decision as to which observations to reject in the spring of 1912—the largest spring flood in the period 1911–15—makes it desirable to supplement the derivation of these values by other studies based upon more observations, particularly in years which are more representative of average conditions.

The following comments are pertinent with reference to Table 59*b*:

The *computed net melting*, computed as mentioned above,

(1) had a minimum value in December, January and February of each year;

(2) had a lowest monthly value of −0.81 inch in December 1913, or −0.026 inch per day, which means that there was a subtraction from storage by water turning into ice at that average equivalent rate in that month;

(3) had a maximum value in April or May of each year;

(4) had a highest monthly value of 10.87 inches in April 1913, or 0.362 inch per day;

(5) had an average annual value of 16.48 inches.

The Observed Run-off—

(1) had a minimum value in February, August or September of each year;

(2) had a lowest monthly value in February 1914 and September 1915 of 0.25 inch, or 0.0083 inch per day;

(3) had a maximum value in May of each year;

(4) had a highest monthly value of 2.94 inches in May 1912, or 0.095 inch per day;

(5) had an average value of 0.017 inch per day.

The Evaporation From Land, computed with $E_l/E_w=2.3$,

(1) had a minimum value in December, January and February of each year;

(2) had a lowest monthly value of 0.48 inch in December 1914, or 0.016 inch per day;

(3) had a maximum value in June, July and August of each year;

(4) had a highest monthly value of 4.74 inches in June 1915, or 0.158 inch per day;

(5) had an average value of 0.06 inch per day.

RELATION OF S_c TO MEAN, MEDIAN AND MODE OF FREQUENCY DISTRIBUTION

In the formulas for normal stream-flow, S_c expresses the flow which would occur at the end of a long period, 257 days, during which just enough rain fell each day to equal the evaporation plus the stream-flow on each day. During such a period the total amount of water in storage in the drainage area, underground, on the surface, and in the lakes and streams would not vary. In general on any day such an equality does not exist between rainfall on the one hand—the gain in storage—and the evaporation plus stream-flow on the other hand—the loss in storage.

Defining S_c in different words, it is that flow which would exist after a long period during which all of the influences tending to increase the stream-flow are exactly balanced by those tending to decrease the flow. It is then that normal stream-flow which should have the greatest frequency or the one which would be most probable. S_c, then, corresponds to the mode of the frequency distribution of the stream-flows.

According to the laws of stream-flow developed in this investigation, the stream-flow will always be the same under identical past meteorological conditions. Since the meteorological conditions, however, are accidental in nature, the improbability that they will repeat themselves according to fixed laws is great, hence the stream-flow which depends on the weather may be classed as accidental in nature. It follows, then, that the difference between any stream-flow greater or less than S_c and S_c takes on the nature of an accidental error, in which S_c is the most frequent value of the measured quantity.

It is well known that stream-flow plotted as a frequency distribution nearly always assumes the form of a skewed curve. Imagine such a curve superposed upon the symmetrical or Gaussian probability curve with their modes falling on the same vertical line. Since the symmetrical probability curve represents the law of distribution of errors which are purely accidental in nature, it follows from preceding statements that there should be a simple relation existing between the symmetrical curve and the curve of actual stream-flow frequency. It was thought that the supposed relation existing between the two curves could be most simply expressed as a ratio.

Omit from consideration stream-flows smaller than S_c. It is desired to obtain the ratio of the curve of actual stream-flow frequency for discharges greater than S_c to the normal probability curve. In order to derive this ratio, the abscissas to the two curves must be expressed in the same unit. The unit used is the probable error of a single observation, which is given by the formula

$$p.\ e. = 0.6745\sqrt{\frac{\Sigma(v^2)}{N'-1}} \dots\dots\dots\dots (94)$$

in which v is the difference between any observed stream-flow greater than S_c and S_c, and N' is the total number of observations of stream-flow greater than S_c. By means of equation (94) the unit of abscissas of the curve of actual frequency distribution can be computed, and by actual count the frequency of the flows greater than S_c which are between successive multiples of the p.e. can be obtained. If we now assume that the area under each curve to the right of S_c is unity, the ordinates to the symmetrical probability curve for various multiples of the p.e. can be obtained from the well-known formula

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{\rho \frac{a}{r}} e^{-x^2} dx \ldots\ldots (95)$$

in which a is the error, r the probable error of a single observation, $\rho = 0.476936$, and $\Phi(x)$ the probability that the error will be less numerically than the limit which is expressed in terms of the probable error.

In order to avoid irregularities in the data it is desirable to smooth out the actual histogram of observed stream-flow by some form of regular curve. In this investigation the Pearsonian curves were used, and it has been found that with the size of class interval used in these studies the data can be represented by the Type III curve of which the general equation is

$$y = y_0 \, e^{-\gamma x} \left(1 + \frac{x}{a}\right)^{\gamma a} \ldots\ldots (96)$$

in which y is the computed frequency of the stream-flow, x, measured in terms of p.e. of equation (94), y_0, γ and a are constants to be derived by the method of moments from the observed frequency distribution, and e is the base of Naperian logarithms.

On Streams A and B, the S_e discharge was known, as determined by the least-square computations. With this known, the p.e. could be computed by equation (94). This p.e. divided into D' measured from S_e as origin is the $\frac{a}{r}$ of equation (95) and divided into D' measured from the mode of (96) as origin is the x of equation (96). From the computations yielding equation (96), the mean, median and mode of the actual frequency distribution could be estimated. Let the ratio of S_e to mean, median and mode be defined as follows:

$$\left.\begin{aligned} \frac{S_e}{\text{Mean}} &= K_1 \\ \frac{S_e}{\text{Median}} &= K_2 \\ \frac{S_e}{\text{Mode}} &= K_3 \end{aligned}\right\} \ldots\ldots (97)$$

In which K_1, K_2 and K_3 are assumed to be constants determinable from the data on Streams A and B. With these constants known, three values of S_e can be computed for any other stream in a moist climate from the mean, median and mode of its frequency distribution. The most probable value of S_e for the stream is taken to be the arithmetic mean of the three separately determined values.

In order to make the foregoing specific, an example of the computations on Stream A will be presented, and subsequently a summary will be given of the results obtained on other streams.

The order of procedure and the results derived on Stream A are those given in the following numbered paragraphs:

(1) The discharge observations were arranged in ascending order of magnitude in increments of one in the units used (or 0.001 cubic foot per second). In order to identify particular discharge magnitudes with the time of year on which they occurred, the listing of the discharges was made as a table of columns of dates, at the top of each column being placed a number, and under these numbers were written the dates when the corresponding discharges were observed.

(2) The tabulations described in the preceding paragraph were summarized by writing in a vertical column the discharge values in ascending order of magnitude and, in an adjacent column, numbers indicating the total number of times that the corresponding discharge had been observed.

(3) In a third column was written the differences between the S_c discharge, 108, and the discharge listed in the first column mentioned in (2), limiting the values used to those greater than S_c. These differences are the v's of equation (94).

(4) In a fourth column, the squares of the values of v of column 3 were written.

(5) In a fifth column, the products of the values in columns 2 and 4 were written. The sums of these values became the $\Sigma(v^2)$ of equation (94). On Stream *A*, this sum was 52,328,754.

(6) The total number of observations of stream-flow on Stream *A* greater than 108 was 700 out of a total of 1,759 values of daily discharges during the time March 9, 1911, to December 31, 1915, inclusive. It appeared that the discharges preceding March 9, 1911, may not have been as accurately determined as the later ones, hence they were rejected. The 700 is the N' of equation (94), which, substituted therein with the above value of $\Sigma(v^2)$ gives $p.\ e. = 0.6745\sqrt{\frac{52328754}{700-1}} = \pm 184.6$

(7) This probable error, ±185, was arbitrarily taken as the class-interval to use in evaluating the constants of equation (96). In Table 60, the third column shows the total number of observations of stream-flow between the limits stated in the first column in increments of 185. The second column shows the deviation of each group in the third column from an assumed mean of 277.5. The values in the fourth to seventh columns inclusive are the products of the values in, respectively, columns 2 and 3, columns 2 and 4, columns 2 and 5 and columns 2 and 6. The sum of the products in columns 4 to 7, inclusive, are the first to fourth moments, respectively, of the data about the vertical through the assumed zero of 277.5.

TABLE 60—*Moments of frequency curve of Stream A*

Limits of discharge (inclusive). Unit = 0.001 c.f.s.	Deviation from 277.5	Observed frequency	Moments			
			1st	2d	3d	4th
0– 185	−1	1,410	−1,410	+1,410	−1,410	+ 1,410
185– 369	0	234	0	0	0	0
370– 554	+1	52	+ 52	+ 52	+ 52	+ 52
555– 739	+2	39	+ 78	+ 156	+ 312	+ 624
740– 924	+3	13	+ 39	+ 117	+ 351	+ 1,053
925–1109	+4	3	+ 12	+ 48	+ 192	+ 768
1110–1294	+5	2	+ 10	+ 50	+ 250	+ 1,250
1295–1479	+6	1	+ 6	+ 36	+ 216	+ 1,296
1480–1664	+7	5	+ 35	+ 245	+1,715	+12,005
Totals		1,759	−1,178	+2,114	+1,678	+18,458

These moments give a mean discharge of 153.6, and a value of K, the Pearsonian criterion by which to judge the type of equation which best represents the data, of -16.4, which indicates a Type III curve.

The constants of equation (96) evaluated from the first to third moments inclusive of Table 60 are

$$\left.\begin{aligned} y_0 &= 619.84 \\ \gamma &= +0.544346 \\ a &= -1.426997 \\ \gamma a &= -0.776781 \end{aligned}\right\} \quad \dots\dots\dots\dots (98)$$

which, substituted in equation (96) gives

$$y = +619.84 e^{-0.544346x} \left(1 - \frac{x}{1.426997}\right)^{-0.776781} \dots\dots\dots\dots (99)$$

in which y is the frequency of the discharge, x, of Stream A, the unit of x being 185. The origin of x is the mode, which, calculated from the moments of Table 60, is at -186.

(8) By means of equation (99), the frequency of occurrence of any discharge may be computed, and it is desired to find the ratio of this discharge frequency for discharges greater than S_c to the normal frequency of those discharges. In column 1 of Table 61 are shown the discharges of Stream A greater than S_c in increments of 0.5 the p.e. or 92.5 (shown alternately as 92 and 93). In column 2 is shown the increments, a/r, of equation (95) or $(D' - S_c)/185$. In column 3 is shown the x of equation (99), or $\dfrac{D' - (-186)}{185}$. Any figure in column 4 is the total number of observed stream-flows between the limits shown in column 1 from the discharge preceding the one opposite it to one less than the discharge following the one opposite it; thus there were 527 discharges between 0.108 and 0.292 cubic foot per second, inclusive. In column 5 are shown the frequencies of discharges between integral increments of a/r as computed by equation (99) by the method of areas, using three ordinates in each area, and computing the area by Simpson's formula. As 185 was used in the unit of x in deriving equation (99), the sum obtained by adding every other value in column 5, beginning with the first, should equal the total number of observed flows greater than $S_c = 108$, or 700. In column 6 is shown the products of $\rho = 0.476936$ into column 2. These are the upper limits of the probability integral, equation (95). Column 7 shows the values of equation (95) for the limits shown in column 6. These represent the total area under the symmetrical probability curve from the origin, or S_c, to the limit shown in column 6 on both sides of the origin. In column 8 are shown the increments of column 7 between alternate limits shown in column 2; thus, the probability of an accidental error occurring between the limits one and two times the probable error is 0.32265. In column 9 is shown the total number of accidental errors in 700 which will occur between alternate limits shown in column 2; thus 350 + and − errors will occur between 0 and 1 time the probable error. The total obtained by adding every other value in column 9, beginning with the first, should equal 700. The ratio of calculated actual frequency to the symmetrical frequency is given by dividing the values in column 5 by those in column 9, and is shown in column 10.

The frequency curve calculated from equation (99) and shown in column 5, Table 61, is shown on Plate 20, as curve 2. On this plate the vertical scale on the

TABLE 61—*Frequency of discharge of Stream A, and frequency ratio*

(1) Observed stream-flow D' in 0.001 c.f.s.	(2) $\frac{a}{r}$ of (95) $=\frac{D'-108}{185}$	(3) x of (99) $=\frac{D'+186}{185}$	(4) Observed frequency	(5) Frequency calculated by (99)	(6) x of (95) $=\rho\frac{a}{r}$	(7) $\Phi(x)$	(8) $\Delta\Phi(x)$	(9) $\Delta\Phi(x)\times700$	(10) Ratio $\frac{\text{Col. (5)}}{\text{Col. (9)}}$
108	0	1.589,189	....	....	0	0	....	....	....
201	0.5	2.091,892	527	*511.2	0.2384681	0.26407	0.50000	350.00	1.46
293	1.0	2.589,189	....	$\underline{195.9}$	.476936	.50000	0.42426	296.98	0.66
386	1.5	3.091,891	89	108.6	.71540	.68833	0.32265	225.86	0.48
478	2.0	3.589,189	....	$\underline{66.4}$	.95387	.82265	0.21992	153.94	0.43
571	2.5	4.091,891	42	42.7	1.19234	.90825	0.13432	94.02	0.45
663	3.0	4.589,189	....	$\underline{28.3}$	1.43081	.95697	0.07351	51.46	0.55
756	3.5	5.091,891	27	19.2	1.66928	.98176	0.03605	25.24	0.76
848	4.0	5.589,189	....	$\underline{13.2}$	1.90774	.99302	0.01583	11.08	1.19
941	4.5	6.091,891	7	9.17	2.14621	.99759	0.00623	4.36	2.10
1033	5.0	6.589,189	....	$\underline{6.46}$	2.38468	.99925	0.00220	1.54	4.20
1126	5.5	7.091,891	1	4.57	2.62315	.99979	0.00070	0.49	9.33
1218	6.0	7.589,189	....	$\underline{3.26}$	2.86162	.99995	0.00020	0.14	23.3
1311	6.5	8.091,891	2	2.34	3.10009	.99999		....	....
1403	7.0	8.589,189	....	$\underline{1.68}$	3.33855	....	....	....	....
1496	....	9.091,891	2	1.21	....	....	....	....	....
1588	....	9.589,189	....	$\underline{0.880}$	....	....	....	....	....
1681	....	10.091,891	3	0.640	....	....	....	....	....
1773	....	10.589,189	....	$\underline{0.467}$	....	....	....	....	....
1866	....	11.091,891	0	0.341	....	....	....	....	....
1958	....	11.589,189	....	....	....	....	....	....	....
			700	700.†				699.97	

* This value was actually computed to be 505.8. It was arbitrarily increased 5.4 or to 511.2 (1 per cent) to make the total area computed from equation (99) equal the total observed area, 700.

† Sum of values other than ones underlined.

left indicates the frequency of discharge of Stream *A*. The lower scale of abscissas is the discharge of Stream *A* in 0.001 cubic foot per second. S_c is shown at the discharge 108 of Stream *A*. With this S_c as origin, the upper scale of abscissas is the discharge, D', confined to values greater than S_c, in terms of the p.e., which, for Stream *A*, is 185. This is also the scale of abscissas of the symmetrical probability curve in terms of the increment a/r.

Curve 1 on Plate 20 shows the observed histogram of Stream *A*, which is shown in column 4 of Table 61. The curve (equation (99)) starts at 0.078 c.f.s., and is shown above that point as a vertical line.

Curve 3 on Plate 20 is the symmetrical probability curve plotted from the values in column 9, Table 61.

Curve 4 (plate 20) is the ratio of the curve of stream-flow frequency calculated from equation (99) for discharges greater than S_c, to the symmetrical probability curve. This curve is plotted from the values in column 10, Table 61. The vertical scale of curves 4, 5, 6 and 7 is shown at the right of Plate 20.

TABLE 62—*Miscellaneous data on some streams*

Name of river	Length of period studied	Limits of discharge in period studied		Ratio $\frac{\text{max.}}{\text{min.}}$ discharge	Drainage area (sq. mi.)	Annual rainfall (in.)	Location of gaging station	Discharge measured by—
		Max.	Min.					
		(c.f.s.)	(c.f.s.)					
A............	1911–15	1.966	0.057	34	0.348	20	Wagon Wheel Gap, Colo.	U.S.W.B. U.S.F.S.
B............	1911–15	1.645	0.041	40	0.313	20	Wagon Wheel Gap.......	U.S.W.B. U.S.F.S.
Dix..........	1918–20	23,000	1.2	19,200	395	45	Near Burgin, Ky.........	U.S.G.S.
Cumberland..	1918–20	43,500	41.0	1,060	2,040	40	Cumberland Falls, Ky...	U.S.G.S.
Delaware.....	1918–20	50,900	685.0	74	3,070	50	Port Jervis, N. Y.........	U.S.G.S. U.S.W.B.
Kankakee....	1913–15	2.60*	0.02*	130	4,600	36	Kankakee, Ill............	Power Co.

* Unit is c.f.s. per square mile.

In a manner similar to that just described in the numbered paragraphs 1 to 8, inclusive, for Stream *A*, the frequency ratio curves 5, 6 and 7, Plate 20, were worked out for Stream *B*, the Delaware River at Port Jervis, New York, and for the Cumberland River at Cumberland Falls, Kentucky, respectively. The numerical values of these ratios are shown in Table 66, together with their weighted mean values, which were obtained by assigning the values the weights 2, 2, 1, 1, respectively in the order in which they appear in the table, to take account of the difference in the amount of data used, and to account for the fact that S_c was determined on the first two streams from the least-square computations. Some characteristics of these streams are shown in Table 62. The histograms of the frequency distributions of the flows of these streams are not shown on Plate 20, neither are their fitting curves.

The frequency distributions of these streams all required Pearson's type III curve for best representation. The values derived for the various constants in the general formula, equation (96), are shown in Table 63. Note that the criterion K is numerically greater than 4 in every case. Note also the rather large skewness, especially on Streams *A* and *B*.

TABLE 63—*Values of constants, etc., in equation (96) for various streams*

Name of river	y_0	γ	a	γa	Curve begins	Skewness	Number of observations used	Criterion K
					(c.f.s.)			
A	619.84	0.544346	−1.426997	−0.776781	0.078	2.12	1,759	−16.4
B	332.82	.425155	−1.990404	−0.846230	0.075	2.17	1,767	−48.9
Cumberland	120.63	.196588	−3.606625	−0.709019	818	1.85	731	− 5.84
Delaware	139.01	.189615	−3.412353	−0.647034	1820	1.68	731	− 8.48

In the first three columns of Table 64 are shown the mean, median and mode as estimated from the fitting curve of the frequency distribution of discharges on each stream. The mode was negative in every case. Using equation (97), in connection with the values of S_c shown in Table 51, page 195, the values of K_1, K_2 and K_3 shown in columns 5, 6, and 7, Table 64, were computed for Streams *A* and *B*. The mean of the two values in each case was taken as the best value of these constants obtainable from this investigation. These means are

TABLE 64—*Mean, median and mode, etc., of frequency distributions*

River	Mean from fitting curve	Median by inspection	Mode from fitting curve	K_1	K_2	K_3
	(c.f.s.)	(c.f.s.)	(c.f.s.)			
A	0.154	0.096	−0.186	0.70	1.12	−0.58
B	0.135	0.089	−0.258	0.71	1.08	−0.37
Cumberland	3780	1890	− 6390			
Delaware	5540	3580	− 5010			

$$\left.\begin{aligned} K_1 &= +0.70 \\ K_2 &= +1.10 \\ K_3 &= -0.48 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (100)$$

from which the S_c of the Cumberland and Delaware Rivers are estimated to be, respectively, using the mean, median and mode in Table 64, 2,600 and 3,410 cubic feet per second. Using the mean *K*'s of equation (100), the estimated S_c of Streams *A* and *B* are, respectively, 101 and 105, in error by −7 per cent and 9 per cent, respectively.

The variation of the three separate values of S_c obtained for each stream from the mean of the three values for each stream is given in Table 65.

TABLE 65—*Variations of S_c from mean value*

River	Mean discharge	Median discharge	Mode discharge
	p. ct.	*p. ct.*	*p. ct.*
A	7 above	5 above	12 below
B	10 below	7 below	18 above
Cumberland	2 above	20 below	18 above
Delaware	14 above	16 above	30 below

From the above table it is seen that the maximum error in an individual value is 30 per cent, and it is estimated that the maximum error in the mean value obtained on any stream with two years of record of discharge will be about ±10 per cent.

The S_c estimated for any stream by this device becomes the R'' of equation (59), page 148. A least-square adjustment based upon, say, 200 daily observations on the stream should serve to give the correction R of equation (60) to this R'', with a probable error which should be nearer 5 per cent than 10 per cent of the true S_c, $R+R''$. The constant, S_c, is rather difficult to determine for a stream from the least-square computations alone when one knows nothing of its probable magnitude. Incidentally, therefore, by estimating it in this manner, one can derive a quick, accurate correction to it to get the true value.

The frequency-ratio curves on Plate 20 all follow the same general course. They show that the observed frequency of discharge of the streams as smoothed-off by Pearson's curves all bear in general a similar relation to the symmetrical probability curve. The ratio curves for Streams A and B tend to follow each other more closely than the other two, which is to be expected from the comparative amounts of data used in establishing them (see Table 63). The weighted-mean values of these ratios shown in the last column of Table 66 are the best values obtainable in this investigation, and are assumed to be constant for any stream in a moist climate. The weighted-mean values of the ratios are shown plotted in the upper left-hand corner of Plate 21. From the weighted-mean values, it appears that the minimum ratio is about 0.40, and it occurs at a discharge of about 2.0 times the p.e. plus S_c. The curve at a p.e. of 6 or greater rises very rapidly. At the two points where this ratio curve is unity are the places where the corresponding symmetrical probability curve would intersect the fitting frequency curve. These two intersections occur between 0 and 0.5 p.e., and between 3.5 p.e. and 4.0 p.e.

TABLE 66—*Weighted mean frequency ratios*

a/r	Frequency ratio				
	Stream A	Stream B	Cumberland River	Delaware River	Weighted mean
0	....	....	....	....	....
0.5	1.46	....	1.08	0.74	1.18
1.0	0.66	0.53	0.68	0.44	0.58
1.5	0.48	0.38	0.53	0.34	0.43
2.0	0.43	0.36	0.49	0.32	0.40
2.5	0.45	0.40	0.53	0.34	0.43
3.0	0.55	0.50	0.64	0.42	0.53
3.5	0.76	0.74	0.92	0.57	0.75
4.0	1.19	1.22	1.46	0.91	1.20
4.5	2.10	2.30	2.63	1.66	2.18
5.0	4.20	5.00	4.80	3.40	4.43
5.5	9.3	11.0	12.4	7.4	10.1
6.0	23.3	23.0	28.0	18.5	24.8
6.5	....	....	....	....	....

By means of these ratios, one can estimate the probability of occurrence of any flood within the limits of the ratios on any stream as soon as the S_c of the stream and its p.e. are known. For example, referring to column 8, Table 61, according to the laws of probability, 2 accidental errors out of 10,000 will occur between 5.5

and 6.5 times the p.e. If N' be the total number of observations on a stream greater than its S_c, then, on that stream, $0.0002 \times N' \times 24.8$ will be the probable number of stream-flows between 5.5 p.e.$+S_c$ and 6.5 p.e.$+S_c$. Thus, on Stream *A*, $0.0002 \times 700 \times 24.8 = 3.47$, which is the probable number of flows out of every 700 greater than S_c which is to be expected between a discharge of $(5.5 \times .185 + 0.108 =)1.126$, and $(6.5 \times 0.185 + 0.108 =)$ 1.311 c.f.s. The actual number observed was 2.

From the definitions of S_c and the mode, one would expect them to have the same value or that the value of K_3 in (100) would be unity. The derived mode from the Type III Pearsonian curve, being negative in every case, has no meaning in this sense. The modes determined by inspection are shown in the lower left-hand corner of Plate 21 based upon the discharges for 1911–13, and in the lower right-hand corner of Plate 22 based upon the discharges for 1911–15. Compare first the modes based upon 1911–13 with the derived values of S_c. For Stream *B*, the mode determined by inspection is 89. The value of S_c is 96. This gives a value of K_3 of $(96/89=)1.08$, which is 8 per cent larger than the value to be expected from theory. For Stream *A*, the value of K_3 based upon 1911–13 is $\left(\frac{108}{86}=\right)1.26$, or 26 per cent too large. Compare next the modes based upon the observed discharges for 1911–15 as determined by inspection. For Stream *B*, K_3 is $\left(\frac{96}{76}=\right)$ 1.26; and for Stream *A*, K_3 is $\left(\frac{108}{76}=\right)1.42$.

The proper interpretation of these discrepancies appears to be that the derived values of S_c are too large in both cases. Note that the value of K_3 for Stream *B* was nearer the value to be expected from theory than that of Stream *A*, corresponding to the known errors in the Stream *A* computations produced principally by the use of too large a value for $M(=+8.39)$. Note that the values of K_3 for both streams were nearer the truth when based upon the data for 1911–13 than when based upon the data for 1911–15, corresponding also to the positive knowledge that $M=+8.39$ (used on Stream *A*) is too large, and with the suspicion that the final values of *C*, *F* and *M* used on Stream *B* give a computed net melting which is possibly too large.

In the case of a frequency distribution of stream discharge which is skewed very little, from which the derived mode by Pearson's Type III curve would presumably come out positive, the procedure would be to take $K_3=1$ in (100) in this case. For intermediate cases, in which the mode derived by Pearson's Type III curve would give absurd values of S_c computed from $K_3=-0.48$, the proper procedure would be to reject such a value and base the S_c upon the estimated mean and median, only.

GENERAL RÉSUMÉ OF THEORY

In the preceding pages there has been presented in detail this method of estimating stream-flow. In the pages which follow it is proposed to give, first, at the expense of some repetition, a résumé of the theory on which this method is based; second, to present certain conceptions which will aid one in applying this method to other streams; third, to state briefly some possible applications of this research; and fourth, to present a general summary of the conclusions reached.

If one understood what happens to each portion of water in a drainage area between the time it falls as rain and the time it goes out of the drainage area as evaporation or as stream-flow he certainly would understand the laws of stream-flow. The water in the drainage area during the period named is believed to be underground as a rule. Only a very small proportion of the water which appears as stream-flow travels over the surface of the ground on all, or much, of the distance from the point where it falls to the stream. As a rule its trip to the stream, if it escapes evaporation, is a slow underground trip occupying many days. The ground in the drainage area, with its constant topographic relief and its constant underground natural drainage system, is the controlling part of the mechanism which gathers in the rain and transmits it to the points where it is evaporated, or to the stream. The ground is sensibly constant in such of its characteristics as affect stream-flow, except as changes are produced by man.

In any case in which man does not, by dams or artificial drainage systems, or other interference with nature, produce changes in the stream-flow, all of the influences upon stream-flow which do not change with lapse of time are expressed by the 23 constants of the formulas for each of Streams A and B—

$$S_c,\ \text{the } R''\text{s, the } R'_f\text{'s, } C,\ F,\ M,\ G,\ T'' \text{ and } \frac{E_l}{E_w}.$$

Such influences are the topography, the geology, the surface conditions, etc. These derived constants are the autograph of the particular stream. They express its character. The constants will be different for each stream because the topography, geology, underground drainage system, etc., are different for each stream.

The meteorological conditions change rapidly and irregularly from day to day. These changes are observed and are put into the formulas as the r's, the t's, etc. The formulas as a whole state that the flow of a given stream will always be the same under identical past meteorological conditions, and they express the variations of the flow in terms of the variations in meteorological conditions.

Review the definition of S_c given on page 211. Assume here, again, for convenience, that on a specific day there is rain or net melting sufficient to produce a gain in storage and that thereafter for 256 days there is no change of storage on any day, the rainfall or net melting being on each day just sufficient to equal the evaporation plus stream-flow. Then, according to the theory developed by this investigation, the influence of the specific day in increasing the flow during later days would be that indicated by the terms $r_1R'_1$, $r_2R'_2$, $r_3R'_3$, . . . $r_{10}R'_{10}$; the r_1 (gain in storage) of the specific day becoming the r_2 of the following day, the r_3 of the next day following, and so on. In the assumed case rain or net melting would be added each day, uniformly distributed over the whole drainage area, and water would be subtracted each day by evaporation equivalent to a layer uniformly distributed over the area. In sharp contrast the water subtracted by stream-flow each day from the drainage area comes at first from the stream itself and areas very close to its banks; later from slightly more remote areas and only after many days from the most remote areas. Under these circumstances the added flow (added to S_c) produced by the increase of storage of the specific day will tend to be relatively large while it is being drawn from the near areas, will decrease as the areas drawn upon become more remote and will tend to become relatively very small after the lapse of many days. The R''s and R'_f's expressed as percentages, therefore, will tend to be relatively large soon after the specific day and decrease gradually toward a minimum. Under the conditions described the total storage in the drainage area has remained a constant. But its horizontal distribution over the drainage area has gradually changed in the sense that more has remained accumulated in the parts of the area remote from the stream and that there is less storage near the stream.

Assume, on the other hand, that on a specific day there is neither rain nor net melting and consequently a decrease in storage, and that thereafter for 256 days there is no change in storage in any day, the rainfall or net melting on each day being just sufficient to equal the evaporation plus stream-flow. Then, according to the theory developed by this investigation, the reverse of what has just been described will take place. The influence of the specific day in decreasing the flow of later days will be indicated in turn by negative values of $r_1R'_1$, $r_2R'_2$, $r_3R'_3$, . . . $r_{10}R'_{10}$. At the end of the 256-day period the total storage will be unchanged. But its horizontal distribution will be changed in the sense that there will be less storage in areas remote from the stream and more near the stream.

In the actual case which occurs in nature, there is either an increase or a decrease in storage on each day. According to the theory of this investigation, each day initiates a train of events like one or the other of the two just described. The actual stream-flow of any day is the aggregate of 257 such trains. Such an aggregate is represented by the formula given for the normal stream-flow.

In an ideally perfect theory, of course, infinity should be substituted for 257 and the series of such terms as $r_1R'_1$, $r_2R'_2$, $r_3R'_3$, . . . should be infinite. To date it has not been found feasible to trace the influences back more than 257 days, that is, to determine any term more remote than $r_{10}R'_{10}$ in the formula.

The theory embodied in the flood-flow formula is (*a*) that a flood-flow is initiated only when the ground near the surface is glutted with water; (*b*), that the best available index of glutting is the size of the stream-flow on the preceding day combined with the intensity of the rainfall on the current day; and (*c*), that when the r_1 for a given day is above the value, G, which produces a glut for that day, a flood-flow is produced for 33 days thereafter, following such a law as is embodied in the formula. The constants of the flood-formula R'_{f1}, R'_{f2}, R'_{f3}, . . . R'_{f7} are to be derived from the observations. It is expected that expressed as percentages the first three will ordinarily be greater than the last three on any stream to which this method is at present limited; that is, the summit of the flood-flow is expected to occur very soon after the maximum rain or net melting which produced it, and thereafter the flood-flow will gradually diminish to zero at the end of a period which for Streams A and B is not less than 33 days long.

It is believed that flood-flow is produced mainly by certain portions of the stored water traveling an unusual proportion of the trip toward the stream over the surface of the ground, or very near the surface, and so producing an unusual horizontal distribution of the underground storage. It is not believed that in general a large proportion of the flood-flow travels the whole trip to the stream over the surface.

The theory embodied in the freezing-melting theory is (*a*) that in general there is a constant tendency for melting to occur on the lower side of the mass of snow, ice and frozen earth, which tendency is offset partially or wholly by a tendency to freeze, which is proportional to the negative temperature reckoned from the critical point, T''; (*b*) that the melting at the surface is proportional to the positive temperature reckoned from the critical temperature T''; and (*c*) that the critical temperature, T'', is below 32° F. by somewhat less than the daily range of temperature because a rapid melting during a few hours must be offset by slow freezing during many night hours if the net melting is to be zero for the 24 hours.

The belief, embodied in the theory that a large part of the travel of water is underground from where it originally fell to the spot where it is evaporated or delivered to a stream, was confirmed during the progress of the investigation by the constants as derived from the observations. Referring to Table 51, page 195, note that for Stream *A* the observations as embodied in the derived constants show that of any increase in storage which does not produce flood-flow, only 9.0 per cent appears as extra stream-flow in 257 days if during that period the storage is held constant. On Stream *B*, only 4.1 per cent appears as extra stream-flow in 257 days. Of that portion of an increase of storage which is above the limit produced by glutting, only 19.0 per cent appears as extra stream-flow in 33 days on Stream *A*, and 29.6 per cent on Stream *B*. These percentages would each necessarily be much higher if much of the travel of the water were on the surface. The phrase "extra stream-flow" is meant to specify that part of the flow which is an excess above the constant portion of the flow, S_c.

APPLICATIONS TO OTHER STREAMS

SOME GENERAL CONCEPTIONS WITH REFERENCE TO SLOPE- THALWEG- AND STREAM-TRAVEL

If the theory, the understanding so far developed, is correct it should apply to other streams, but the constants, S_c, the R''s, the R'_f's, G, and $\frac{E_l}{E_w}$ should differ for other streams from those derived on Streams *A* and *B*. Possibly C, F, M and T'' would differ also, but this is not certain. The differences in the constants should be expressible in terms of topography, geology, surface conditions, drainage conditions, etc., that is, in terms of the constant characteristics of the drainage area, if one had complete knowledge. Some other streams besides Streams *A* and *B* have been partly studied.[1] Some slight progress has been made in interpreting the constants in terms of the characteristics of the drainage area. For example, from a study of the topographic map of Wagon Wheel Gap, Plate 7, the conclusion was reached from pure theory that the maximum R' for Stream *B* should be less than for Stream *A* and should come later. The computations later showed that conclusion to be true.[2]

In order that the reader may have full advantage of the use of the ideas embodied in the theory just mentioned in enabling him to estimate the R''s on any stream, those ideas are herewith presented briefly.

There are three parts to the travel of a given particle of water from its first contact with the ground as rain to its arrival at a point of measurement as stream-flow during which its performance is probably decidedly characteristic of each part as distinguished from the others, as follows:

Travel underground, not under a well-defined thalweg. Call this *slope travel.*
Travel underground, under a well-defined thalweg. Call this *thalweg travel.*
Travel in a stream (temporary or permanent) flowing on the surface of the ground. Call this *stream travel.*

The following general propositions with reference to the three paths of travel are believed to be true.

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[1] These are indicated in Table 62, page 216.
[2] See Plate 8 and Table 51.

SLOPE TRAVEL

Slope travel is uniformly distributed over a large area in such manner that the rate of travel is most increased at first at the lower end of such travel, and becomes a progressively smaller increase. As a consequence it follows that the maximum R' (expressed as a percentage) tends to be greater the smaller the average slope travel for a stream, and to be greater the more rapid the slope travel. For any given line of slope travel there is a *normal* profile for the water surface. S_c is the discharge corresponding to the normal profile. This normal profile will be approached by a *stable* profile whenever the r_1, r_2, r_3, . . . r_n for a given day are all zero, and will be reached if at that time the total storage in the drainage area is equal to the average storage of a long period. All profiles are nearly coincident at their lower ends with the normal profile, and the discharge corresponding to a stable profile is a close approach to S_c.

When a stable profile is reached, for any short section of the profile, the flow at the lower end of the section will equal the flow along the profile at the upper end of the section, plus the total amount of water received vertically from above. The changes in the water received vertically from above tend to be the same simultaneously over the whole drainage area, and hence tend to raise (or lower) all parts of all profiles by the same amount simultaneously. After the period necessary to establish a stable profile the remaining R''s tend to be more nearly alike; that is, the discharge expressed by these R''s is merely a function of the total storage and not of the time when that storage was accumulated or reduced.

Whenever any part of a profile is changed, the rate of flow at that part changes in proportion to the quantity h/p, in which h is the head (or vertical travel) and p is the corresponding length along the profile. (Note that the maximum value of h/p is 1.0.) It follows that in any given section the rate of flow may, as a first approximation, be taken proportional to h in that section. Under all steep slopes the percentage variation in h tends to be small, because h is large in proportion to the change of level of the water surface. Consider the cross-sections of divides as shown in the upper right-hand corner of Plate 21. The percentage variation in h will be smaller in (*a*) than in (*b*) if the variation of the water level at the upper end of the slope is less in (*a*) than in (*b*). It will be smaller in (*b*) than in (*c*) if the variation in water level at the lower end of the slope is less in (*b*) than in (*c*). Under all flat slopes, the percentage variation in h tends to be large. It tends to be especially large and prompt at and near the lower end of such slopes. It follows then that the rate of flow under steep slopes tends to be relatively constant; and that the rate of flow under flat slopes tends to be relatively variable. The greater the percentage of slope travel which is above the point of steepest slope, the steadier will be the flow and the longer the change of flow due to a given r_1 will last. Also the greater the height, h, involved in the steep slope, the steadier the flow.

Of the five cases shown on Plate 21, the order of steadiness of flow would probably be, steadiest first, (*c*), (*d*), (*b*), (*a*), (*e*).

The preceding remarks apply to slope travel alone. To estimate the relations of the various R''s one must also take thalweg and stream travel into account.

THALWEG TRAVEL

Thalweg travel is much more rapid than stream travel, because under a well-defined thalweg at the depth of rapid travel of water the soil or rock is more porous than under the slopes, chemical and mechanical erosion having developed porosity there. Thalweg travel affects the *time of delivery at the maximum rate* at the stream-flow measuring station very decidedly, but influences the rate of that maximum (maximum R') but little, that little being, however, in the direction of a reduction.

Closely related to thalwegs are underground paths of small resistance to flow. In the formation of such paths, the mechanical erosion produces extension of such paths from the downstream end upward, since the rate of such erosion is apt to be greater the more rapid the movement of the water and the greater the total amount of water involved, and, since the erosion of this character may be negative, a plugging up in regions where the rate of movement is small.

The chemical erosion tends to be greatest where there is water which has been in contact with the earth but a short time; that is, near the beginning of the underground travel, and at and near streams where there is the most active flow of underground water, i.e., near and at underground paths of small resistance to flow. The chemical erosion at a given point will tend to be proportional to the departure from chemical saturation of the water at that point, and the total amount of water per year departing from that point. Chemical erosion may in some cases cause a progressive extension of paths of small resistance to flow from the upstream end downward.

The existence of thalwegs favors the development of underground paths of small resistance to flow by concentrating underground flow in certain paths under the thalwegs, and by producing locally large differences of head underground. Conversely, the existence of underground paths of small resistance to flow favors the development of thalwegs by favoring rapid local underground chemical and mechanical erosion, which in time produces a local lowering of the surface and thereby starts a thalweg, which tends to accentuate itself by surface erosion; and by favoring a concentration of underground flow to the point at which it produces surface flow, intermittent and steady, which in turn tends to produce surface erosion which favors a gradual upstream extension of a thalweg from the temporary or permanent headwaters of the stream.

It follows that close dissection of the topography of a region by streams and thalwegs is strong presumptive evidence of close dissection of that region by underground paths of small resistence to flow, and vice versa.

STREAM TRAVEL

Stream travel predominates in the control of the lag of the maximum R' behind the corresponding rainfall, and reduces the size of the maximum R'. The rate of travel of the crest of a flood wave is not fixed by accelerations, except in great expansions of the river—lakes—in which there is a negligible current. The average rate of travel of the crest of flood waves was found statistically to be 44 miles per day, or 1.8 miles per hour, or 33 minutes to the mile. The wave controlled by accelerations, the tidal wave, would move at double this speed even though the depth were only one foot, or at that rate if the depth were 0.25 foot. The average rate of travel of the crest of a flood-wave throughout its trip downstream is substantially a constant for all streams. It is fixed in some way by the Chezy formula expressing the relation between slope of surface, hydraulic radius or resistance and velocity of flow. In other words it is fixed primarily by the internal and boundary resistance of the water.

The average rate of travel of 44 miles per day for the crest of a flood wave in a river was determined by a statistical study of the daily river stages as published by the Weather Bureau. In all, 15 rivers, located in all parts of the United States, were studied. The value 44 miles per day is based upon a study of 117 cases, each case being the observed rate of travel of the crest of the flood from one gaging station to the next one down stream on one river. A similar study of 109 cases of the rate of travel of the end of the flood wave fixed it at 25 miles per day. In some cases the rate of travel appeared to be infinite; that is, the summit of the crest or end of the wave, as the case may be, appeared to arrive at two stations several miles apart on the same day. In other cases there appeared to be a negative rate of travel; that is, the crest or end of the flood wave appeared to arrive at the lower of two gaging stations before it arrived at the next station upstream. Such cases as these two were rejected in arriving at the values 44 and 25 stated above.

In this investigation the laws of stream-flow as stated were derived from observations on two small watersheds. If this method is applied to a very large watershed, it is assumed that the r_1 corresponding to each rain gage should be set forward according to the following rule:

> If the stream travel from the rain gage to the point of stream measurement is less than 44 miles, no set forward; is between 44 and 88 miles, set forward one day; is between 88 and 132 miles, set forward two days; and so on.

Under this rule it is believed that the *position* of the maximum R' will remain unchanged as the total length of the stream travel increases. It is believed that the *size* of the maximum R' will be affected as follows: It will tend decidedly to get smaller as the stream travel increases from zero to 44 miles, and for increases beyond 44 miles there will be a moderate tendency to get smaller. It will tend to be larger the more nearly each branch stream agrees in length of its stream travel with the total stream travel of the main stream above the mouth of the branch stream.

As a first approximation it may be assumed that the rate of travel of a summit of thalweg travel is one mile per day. On this basis, one mile of length on an average for each thalweg would tend to delay the maximum R' by about one day.

The greater the variety of water travel by slope, thalweg and stream, the broader will be the summit of the R' or run-off curve.

SOME POSSIBLE APPLICATIONS OF THIS RESEARCH

During the course of this investigation several possible applications of the methods and constants developed became evident. It is proposed to set forth here briefly some of these possible applications.

APPLICATION TO INCREASING LENGTH OF RECORD OF FLOW OF A STREAM

Observations of stream-flow are used in connection with the design of future hydro-electric power plants, irrigation projects or water-supply projects in making estimates of the future mean flow, maximum flow and minimum flow. The more accurately it is possible to predict the flow of a stream, the more accurate is it possible to design the works with respect to economy and safety. The decision in the first place as to whether the waters of a stream can be economically utilized frequently hinges upon the amount of water available.

By this method, for any given stream, the procedure would be as follows:

First, compute its constants from the series of stream-flow observations which are available;

Second, compute the stream-flow (by use of the derived constants) for other months and years for which no direct observations of stream-flow are available.

Third, from the total of stream-flow data, both observed and computed, thus made available, make the estimates for the future by use of the methods for that purpose which now exist, and also by using extensions of those methods and new methods based upon the more complete understanding of stream-flow which has been gained.

The new method has the inherent advantage permeating every part of it of a truer and more complete understanding of stream-flow. The gain from such improved understanding is difficult to estimate but it is certainly great.

In applying the second part of the process to compute the stream-flow for months and years from which no stream-flow observations are available, one is free to choose any month or year for which the necessary meteorological observations are available. The meteorological record of the necessary completeness in any locality is ordinarily much longer than the stream-flow record. Therefore there will ordinarily be more possible extension of the computed stream-flow than is feasible or necessary to use. From the many months or years of meteorological record one may, with the constants of the stream before him, select by examination the months and years of extremely large flow, extremely small flow, or average flow, whichever is desired, and so secure from a given amount of computation a much more reliable estimate of the maximum flow, minimum flow, or mean flow, than would otherwise be feasible.

RULES FOR APPLYING THIS METHOD

In deriving the constants of the stream from the stream-flow observations which are available, the order of procedure would be as follows:

(1) Estimate the mean, median and mode discharges by fitting a Pearson Type III frequency curve to the available data. Then by use of the values of K_1, K_2 and K_3 (100) page 217, estimate the S_e of the stream by equation (97) page 212.

(2) The estimated S_e of step (1) becomes the R'' of the general equation (59) page 148. Estimate R''_1, R''_2, R''_3, . . . of equation (59) by comparison with other streams already studied, in connection with the general ideas set forth on pages 222 to 225.

(3) Estimate the ratio $\frac{E_l}{E_w}$ for the stream by use of equation (35a) page 137, in which the net melting is to be computed by equation (92) page 193, and E_w by equation (23) in Part I, page 82.

(4) By use of the ratio $\frac{E_l}{E_w}$ established in step (3), and equations (92) and (23), compute r_1, r_2, r_3, . . . r_n for the stream for the years of available discharge observations. In the case of a watershed on which the distance from the point of stream measurement to the farthest rain-gage is greater than 44 miles, the values of r_1 should be set forward according to the rules stated on page 225. The beginning of the period of decreased rate of melting is to be estimated by method (d), page 157, assuming that the maximum R'_f for the stream will be R'_{f3}, and during the

period of decreased rate of melting, the net melting is to be computed by equation (67), page 156, in which are substituted the final values of C, F, M and T'' from (91), page 193.

(5) Using the residuals of equation (59), viz, $-D$, as the absolute term in (60), derive the corrections to the estimated S_e of step (1) and the estimated R''_1, R''_2, R''_3, . . . of step (2). These corrections will be R_1, R_2, R_3, . . . of equation (60). Adding these to the assumed values according to (61) will give the normal-flow constants of the stream, S_e, R'_1, R'_2, ...

(6) Estimate the flood-flow constants of the stream, R''_{f1}, R''_{f2}, R''_{f3}, . . . by comparison with other streams studied in connection with the general ideas set forth on pages 222 to 225. Obtain the $G-D'$ curve of the stream by studying the residuals of equation (60) and the observed discharge, D'.

(7) Compute r_{f1}, r_{f2}, r_{f3}, . . . from the values of r_1 computed in step (4) and the value of G established in step (6). Use these in equation (79), page 172, together with the R''_f's estimated in step (6) and the residuals of equation (60) as absolute term, taken from days of heavy rain and rapid rates of melting.

(8) Take the residuals of equation (79) as the absolute term in equation (78) and compute the corrections R_{f1}, R_{f2}, R_{f3}, . . . to the assumed values, R''_{f1}, R''_{f2}, R''_{f3}, . . . to get the flood-flow constants of the stream R'_{f1}, R'_{f2}, R'_{f3}, . . . according to (80).

(9) The normal-flow formula obtained in steps (1) to (5), and the flood-flow formula obtained in steps (6) to (8) can now be used to estimate the stream-flow for years for which no direct observations of flow are available, in terms of the observed meteorological elements on the watershed, by the process stated in general terms on pages 138 and 139.

The eight steps outlined above for deriving the discharge formulas of the stream represent the probable least amount of labor which will be required for that purpose. To carry out those eight steps would require about 100 to 200 days of work of one computer. It is probable that additional refinement will be necessary, in addition to those eight steps, as in the case of Solution P, Stream A (see pages 166 to 168), which would probably bring the labor required to nearer 200 than 100 days of work for one computer. It is estimated that with the maximum time stated, the constants for any stream in the eastern two-thirds of the United States, in a region of 20 inches or more of annual rainfall, could be developed with greater accuracy than those now available for Stream B, after the meteorological data and stream-flow data have been gathered for 3 years.

The rate at which the stream-flow can be estimated by use of the discharge formulas after they are derived has not been determined. It appears, from general considerations, however, that less than 10 man-hours of time should be required to compute the daily stream-flow for one month from the meteorological elements after the formulas are derived.

In connection with the estimate of accuracy with which this method may be applied on a new stream, made in the second preceding paragraph, it is noteworthy that the most important part of the duration curve for power estimates on run-of-river power plants is the lower part. This happens to be the most accurately determinable part by this method. In this type of plant very low flows can not be utilized because of insufficient volume of water to turn the turbines. Very high flows can not be utilized because of the decrease in effective head on the turbines due to rise of tailwater. If a run-of-river plant be considered as installed on

Stream *B*, the estimate of power available from the computed mean duration curve (Plate 19) would have been *practically identical* with that made from the observed mean duration curve. For estimates for plants utilizing storage, the daily variation in flow is not so important as the monthly and annual aggregate flows. In this connection note the agreements on Stream *B* (Tables Nos. 57, 59*a*, pages 206 and 207 and footnote on page 230.)

The above rules for applying the method are written in definite form from which it might be inferred that the results will be unfailing and uniform. It is not claimed that the new method of computing the constants of a stream and of estimating the future stream-flow is standardized so far that it may be applied by following a set of rules. It is still in the developing stage. The method and the computation of the constants involves the use of judgment, as distinguished from rules, at many points. The computations involve essentially a process of successive approximations which may be very slow and poor if poor judgment and little knowledge is used.

APPLICATION TO FORECASTING THE FLOW OF A STREAM

There are various cases occurring in engineering practice after a plant utilizing stream-flow is in operation, in which it is desirable to make and act upon a prediction of the stream-flow a few days or weeks in advance. Contracts for the sale of power may thus be made more wisely. Coal may be bought to better advantage in the case of interconnected systems involving both steam and water-power. The new method of estimating stream-flow lends itself admirably to such predictions. Notice that S_c and $R'_{10}r_{10}$ are known four months ahead. S_c, R'_8r_8, R'_9r_9 and $R'_{10}r_{10}$ are known a month ahead, and all terms even of the flood-flow formula having subscripts greater than 5 are known nine days ahead. Predictions four months ahead, one month ahead, or nine days ahead of time would be made by using the known terms of the formula and supplementing them by average values of the remaining terms for the corresponding portions of the year, based upon past experience.

Since the energy output of a turbine varies as the three-halves power of the head, a small gain in head is of relatively more importance in a low-head than in a high-head plant. The prediction method stated in the preceding paragraph will be especially useful on the low-head plant, in rendering possible the maintenance of a maximum head and at the same time wasting as little water as possible over the spillway.

In the case of a high-head plant utilizing storage, it is probably more important to know how much water a given rain will put into the reservoir, or how much water will run into it as the result of a thaw in the spring, than to forecast the flow from day to day, inasmuch as the daily flow may change the reservoir content but little. This method lends itself admirably to forecasts of this kind also. Take the case of the rain of October 4–5, 1911 (see plate 13). On October 5, the total amount of water which the rains of October 4 and 5 would deliver to Stream *B* by surface travel could have been forecasted by equation (86), page 177, with an error of less than 8 per cent. Inasmuch as the flow by percolation was relatively insignificant in this case, the estimated total flow during the month following October 5 due to this rain would have been but slightly in excess of that estimated by equation (86) alone. If the temperature should rise suddenly in the spring, when the ground is covered with snow, the amount of water (melted snow) to be delivered to the reservoir could be similarly predicted by equation (86), in which the net melting is estimated by equation (92).

For a low-head plant fed by a shallow reservoir of large area, such as the one at Keokuk, Iowa, a change of elevation of the reservoir surface of a few hundredths of a foot is highly important from the operating standpoint. In such a case it is pertinent to note that the determination of the elevation of the reservoir surface at any hour may be considerably in error because of the effects of winds and barometric pressures upon it. It is claimed that in such a case, the highest attainable efficiency in use of water could be effected by taking account of these effects, and by forecasting the yield to the reservoir by methods developed in this investigation.

APPLICATION TO DETERMINATION OF EFFECTS OF FOREST COVER ON RUN-OFF

If the denudation of a watershed of its forest cover has any effect upon run-off, that effect should be expressible in terms of the physical constants of the stream, S_c, the R''s, the R'_f's, C, F, M, T''', G and E_l/E_w. If the presence or absence of forests makes no difference upon run-off, the stream-flow formulas derived from parallel observations of stream-flow and the weather elements during an epoch before denudation should be identical with the formulas derived after denudation. If the character of the run-off varies with variation in the amount of vegetal cover, that variation should be expressible qualitatively and quantitatively by the methods developed in this investigation.

It would be too much to claim that, in its present stage, the method of analysis would be superior to the excellent studies recently completed at Wagon Wheel Gap, Colorado, for the purpose of settling that question, by the Weather Bureau and Forest Service. Its present degree of accuracy may not be great enough to warrant such a claim for such a scientific application. Indications are that the accuracy can be increased, however, and in this sense the method has potential merit which should make it a useful instrument by which to corroborate those conclusions. It certainly has the advantage over the Wagon Wheel Gap method in being applicable to many watersheds in any past time for which the necessary, ordinary observations are available.

SUMMARY OF GENERAL CONCLUSIONS TO PART II

The principal conclusions to the stream-flow part of this investigation are here briefly summarized for the convenience of anyone who wants to get a general view of the results of this part of the investigation. Page references enable those interested to look up the basis of particular conclusions.

General formulas have been derived, which express the relationship between the daily flow of a perennial stream in a moist climate, on the one hand, and the meteorological elements of rainfall, snowfall, vapor-pressure, air temperature, and wind velocity observed on its watershed, on the other hand. The principal ones of these formulas are equations (33) and (37). They involve, indirectly, other formulas, such as equations (34), (35*a*), (36) and (67) (pages 135 to 139, 156 to 158).

Specific formulas have been developed for Streams *A* and *B*, Wagon Wheel Gap, Colorado, which enable one to estimate the daily flow of those streams with a fair degree of accuracy, from the five weather elements stated in the preceding paragraph as observed on their watersheds. These formulas are, for Stream *A*, equations (76) and (87), pages 168 and 178, respectively; and for Stream *B*, equations (77) and (86), pages 171 and 177, respectively. The degree of accuracy with which it is possible to estimate the flow of these streams from their respective formulas in terms of the observed weather elements is stated in Table 54 to 59*a*, pages 205 to 207.[1]

The ratio of the frequency curve of discharge of a stream, for discharges greater than S_c (the constant part of the flow), to the normal or Gaussian frequency curve, is believed to be the same for all streams in moist climates (pages 211 to 219).

The larger part of the water which reaches a stream travels underground to the stream from the point where it falls as rain, or occurs as melted snow. Only a small part of it travels over the surface of the ground for all or much of the trip to the stream. On Stream *A*, for example, of any increase in storage in the ground which does not produce a flood-flow, only 9.0 per cent appears as extra stream-flow in 257 days, if during that period the storage is held constant. On Stream *B*, only 4.1 per cent appears as extra stream-flow in 257 days. Of that portion of an increase in storage which is above the limit produced by glutting, only 19.0 per cent appears as extra stream-flow in 33 days on Stream *A*, and 29.6 per cent on Stream *B*. These percentages, 19.0 per cent and 29.6 per cent, would each necessarily be much higher if much of the travel of the water were on the surface of the ground. The phrase "extra stream-flow" is meant to specify that part of the flow which is an excess above the constant part of the flow, S_c (pages 163 to 168, 170 to 171, 172 to 176, and 178).

When there is an abundance of snow and ice available on a watershed for melting, there is a constant addition to the stored *water* in the ground equivalent to 0.0397 inch of depth per day when the mean air temperature on the watershed for the 24 hours is 28° F. For each day for each degree that the mean air temperature for the day is less than 28° F. there is a subtraction of water from storage due to freezing at the rate of 0.00457 inch of depth per day. For each day for each degree that the mean air temperature for the day is greater than 28° F. there is an addition to the stored water in the ground due to melting at the rate of 0.0519 inch of depth per day. The rate of addition to storage by melting is therefore about 11 times greater than the rate of subtraction therefrom by freezing (pages 192 to 194).

The average rate of travel of the crest of a flood wave throughout its trip downstream is substantially a constant for all streams. It was found statistically to be 44 miles per day, or 1.8 miles per hour, or 33 minutes to the mile. It is not fixed by accelerations, except in great expansions of the river—lakes—in which there is a negligible current (pages 224 and 225).

This research may be applied to the following, among other, problems: (*a*) the problem of increasing the length of record of flow of a stream as a basis for greater accuracy in the design of engineering works for power, irrigation, navigation and sanitation, (*b*) the problem of forecasting the flow of a stream as a basis for increasing the economy of operation of hydro-electric power plants, (*c*) the problem of determining the effect of forest cover on the run-off from watersheds (pages 225 to 229).

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[1] The accuracy stated does not represent the best that can be done by the methods developed in this investigation. The figures in Tables 54 to 59a are taken from the computations representing various stages in the evolution of the theory. If the theory as finally evolved, together with the best technique developed, were used, the accuracy stated could be increased. It appears that the annual aggregate stream-flow may be computed from the weather elements observed on its watershed with an error of less than 10 per cent on most of the years.

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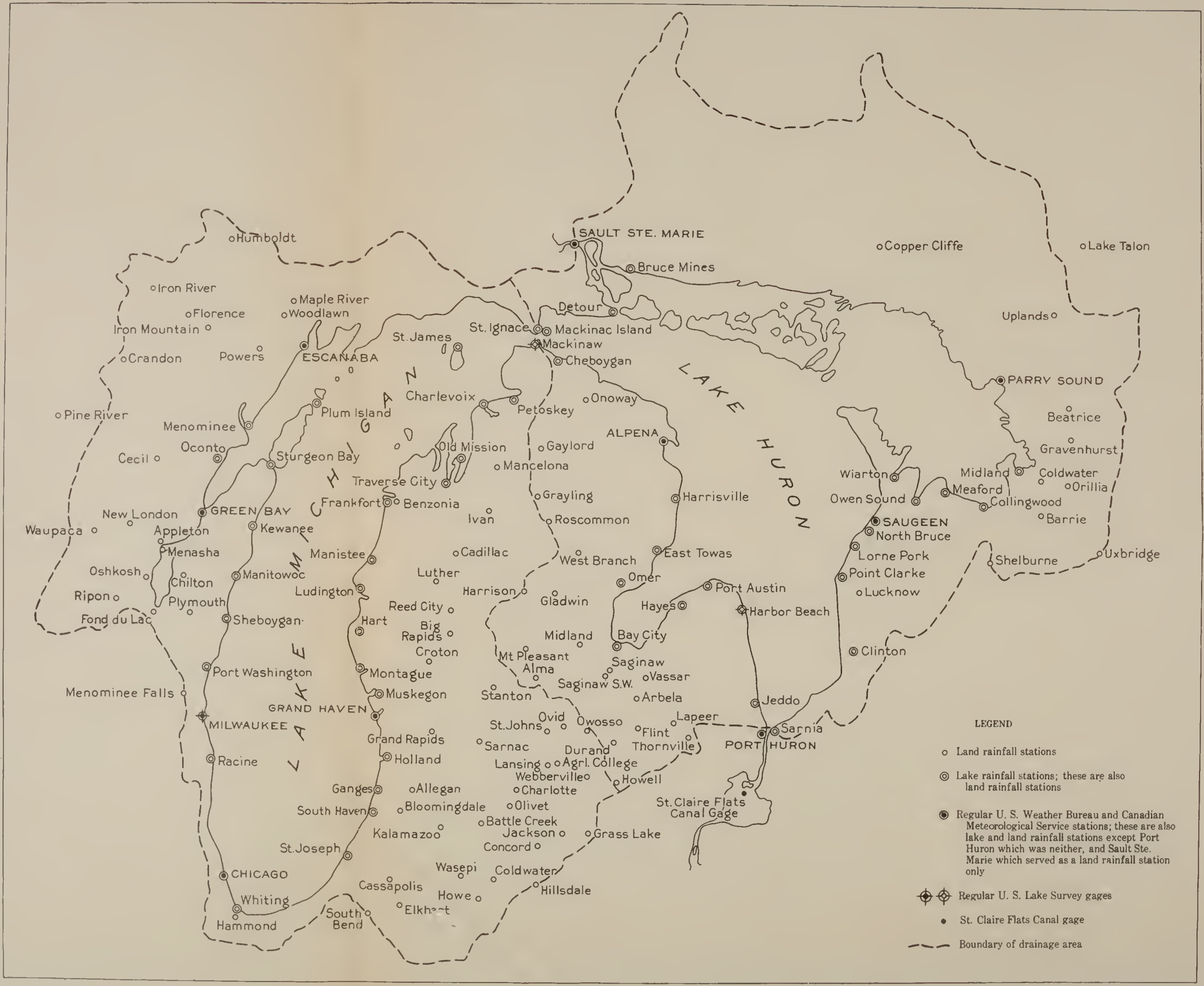

Lake Michigan-Huron Drainage Area, Showing Meteorological and Hydrographic Stations.

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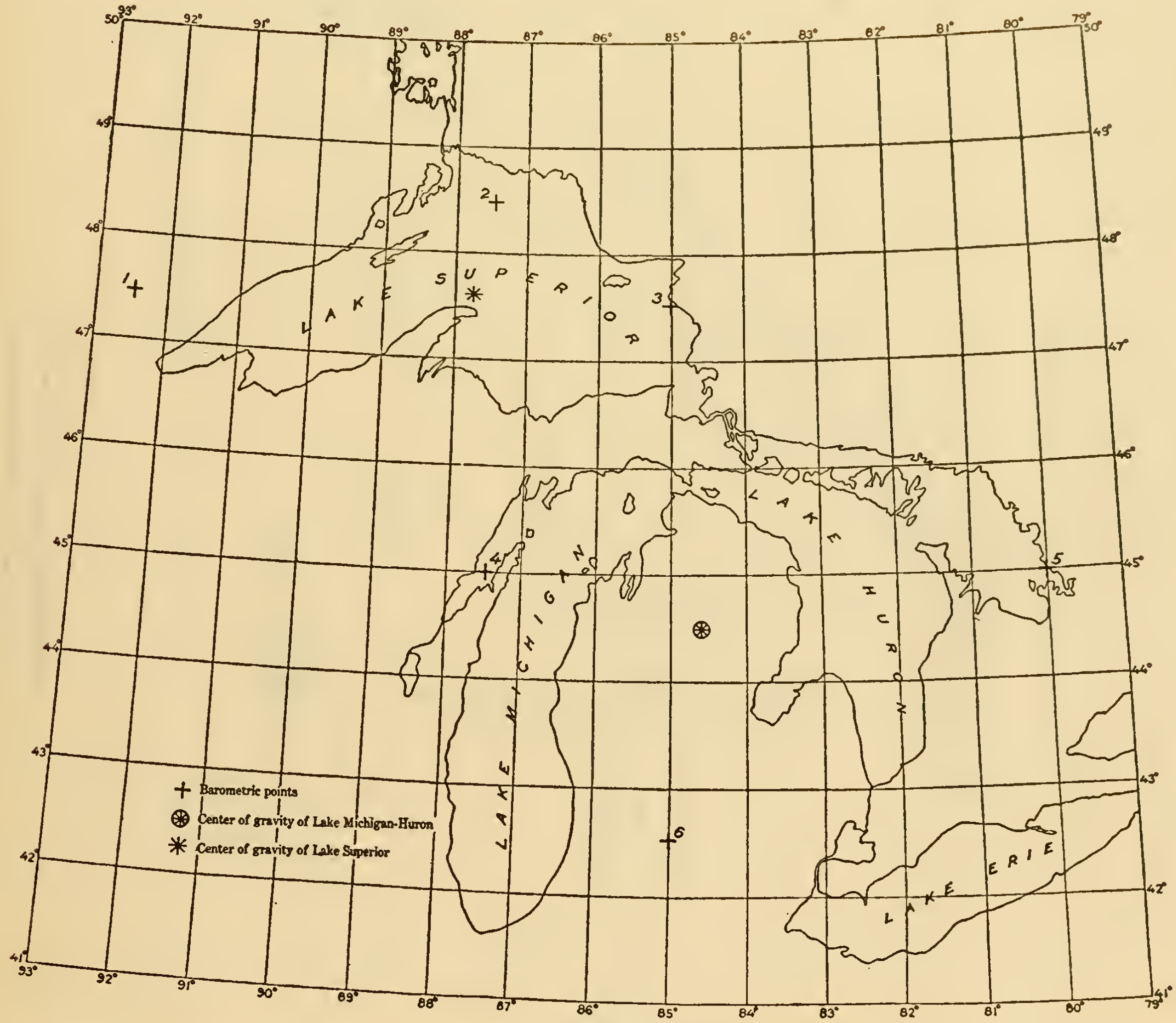
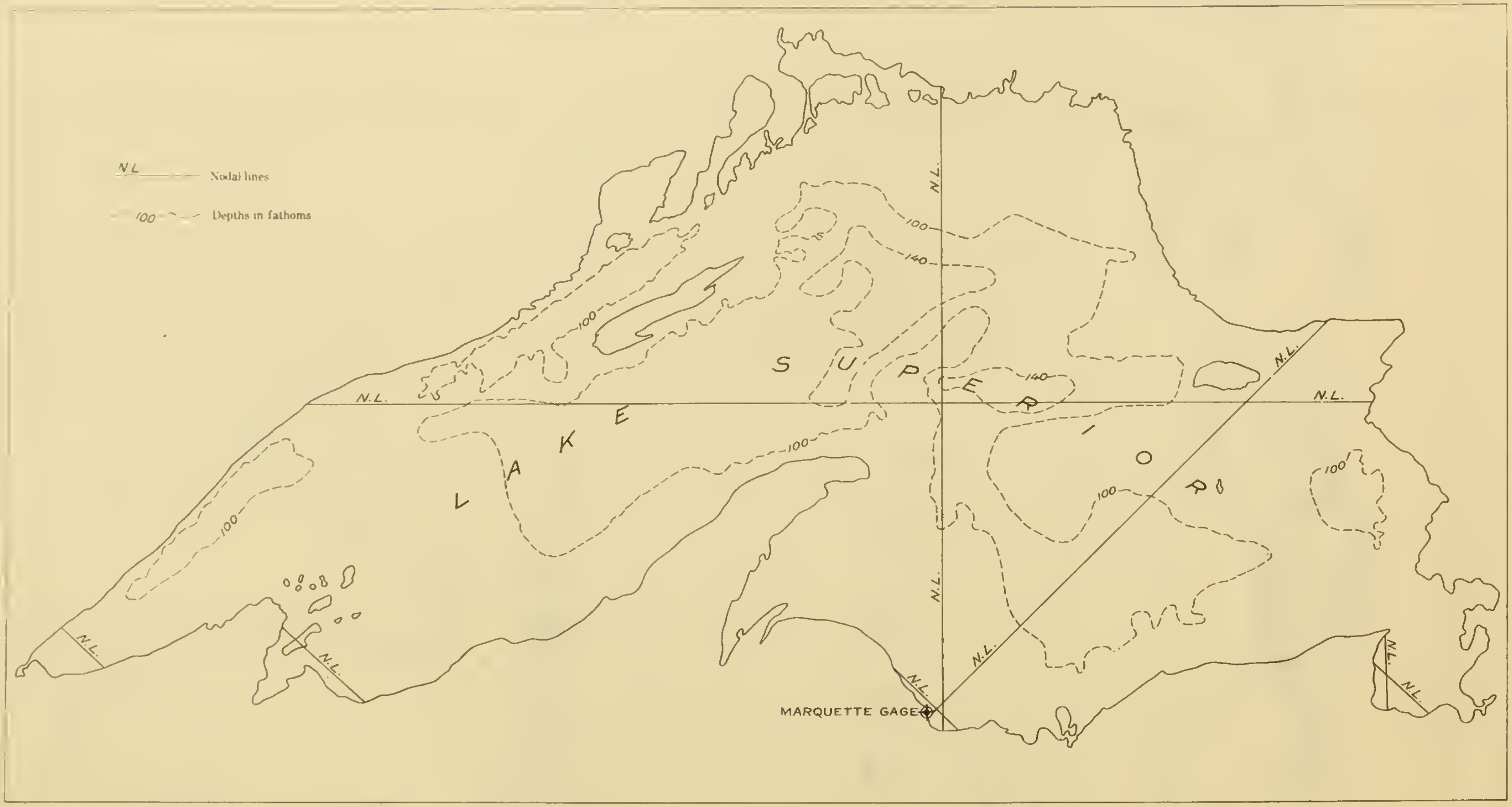


Barometric Points around and Centers of Gravity of Lakes Superior and Michigan-Huron.



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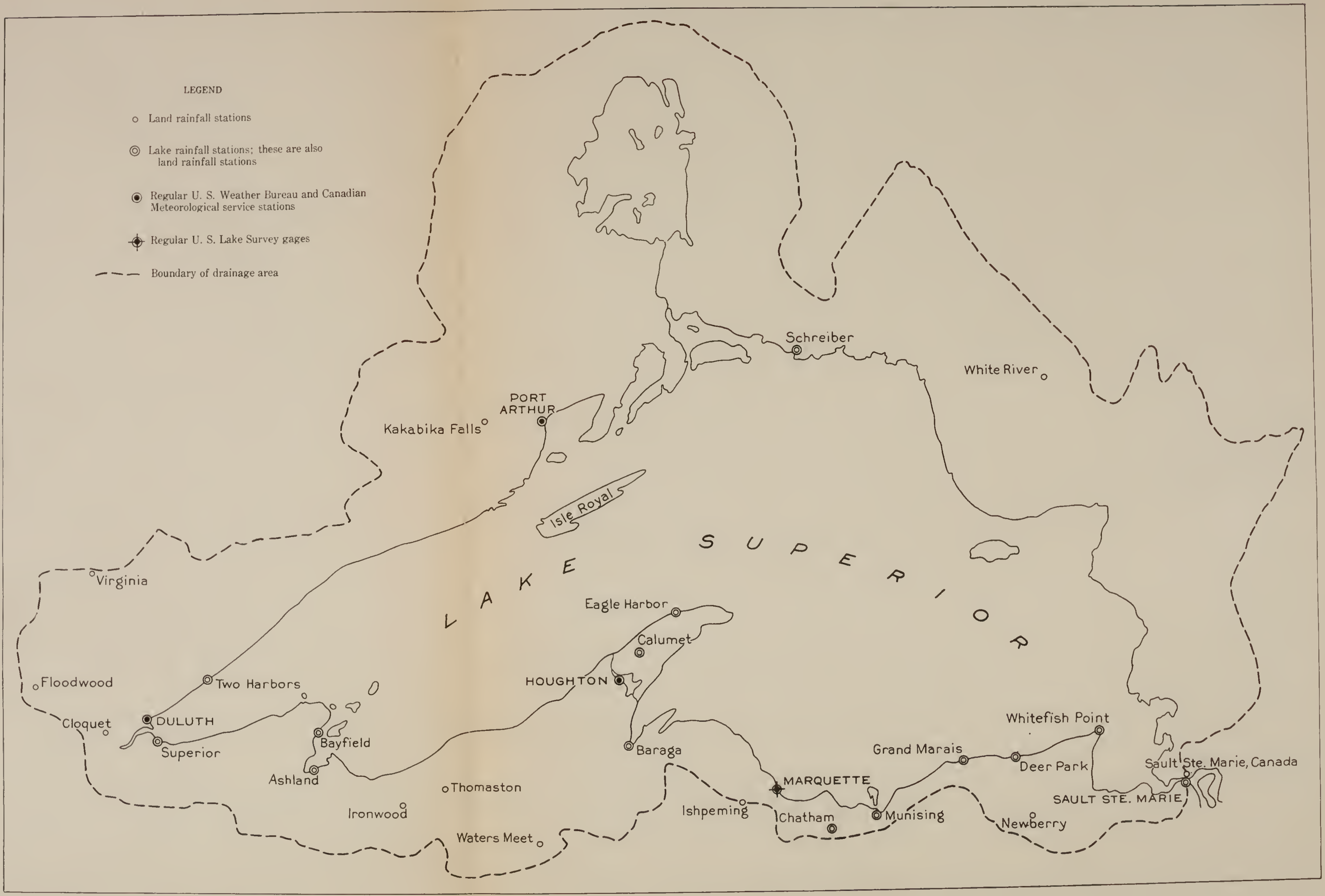

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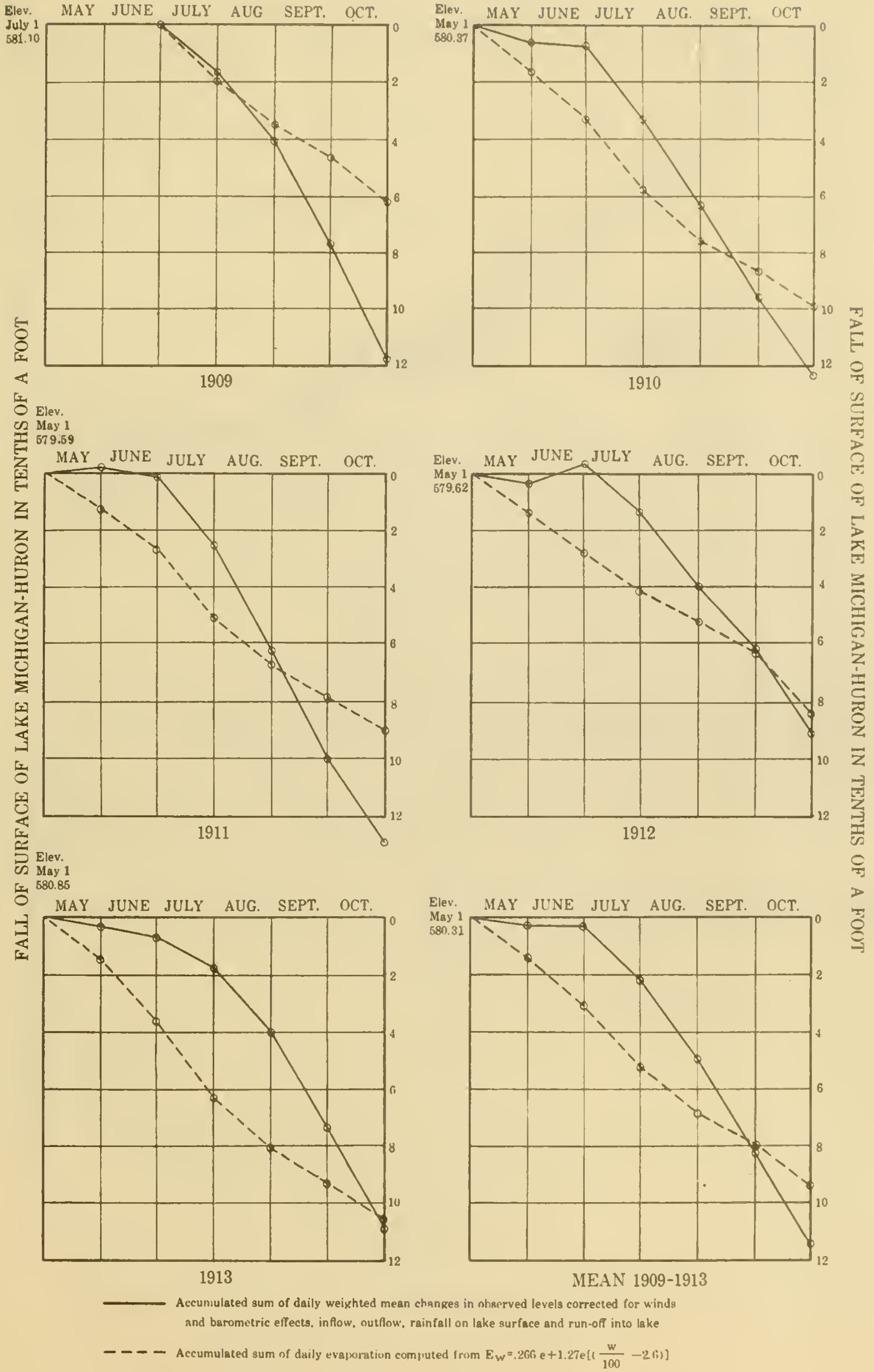

Mass Curves of Computed Evaporation, and of Observed Fall of Surface of Lake Michigan-Huron.

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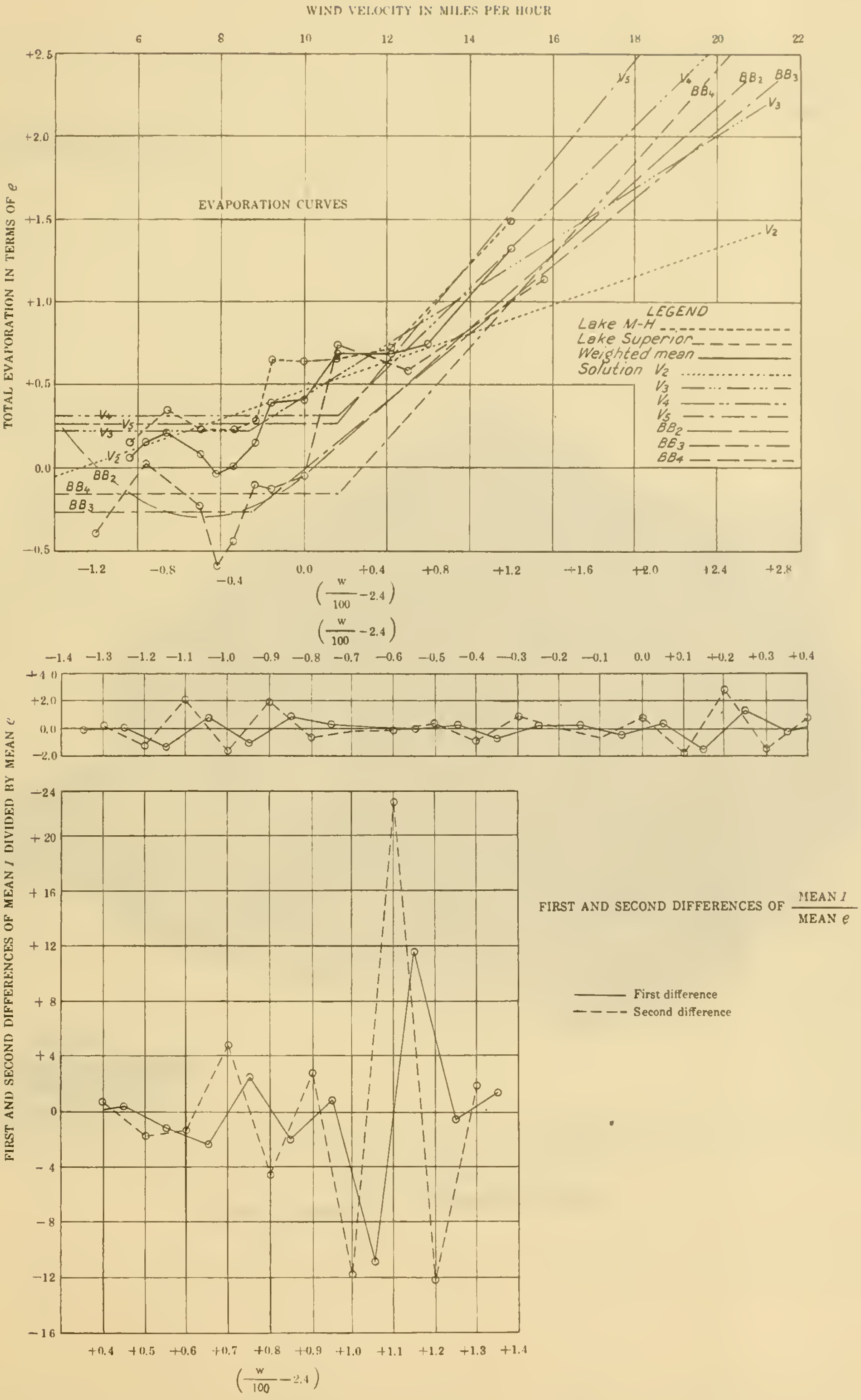

Evaporation Curves. First and Second Differences of $\frac{\text{Mean } I}{\text{Mean e}}$.

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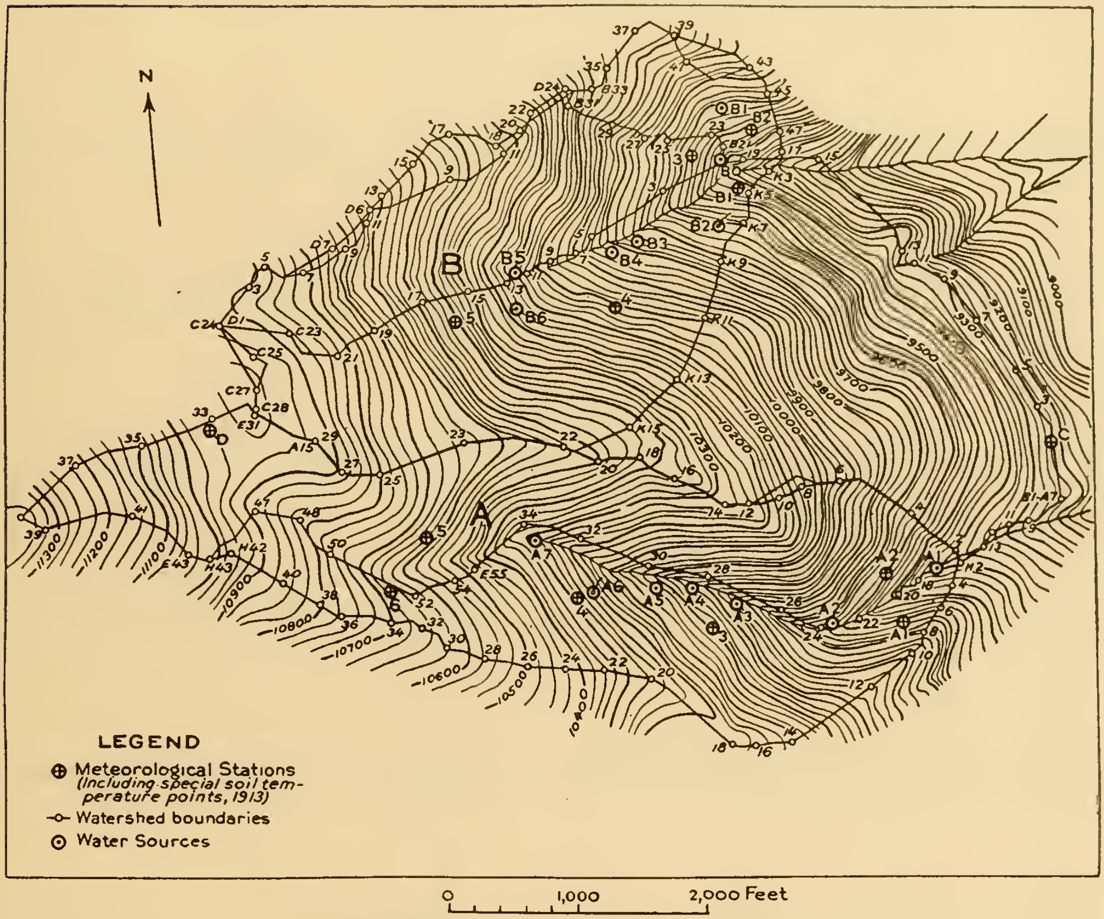
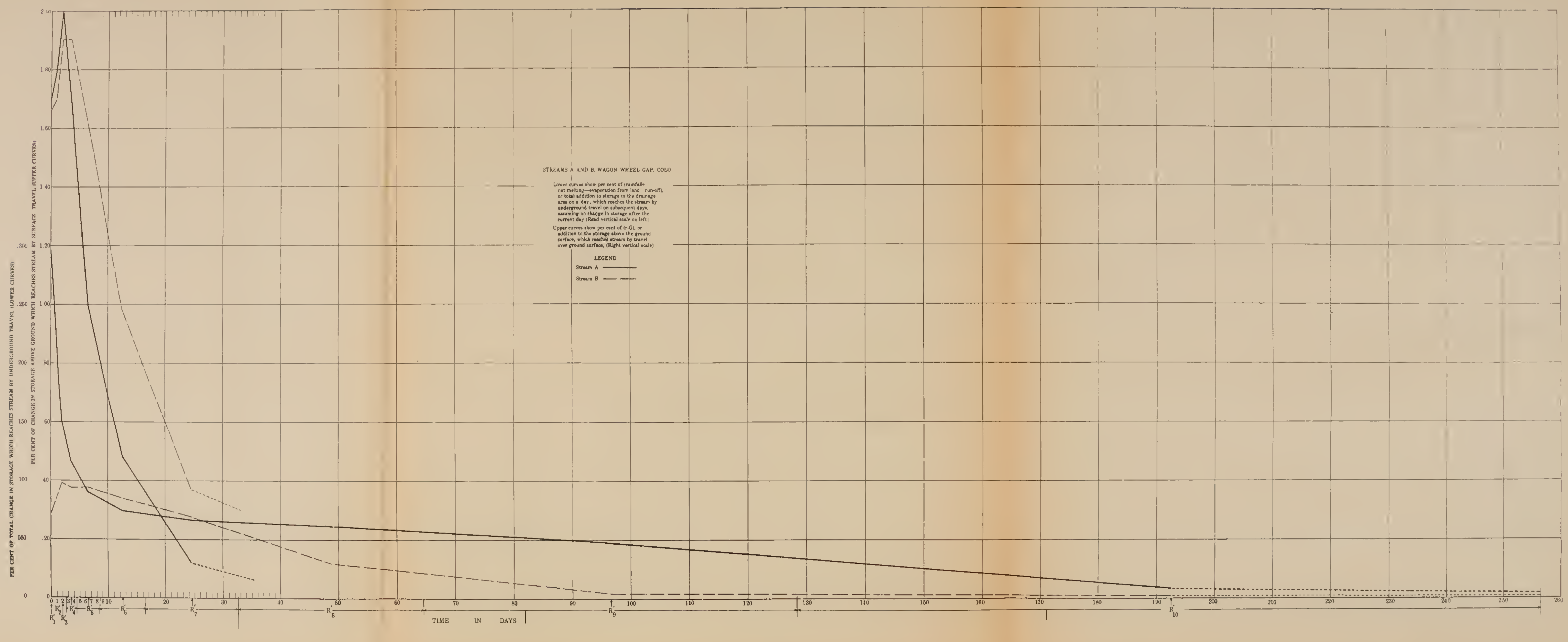


Area, Topography, etc., of Watersheds A and B.



STREAMS A AND B, WAGON WHEEL GAP, COLO

Lower curves show per cent of (rainfall+ net melting—evaporation from land run-off), or total addition to storage in the drainage area on a day, which reaches the stream by underground travel on subsequent days, assuming no change in storage after the current day (Read vertical scale on left)

Upper curves show per cent of (r-G), or addition to the storage above the ground surface, which reaches stream by travel over ground surface, (Right vertical scale)

LEGEND

Stream A ———

Stream B — —

PER CENT OF TOTAL CHANGE IN STORAGE WHICH REACHES STREAM BY UNDERGROUND TRAVEL (LOWER CURVES)
PER CENT OF CHANGE IN STORAGE ABOVE GROUND WHICH REACHES STREAM BY SURFACE TRAVEL (UPPER CURVES)
.300
.250
200
150
100
050
0
2.00
1.80
1.60
1.40
1.20
1.00
.80
.60
.40
.20
0
0
1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260
R'1
R'2
R'3
R'4
R'5
R'6
R'7
R'8
R'9
R'10
TIME IN DAYS

Run-off curves of Streams A and B.

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Hydrographs, etc., of Stream A for 1912.

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Hydrographs, etc., of Stream A for 1913.

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CURVES
PERTAINING TO
STREAM A, WAGON WHEEL GAP, COLO.
SHOWING
Observed flow
Computed normal flow
Computed (normal + flood) flow
Change in storage, r_1
Constant part of flow, S_c
Duration curve of observed flow
Duration curve of computed flow
1914
STREAMFLOW IN 0.001 C.F.S.
CHANGE IN STORAGE IN 0.01 inch
500
450
400
350
300
250
200
150
100
50
0
+60
+50
+40
+30
+20
+10
0
−10
−20
−30
−40
−50
−60
Duration curve of observed flow
Duration curve of computed flow
Computed (normal + flood) flow
Change in storage, r_1
Constant part of flow, S_c
Computed normal flow
Observed flow
0.001 C.F.S.
700
600
JANUARY 1914
FEBRUARY
MARCH
APRIL
MAY
JUNE
JULY
AUGUST
SEPTEMBER
OCTOBER
NOVEMBER
DECEMBER 1914

Hydrographs, etc., of Stream A for 1914.

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Hydrographs, etc., of Stream A for 1915.

THE COLUMBIA PLANOGRAPH CO. WASHINGTON, D. C.

Hydrographs, etc., of Stream B for 1911.

THE COLUMBIA PLANOGRAPH CO., WASHINGTON, D. C.

Hydrographs, etc., of Stream B for 1912.

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CURVES
PERTAINING TO
STREAM B, WAGON WHEEL GAP, COLO.
SHOWING
Observed flow
Computed normal flow
Computed (normal + flood) flow
Change in storage, r₁
Constant part of flow, S꜀
Duration curve of observed flow
Duration curve of computed flow
1913
STREAMFLOW IN CUBIC FEET PER SECOND
CHANGE IN STORAGE IN 0.01 inch
500
450
400
350
300
250
200
150
100
50
Duration curve of observed flow
Duration curve of computed flow
Computed (normal + flood) flow
Observed flow
Change in storage, r₁
Constant part of flow, S꜀
Computed normal flow
JANUARY 1913
FEBRUARY
MARCH
APRIL
MAY
JUNE
JULY
AUGUST
SEPTEMBER
OCTOBER
NOVEMBER
DECEMBER 1913

Hydrographs, etc., of Stream B for 1913.

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CURVES
PERTAINING TO
STREAM B, WAGON WHEEL GAP, COLO.
SHOWING
Observed flow
Computed normal flow
Computed (normal + flood) flow
Change in storage, r_t
Constant part of flow, S_c
Duration curve of observed flow
Duration curve of computed flow
1914
Duration curve of observed flow
Duration curve of computed flow
Computed (normal + flood) flow
Change in storage, r_t
Constant part of flow, S_c
Computed normal flow
Observed flow
STREAMFLOW IN .001 CUBIC FOOT PER SECOND
CHANGE IN STORAGE IN .01 inch
500
450
400
350
300
250
200
150
100
50
0
0.001 C.F.S.
JANUARY 1914
FEBRUARY
MARCH
APRIL
MAY
JUNE
JULY
AUGUST
SEPTEMBER
OCTOBER
NOVEMBER
DECEMBER 1914

Hydrographs, etc., of Stream B for 1914.

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CURVES
PERTAINING TO
STREAM -B- WAGON WHEEL GAP, COLO.
SHOWING
Observed flow
Computed normal flow
Computed (normal + flood) flow
Change in storage, r_1
Constant part of flow, S_c
Duration curve of observed flow
Duration curve of computed flow
1915
STREAMFLOW IN 0.001 CUBIC FOOT PER SECOND
CHANGE IN STORAGE IN 0.01 inch
500
450
400
350
300
250
200
150
100
50
0
+90
+80
+70
+60
+50
+40
+30
+20
+10
0
−10
−20
−30
Duration curve of computed flow
Duration curve of observed flow
Computed (normal + flood) flow
Change in storage, r_1
Constant part of flow, S_c
Computed normal flow
Observed flow
0.001 C. F. S.
800
700
600
500
JANUARY 1915
FEBRUARY
MARCH
APRIL
MAY
JUNE
JULY
AUGUST
SEPTEMBER
OCTOBER
NOVEMBER
DECEMBER 1915

Hydrographs, etc., of Stream B for 1915.

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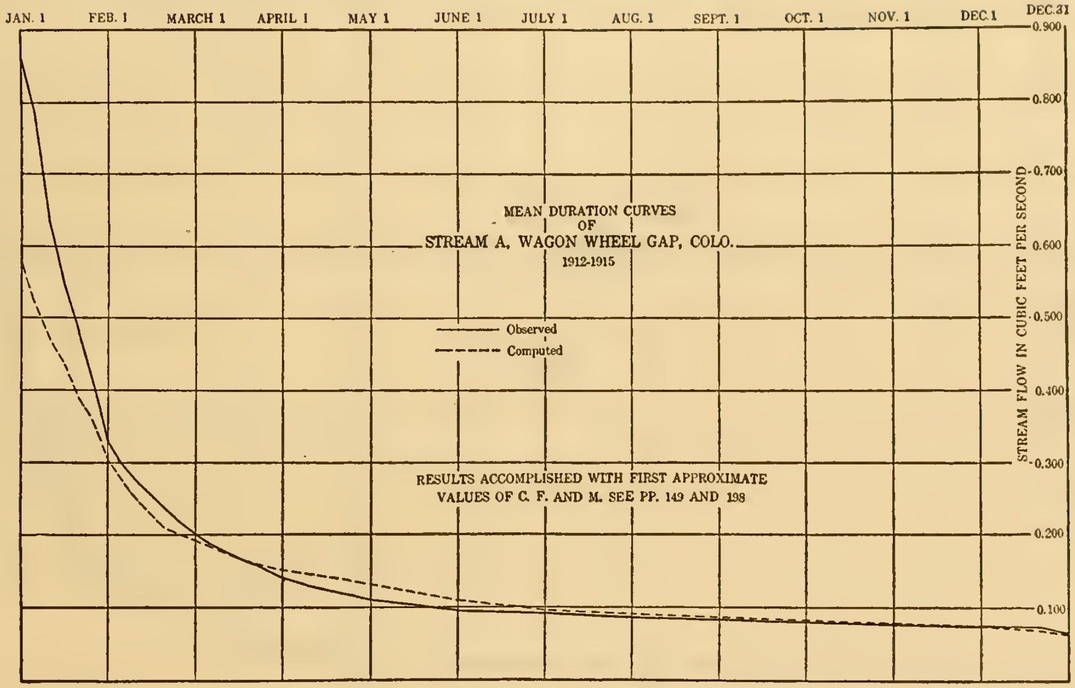
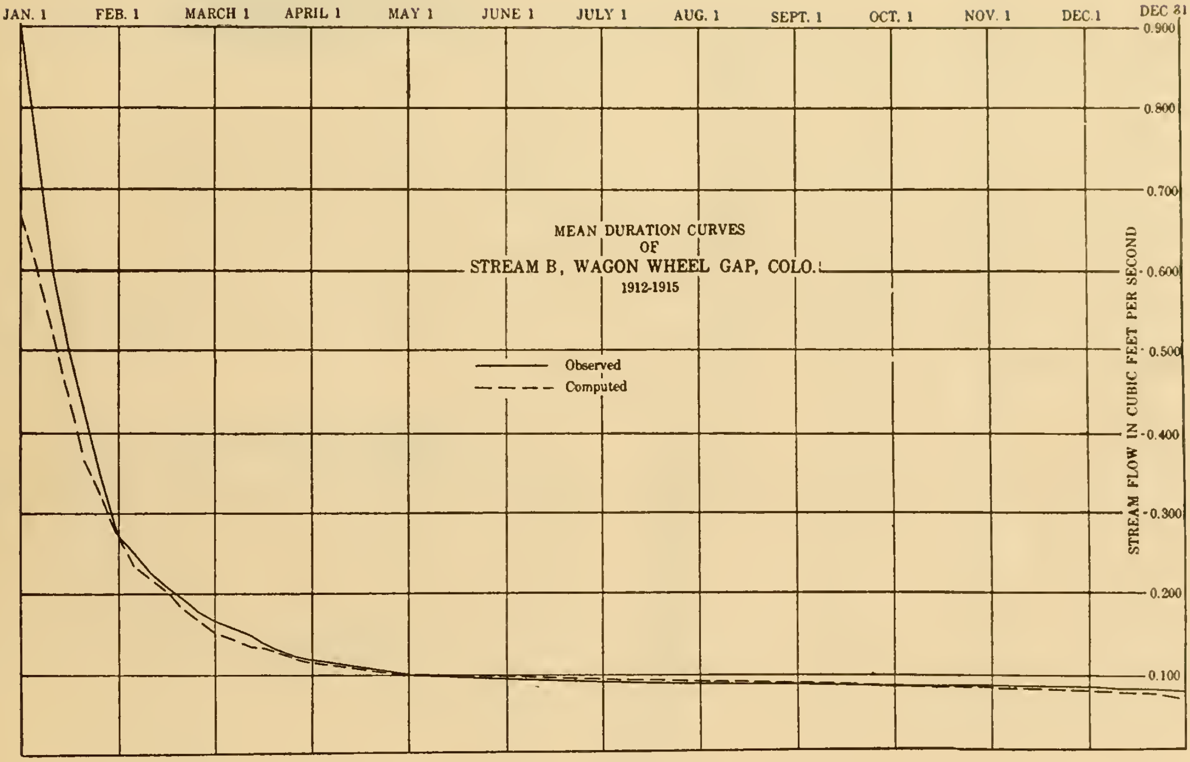
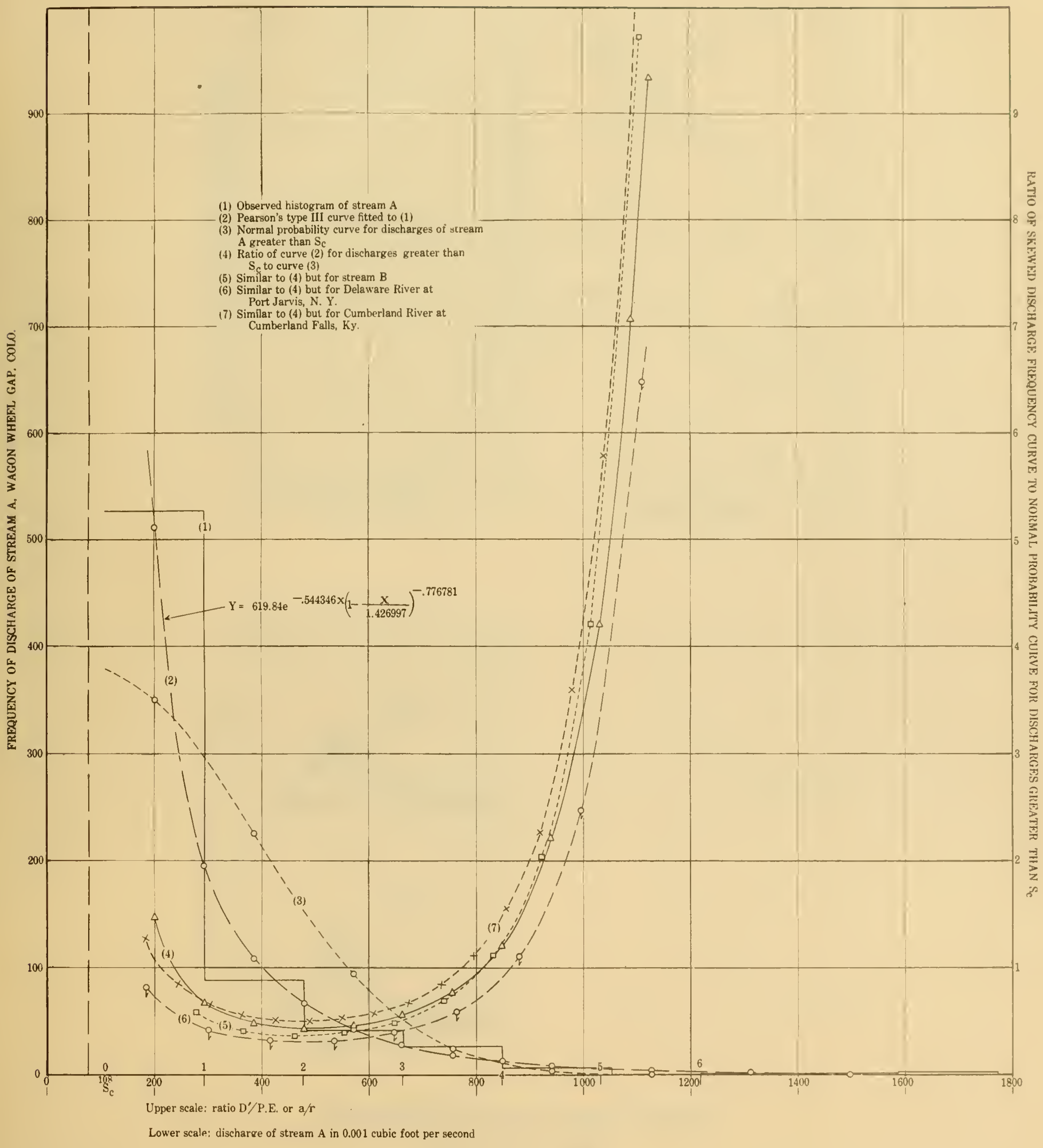


Mean Duration Curves of Stream *A*, 1912–1915.



Mean Duration Curves of Stream *B*, 1912–1915.



Frequency Ratio Curves.

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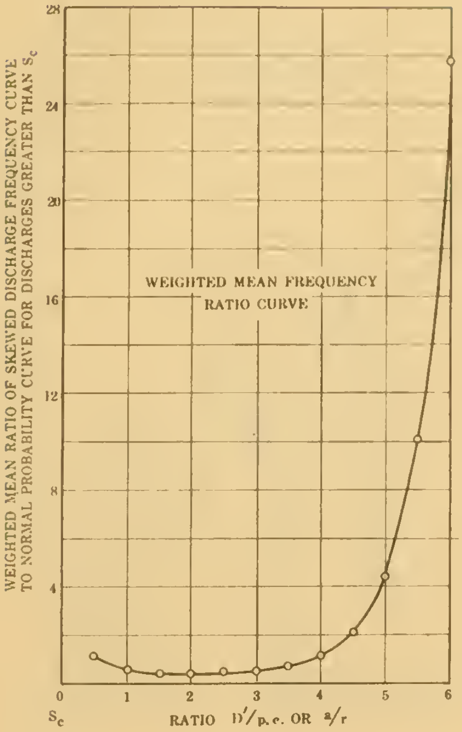
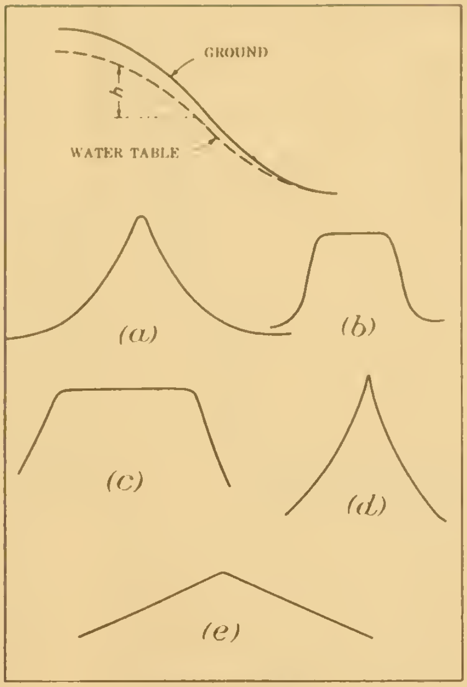
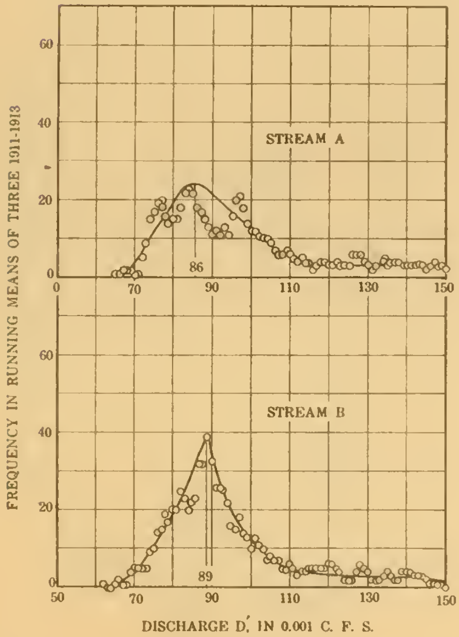




PROFILES OF DIVIDES



Mean Frequency Ratio Curve. Cross-Sections of Divides.

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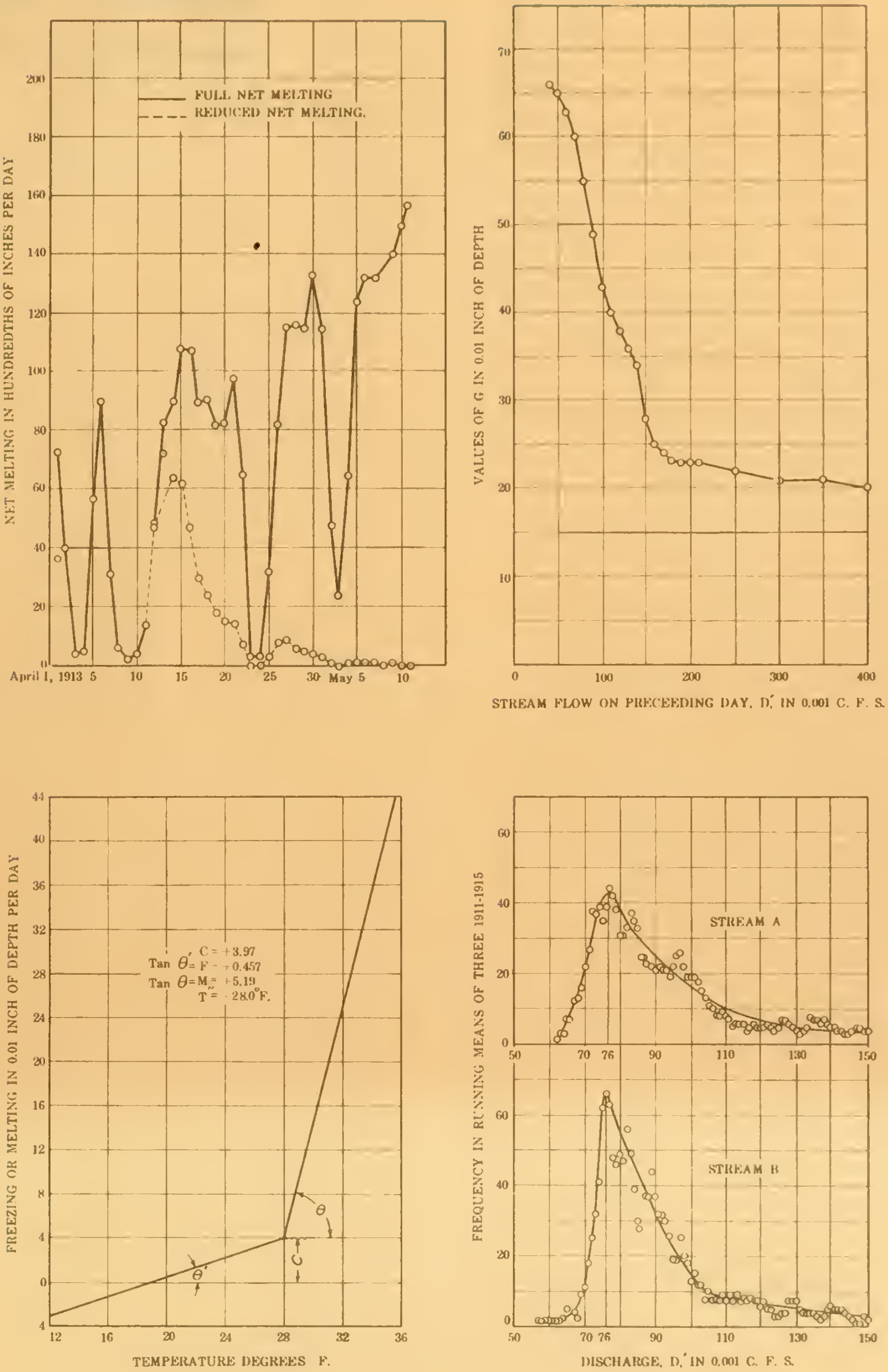

Net Melting, G — D′, Freezing-Melting, and Frequency Curves.

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